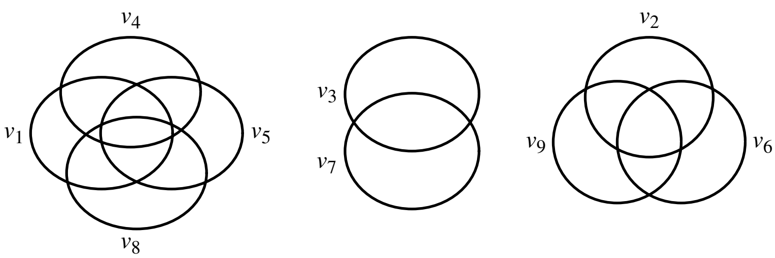
# F

**factor–** In an **analysis of variance** or **regression analysis,** a factor is an **independent variable** that is presumed to influence the **response variable.** In an **experimental design,** a factor is a **variable** that represents a possible source of **variation** of a quantity under investigation and must be controlled. In **factor analysis,** a factor is a **linear combination** of related variables that are expected to have some special affinity among them.

**factor analysis–** An advanced **multivariate technique** for analyzing the relationships among a large set of items or indicators to delineate the **factors** or dimensions that underlie the **data.** Factor analysis is performed by expressing **observed variables** as a **linear combination** of a smaller number of **variables,** known as factors or **latent variables,** which are of special relevance in the context of the investigation. In its initial stages, the




| Factor 1 | Factor 2 | Factor 3 |
|---|---|---|
| $v_1$ | $v_3$ | $v_2$ |
| $v_4$ | $v_7$ | $v_6$ |
| $v_5$ | | $v_9$ |
| $v_8$ | | |

Schematic illustration of factor analysis where nine variables are reduced to three factors analysis is known as exploratory factor analysis, in contrast to the confirmatory factor analysis that is performed to test a set of common factors for consistency with the **correlations** of the observed variables. Thus, an exploratory factor analysis assesses adequacy of the number of factors postulated in the **model** in order to provide an explanation of the observed correlations between the items while a confirmatory factor analysis assesses whether the correlations between the items can be adequately explained by a given factor model. It is frequently used in the analysis of rating scales and questionnaires.

**factorial–** A mathematical operation in which an integer is multiplied by all the integers equal and smaller than it up to the integer 1. It is symbolized by an exclamation point (!). For example, $3! = 3 \times 2 \times 1 = 6$, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. Also, by convention, $0! = 1$.

$$
\begin{aligned}
1! &= 1 \\
2! &= 1 \cdot 2 = 2 \\
3! &= 1 \cdot 2 \cdot 3 = 2! \cdot 3 = 6 \\
4! &= 1 \cdot 2 \cdot 3 \cdot 4 = 3! \cdot 4 = 24 \\
5! &= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 4! \cdot 5 = 120 \\
6! &= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 5! \cdot 6 = 720 \\
7! &= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 6! \cdot 7 = 5{,}040 \\
8! &= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 = 7! \cdot 8 = 40{,}320 \\
9! &= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 = 8! \cdot 9 = 362{,}880 \\
10! &= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 = 9! \cdot 10 = 3{,}628{,}800
\end{aligned}
$$

The first 10 factorials

**factorial analysis of variance–** The **analysis of variance** from an **experiment** involving two or more **independent variables** that have been cross-classified.

**factorial design–** A design involving two or more **factors,** each being investigated at two or more **levels.** In a factorial design, all levels of a factor occur with all levels of the others. The simplest factorial design involves the use of two factors, each at two levels, resulting in four treatment combinations. The main goal in a factorial design is to determine whether the factors do or do not interact with each other so that any possible **interactions** can be evaluated. The factors being included may be either qualitative or quantitative, and either independent or dependent. Compare *nested design*.

**factorial experiment–** Same as *factorial design.*

**factorial moment–** The $r$th factorial moment about an arbitrary origin $a$ of a **discrete random variable** $X$ with **probability function** $p(x)$ is defined by

$$\mu'_{[r]} = E(X - a)^{[r]} = \sum_{x=-\infty}^{\infty} (x - a)^{[r]} p(x)$$

where $x^{[r]} = x(x-1)(x-2)\cdots(x - \overline{r-1})$. Factorial moments are used almost entirely for **discrete distributions,** or **continuous distributions** grouped in intervals of a finite length. In statistical theory they are not very useful, but they provide very concise formulas for moments of certain discrete distributions, such as the binomial, which have **probability mass** distributed at equally spaced values.

**factorial moment generating function–** A function of a **variable** $t$ associated with the **probability distribution** of a **discrete random variable** $X$ distributed at equally spaced values, taken to be 0, 1, 2, . . . , and defined by

$$\eta_x(t) = E(t^X) = \sum_{x=0}^{\infty} t^x p(x)$$

Although not of much theoretical interest, it is useful in the calculation of **factorial moments.**

**factorial product–** See *factorial.*

**factorization theorem–** A theorem in mathematical statistics that is based on the concept of the **likelihood function** and **sufficient statistic.** It provides a necessary and sufficient condition that a **statistic** be sufficient for a **parameter** of interest.

**factor level–** In **experimental design,** a term used to denote the **level** of a **factor** being studied.

**factor loading–** In a **factor analysis** the term is used to refer to the **coefficients** of the **observed variables** on the common **factors.** They are analogous to **regression coefficients** in **multiple regression analysis** and can be interpreted as **correlations** between each variable and each factor.

**factor rotation–** In a **factor analysis** the term is used to describe the process of transforming the **factors** initially extracted in order to make the common factors more clearly defined and simplify their interpretation. The procedure consists of turning axes about the origin until an alternate position is reached. The factors being rotated can be either orthogonal and oblique while taking into account the nature of the resulting solution and case of their interpretation. In orthogonal rotation, loadings are uncorrelated while oblique rotation involves correlated loadings.

**failure time–** Same as *survival time.*

**fair gamble–** In **theory of games,** a game of chance in which the **expected monetary value** of what is being lost is exactly equal to the expected monetary value of what is being received. In a large sequence of such games, the player with larger capital has greater **probability** of winning over his opponent. Compare *unfair gamble.*

**fair game–** Same as *fair gamble.*

**false acceptance error–** Same as *type II error.*

**false negative–** An **error** in a **diagnostic test** that gives a disease-free indication to a person who really has the disease. Compare *false positive.* See also *screening, sensitivity, specificity.*

**false-negative rate–** In a **screening** or **diagnostic test,** the **probability** that the test will yield a negative result when administered to a person who has the disease or condition in question. Compare *false-positive rate.* See also *sensitivity, specificity.*

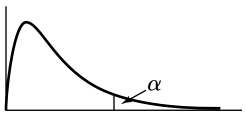
**false positive–** An error in a **diagnostic test** that gives a disease indication to a person who does not have the disease. Compare *false negative.* See also *screening, sensitivity, specificity.*

**false-positive rate–** In a **screening** or **diagnostic test,** the **probability** that the test will yield positive result when administered to a person who does not have the disease or condition in question. Compare *false-negative rate.* See also *sensitivity, specificity.*

**false rejection error–** Same as *type I error.*

***F* distribution–** A theoretical **distribution** which can be described as the distribution of the **statistic** $F = S_1^2 / S_2^2$, where $S_1^2$ is the **variance** of a **sample** of size *m* from a **normal population** with variance $\sigma_1^2$ and $S_2^2$ is the variance of an independent sample of size *n* from a normal population with variance $\sigma_2^2$. The statistic *F* is said to have an *F* distribution with $m - 1$ **degrees of freedom** in the numerator and $n - 1$ degrees of freedom in the denominator. In general, an ***F* statistic** is obtained as the **ratio** of two independent **random variables** each having a **chi-square distribution,** divided by their respective degrees of freedom. The distribution of *F* involves a family of curves each adjusted for the degrees of freedom associated with the two variances being compared. The *F* distribution is also known as the variance ratio distribution. It was first studied by R. A. Fisher, and the ratio *F* was denominated by G. W. Snedecor after the first letter of the originator's name. The distribution is related to the **beta distribution** with $\alpha = \nu_1/2$ and $\beta = \nu_2/2$ where $\nu_1 = m - 1$ and $\nu_2 = n - 1$. The distribution is of fundamental importance in **analysis of variance.** The accompanying tables give **critical values** of the distribution, which denote the values for which the area to its right under the *F* distribution with $\nu_1$ and $\nu_2$ degrees of freedom is equal to $\alpha$.

### *F* distribution table




$F\nu_1, \nu_2, \alpha$

**The entries in this table are values of $F\nu_1, \nu_{2,\,0.01}$ for which the area to their right under the *F* distribution with $\nu_1$, $\nu_2$ degrees of freedom is equal to 0.01**

| $\rightarrow\nu_1$ $\downarrow\nu_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4052 | 4999.5 | 5403 | 5625 | 5764 | 5859 | 5928 | 5982 | 6022 | 6056 | 6106 | 6157 | 6209 | 6235 | 6261 | 6287 | 6313 | 6339 | 6366 |
| 2 | 98.5 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.87 | 26.69 | 26.60 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.20 | 9.11 | 9.02 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.14 | 7.06 | 6.97 | 6.88 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.82 | 5.74 | 5.65 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.03 | 4.95 | 4.86 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.48 | 4.40 | 4.31 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.17 | 4.08 | 4.00 | 3.91 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.25 | 4.10 | 4.02 | 3.94 | 3.86 | 3.78 | 3.69 | 3.60 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.62 | 3.54 | 3.45 | 3.36 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.82 | 3.66 | 3.59 | 3.51 | 3.43 | 3.34 | 3.25 | 3.17 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.66 | 3.51 | 3.43 | 3.35 | 3.27 | 3.18 | 3.09 | 3.00 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.13 | 3.05 | 2.96 | 2.87 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.41 | 3.26 | 3.18 | 3.10 | 3.02 | 2.93 | 2.84 | 2.75 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.31 | 3.16 | 3.08 | 3.00 | 2.92 | 2.83 | 2.75 | 2.65 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.23 | 3.08 | 3.00 | 2.92 | 2.84 | 2.75 | 2.66 | 2.57 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.15 | 3.00 | 2.92 | 2.84 | 2.76 | 2.67 | 2.58 | 2.49 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.69 | 2.61 | 2.52 | 2.42 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.17 | 3.03 | 2.88 | 2.80 | 2.72 | 2.64 | 2.55 | 2.46 | 2.36 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.12 | 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.50 | 2.40 | 2.31 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.07 | 2.93 | 2.78 | 2.70 | 2.62 | 2.54 | 2.45 | 2.35 | 2.26 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.40 | 2.31 | 2.21 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.99 | 2.85 | 2.70 | 2.62 | 2.54 | 2.45 | 2.36 | 2.27 | 2.17 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.96 | 2.81 | 2.66 | 2.58 | 2.50 | 2.42 | 2.33 | 2.23 | 2.13 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.93 | 2.78 | 2.63 | 2.55 | 2.47 | 2.38 | 2.29 | 2.20 | 2.10 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.90 | 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.26 | 2.17 | 2.06 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.87 | 2.73 | 2.57 | 2.49 | 2.41 | 2.33 | 2.23 | 2.14 | 2.03 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.21 | 2.11 | 2.01 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.66 | 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.02 | 1.92 | 1.80 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.84 | 1.73 | 1.60 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.66 | 1.53 | 1.38 |
| $\infty$ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.47 | 1.32 | 1.00 |

(*Continued*)

(*Continued*)

**The entries in this table are values of $F\nu_1, \nu_{2, 0.05}$ for which the area to their right under the $F$ distribution with $\nu_1, \nu_2$ degrees of freedom is equal to 0.05**

| $\rightarrow\nu_1$ / $\downarrow\nu_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.36 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.38 | 2.34 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.11 | 2.06 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.03 | 1.94 | 1.90 | 1.85 | 1.81 | 1.75 | 1.70 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.74 | 1.68 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.64 | 1.58 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.53 | 1.47 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.43 | 1.35 | 1.25 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.32 | 1.22 | 1.00 |

*Source:* Computed by software.

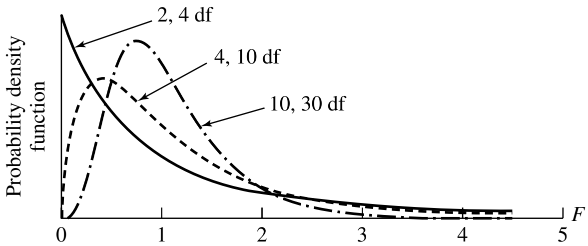
**feasibility study–** Same as *pilot study.*

**fertility–** In **demography,** fertility is used in the sense of actual production or bearing of offspring.

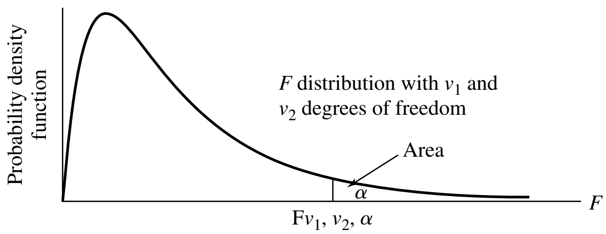
**fertility rate–** Number of live births occurring in a specified period per 1000 women of child-bearing age, i.e., 15 to 49 years. In some countries the child-bearing age is taken as 15 to 44 years. It is calculated as the number of live births actually observed by the female population of child-bearing age (expressed per 1000). It is more refined than the **crude birth rate,** which takes into account the whole population. Further refinements can be made by reporting **ratio** of births for various age groups within 15 to 49 years. See also *age-specific fertility rate.*

**fiducial distribution–** Same as *fiducial probability distribution.*

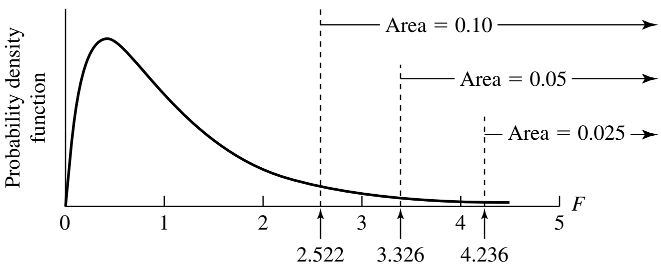
**fiducial inference–** A term first used by R. A. Fisher in 1930 to describe a type of **statistical inference** based on **fiducial probability distribution.** Its objective is to make statistical inference about an unknown **parameter** by deriving its **probability distribution** from the **distribution** of its **estimator** without having first assigned the parameter any **prior distribution.** Many **statisticians** find it a problematic form of **inference** and have commented adversely about it. See also *fiducial interval.*




Probability density curves for *F* distributions with (2, 4), (4, 10) and (10, 30) degrees of freedom




An *F* value having an area equal to $\alpha$ in the right tail




Area to the right of 2.522 is 0.10, etc.

Probability density curve for *F* distribution with (5, 10) degrees of freedom

**fiducial interval–** In the theory of **estimation,** an interval similar to **confidence interval** that can be expected, with a specified **probability,** to contain the value of some unknown **parameter.** The term is also used as a synonym of confidence interval but its conceptual origin is different. Whereas in a confidence interval, the limits of the interval are the **random variables,** in a fiducial interval the parameter is assumed to have a (fiducial) distribution.

**fiducial limits–** The limits of a **fiducial interval** that define the interval.

**fiducial probability–** See *fiducial probability distribution.*

**fiducial probability distribution–** A term used to describe the **probability distribution** of a **parameter** being used in a **fiducial inference.** It is not a probability distribution in the usual sense of the term, but is constructed from the **distribution** of **estimators** and contains all the relevant information in the **sample.**

**field plot–** Same as *plot.*

**finite population–** A **population** of items or individuals that are finite in number.

**finite population correction–** If a **sample** of size $n$ is drawn without replacement from a **finite population** of size $N$, the **standard error** of the **sample mean** $\overline{X}$ can be written as

$$\sigma_{\overline{X}} = \sqrt{\left(\frac{N-n}{N-1}\right)}\frac{\sigma}{\sqrt{n}}$$

where $\sigma$ is the **population standard deviation.** The multiplier term $(N-n)/(N-1)$ in the above formula is sometimes called the finite population correction; whenever $n/N \leq 0.05$, the finite population factor is close to 1 and hence $\sigma_{\overline{X}} = \sigma/\sqrt{n}$.

**finite population factor–** See *finite population correction.*

**first quartile–** The 0.25 **fractile** or 25th **percentile point** in a **data set** below which a quarter of all observations lie. See also *median, quartiles, second quartile, third quartile.*

**Fisher information matrix–** It is the **matrix** obtained as the inverse of the **variance–covariance matrix** of a set of **estimators.**

**Fisher's discriminant function–** See *discriminant analysis.*

**Fisher's exact test–** An "exact" conditional test for analyzing data in a **2 × 2 contingency table.** It is used when the **sample size** is too small (<30) to use the **chi-square test.** It is based on the exact **hypergeometric distribution** of the observed **cell frequencies** within the table. The procedure consists of evaluating the sum of exact hypergeometric probabilities associated with observed cell frequencies and of those deviating more than the observed frequencies under the **hypothesis** of **independence.** The procedure leads to a **conservative test** and has been the subject of controversy among **statisticians.** See also *Yates' correction for continuity.*

**Fisher's ideal index number–** A **consumer price index** obtained as the **geometric mean** of **Laspeyres' index number** and **Paasche's index number.** Laspeyres and Paasche index numbers are biased, if at all, in opposite directions. For example, if the index is one of prices, the former is usually biased upward and the latter downward. Taking the geometric mean provides an **index number** free from the **bias** inherent in them. It is named in honor of the American economist Irwing Fisher (1867–1947). It is calculated by the formula

$$\sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}}$$

where $p_0^i$ = price at base period, $q_0^i$ = quantity at base period, $p_1^i$ = price at first time period, and $q_1^i$ = quantity at first time period.

**Fisher's LSD test–** Same as *least significant difference test.*

**Fisher's scoring method–** Same as *scoring method.*

**Fisher's transformation of the correlation coefficient–** Same as *Fisher's z transformation.*

**Fisher's $z$ transformation–** A **transformation** applied to the **correlation coefficient** $r$ so that it is normally distributed with **mean** zero and **standard deviation** of one. It is given by the formula

$$z = \frac{1}{2}\log_e \frac{1+r}{1-r}$$

The **statistic** $z$ has mean

$$\frac{1}{2}\log_e \frac{1+\rho}{1-\rho}$$

and **variance**

$$\frac{1}{n-3}$$

where $\rho$ is **the population correlation** and $n$ is the **sample size.** The transformation may be used to test a **hypothesis** or construct a **confidence interval** for $\rho$.

**five-number summary–** An **exploratory data analysis** technique that uses the following five numbers to summarize the **data set:** minimum value, **first quartile** (lower hinge), **median, third quartile** (upper hinge), and maximum value. The five-number summary forms the basis for constructing a **box-and-whisker plot.** See also *stem-and-leaf plot.*

**fixed-base index number–** An **index number** with a common base. The base is usually taken as one of the periods, times, or places within the series, not necessarily the first one. It provides a mechanism for a common standard of comparison.

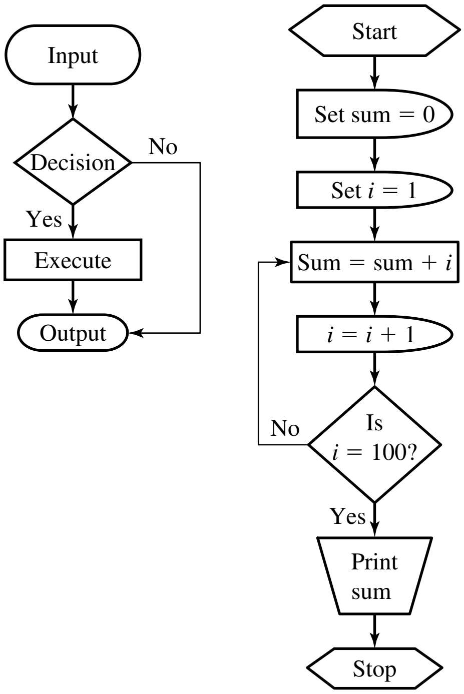
**fixed effects–** A term used to denote **effects** attributable to the collection of **levels** of a **factor** or **treatment** where all the levels of interest are included in a given **experiment** or study. Compare *random effects.*

**fixed-effects analysis of variance–** See *fixed-effects model.*

**fixed-effects model–** An **analysis of variance** or **regression model** in which the **treatment levels** associated with a **factor** are considered to have fixed or **constant** effects. This model is also referred to as **Model I.** In a fixed-effects model all the **treatments** of interest to the researcher are included in the **experiment** or study under consideration. In the context of **meta-analysis,** the term is used to describe a model that assumes that the number of studies being summarized are the only ones of interest to the investigator. In a meta-analysis with fixed-effects model, the results of the combined estimate can be applied to any subject from the **target population** represented by the individual studies. See also *mixed-effects model, random-effects model.*

**fixed factors– Factors** in an **analysis of variance** or **regression model** thought to have a **fixed effect.** Some examples of factors that are usually considered fixed are: type of disease, treatment therapy, gender, and marital and economic status. Compare *random factors.* See also *random effects.*

**flowchart–** A pictorial representation of a system or process that uses certain symbols and conventions to outline all the steps in the process, interrelationships between different steps, and the order in which they are to be executed.




Figures illustrating a flowchart

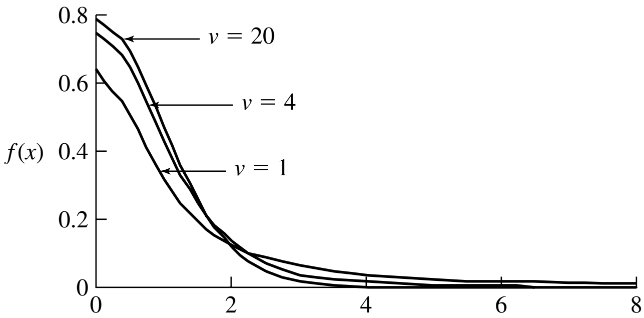
**folded normal distribution–** Same as *half-normal distribution.*

**folded standard normal distribution–** The **probability distribution** of a **random variable** $Z = |X|$, where $X$ has a **standard normal distribution.** Its **probability density function** is given by

$$f(z) = \sqrt{2/\pi}\, e^{-z^{2}2}$$

**folded *t* distribution–** The **probability distribution** of a **random variable** $t' = |X|$ where $X$ has a ***t* distribution** with $\nu$ **degrees of freedom.** It can be shown that the **folded standard normal distribution** is a limiting form of the folded $t$ distribution as $\nu \to \infty$. The folded $t$ distribution is also related to the **chi distribution** by the relation $t' = x/(\sqrt{\nu}\, y)$ where $X$ and $Y$ are independent **chi variables** with 1 and $\nu$ degrees of freedom respectively.

**follow-up–** The process of locating individuals participating in a **longitudinal study** in order to determine outcome measures and other pertinent characteristics at regular intervals of time in the future. In a field **experiment** or **sample survey,** the term is used to describe a further attempt to obtain information on individuals who could not be located in the initial attempt.




Probability density curves for the folded *t* distribution

**follow-up period–** The length of time individuals participating in a **longitudinal study** are kept under observation in order to record outcome measures and other pertinent characteristics.

**follow-up study–** Same as *prospective study.*

**forecast–** See *forecasting.*

**forecasting–** Making statements or **predictions** about an unknown, uncertain, and, generally, future **outcome** or quantity, such as the inflation or interest rate. Forecasting is generally based on past values and employs statistical methods based on **regression model** or **time-series analysis.** The specific value most likely to provide an accurate prediction of a future value is known as the forecast.

**FORTRAN–** An acronym for Formula Translation. A mathematically oriented programming language used for writing **computer programs.**

**forward-looking study–** Same as *prospective study.*

**forward selection procedure–** In **multiple regression analysis,** a method for selecting the best possible set of **predictors** of the **criterion variable.** The method proceeds by introducing the **variables** one at a time according to a prechosen criterion of **statistical significance.** The variable that has the highest **sample correlation** with the criterion variable is selected first and is included in the **model equation** if it meets the criterion. Next, the variable with the highest correlation with the criterion variable, after adjusting for the effect of the first variable included in the model (i.e., the variable with the highest sample correlation coefficient with the **residuals** from step 1) is examined and is included if it meets the criterion. The selection of third, fourth, etc. variables to be included in the **model** proceeds in the same way. The process is continued till the last variable entering the equation does not meet the criterion, or all the variables are included in the model. Compare *backward elemination procedure, stepwise regression.*

**forward solution–** Same as *forward selection procedure.*

**fourfold table–** Same as *2 × 2 contingency table.*

**fractile–** A value in a **data set** below which a certain specified **proportion** of all values lies. Fractiles divide a data set into groups with known proportions of **observations** in each group. It is also called **quantile.** See also *deciles, percentiles, quartiles, quintiles.*

**fractional factorial design–** In a **factorial design** if there are large number of **treatment factors** and the available resources are limited, it may be necessary to use a **replication** of only a fraction of the total number of treatment combinations. In a design involving a fractional replication, some of the **effects** cannot be estimated since they are **confounded** with one or more other effects. Usually, the choice of a fractional replication is made such that the effects considered to be of importance are confounded only with the effects that can be assumed to be negligible. Thus, the design is likely to be useful only when certain high-order **interactions** can be regarded as negligible.

**frame–** A list, map, or other record of the **sampling units** that constitute the available information relating to the population designated for a particular **sampling design.**

**Freeman–Tukey test–** A **test procedure** for testing the **goodness of fit** of a specified **model** or a theoretical **distribution.** The procedure is usually applied on **count** or **frequency data** by comparing the **observed** and the **expected frequencies** under the assumed model. See also *chi-square statistic, goodness-of-fit statistic, goodness-of-fit test,* $G^2$*-statistic, likelihood ratio statistic.*

**Freeman–Tukey transformation–** A **transformation** of the form $\sqrt{x} + \sqrt{x+1}$ proposed by Freeman and Tukey, in order to stabilize its **variance.** It is normally used to a **random variable** having a **Poisson distribution.**

**frequency–** The number of times a given value of an **observation** or a particular type of **event** occurs, or the number of elements of a **population** that belong to a specified group or class. It is also called count. See also *relative frequency.*

**frequency count–** Same as *frequency.*

**frequency curve–** A **graphical representation** of a continuous **frequency distribution** by a smooth curve. The **variate** is marked as the **abscissa,** and **frequency** is shown as the **ordinate.** The frequency curve may be considered a limiting form of the **frequency polygon** as the number of **observations** tends to infinity and the **class width** tends to zero.

**frequency data–** Same as *count data.*

**frequency density–** In a **frequency distribution,** the **ratio** of a **class frequency** to the **class width.** See also *probability density.*

**frequency distribution–** The method of classifying and representing **statistical data** that involve two columns: one listing the categories, **score intervals,** or **events** into which the **data** are sorted and the other indicating the number of items or members in each category. It is customary to list **scores** in descending order, from the highest to the lowest. When values in a **data set** are arranged in ascending or descending order of magnitude, the frequency distribution shows the number of times (**frequency**) that each value occurs. See also *cumulative frequency distribution, cumulative relative frequency distribution.*

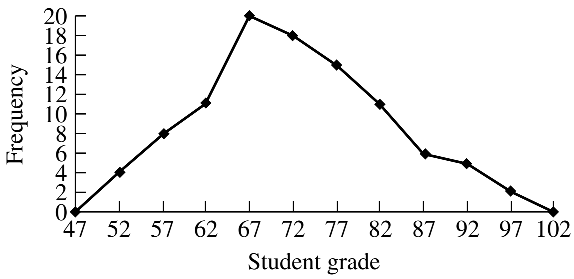
**frequency function–** A mathematical function that gives the **frequency** of a **variate** value $x$ as a function of $x$. For a continuous **random variable** $X$, it is the frequency in an elemental range $dx$. A frequency function is used to describe a **frequency curve.** See also *probability density function, probability function.*

**frequency histogram–** Same as *histogram.*

**Frequency distribution for student grades: hypothetical data**

| Class | Midpoint | Frequency |
|---|---|---|
| 50–54 | 52 | 4 |
| 55–59 | 57 | 8 |
| 60–64 | 62 | 11 |
| 65–69 | 67 | 20 |
| 70–74 | 72 | 18 |
| 75–79 | 77 | 15 |
| 80–84 | 82 | 11 |
| 85–89 | 87 | 6 |
| 90–94 | 92 | 5 |
| 95–99 | 97 | 2 |

**frequency polygon–** A **graphical representation** of a **frequency distribution** in which the **horizontal axis** represents score values or **midvalues** and the **vertical axis** represents **frequency** of occurrence. A dot is placed over each score value at the height representing its frequency of occurrence. These dots are then joined by straight lines to form a polygon. It is useful in comparing two or more frequency distributions.




A frequency polygon of the frequency distribution for student grades

**frequency table–** A **tabular representation** of a **frequency distribution.** See also *cumulative frequency distribution, cumulative relative frequency distribution.*

**frequency theory of probability–** Same as *empirical probability.*

**frequentist–** A believer in the **frequency theory of probability** and **classical statistical inference.**

**frequentist inference–** Same as *classical statistical inference.*

**Friedman's rank test–** A **nonparametric test** procedure used to compare three or more correlated or **matched samples** of **observations** that cannot be compared by means of an ***F* test** in a **randomized block design** either because the **scores** are ordinal in nature or because the **normality** or **homogeneity of variance** assumptions cannot be satisfied. The method consists of ranking observations separately within each **block,** and the **test statistic** is based on the sum of the **ranks** assigned to the individual **treatment groups.** See also *Kruskal–Wallis test.*

**Friedman's two-way analysis of variance–** Same as *Friedman's rank test.*

***F* statistic–** In general, any **statistic** that has an ***F* distribution.** In an **analysis of variance,** the **ratio** of two **mean squares** known as **mean square ratio** follows an *F* distribution. The *F* statistic is also used to compare **variances** from two **normal populations.**

***F* test–** A **statistical test** based on an ***F* statistic.** Two commonly used *F* tests are ***F* test for analysis of variance** and ***F* test for two population variances.**

***F* test for analysis of variance–** The **statistical test** for comparing the **means** of several **populations** used in the **analysis of variance.** Under the **null hypothesis** of no difference between the **population means,** the two **mean squares** (between and within) are approximately equivalent and their **ratio** (***F* statistic**) is nearly equal to 1. In comparison of the means of two **independent groups,** the ***F* test** is equivalent to the **two-sample *t* test.** In **regression analysis,** the *F* statistic is used to test the joint significance of all the variables in the **model.**

***F* test for two population variances–** A test devised by R. A. Fisher to compare the **variances** of two **populations.** It makes its comparisons directly in the form of a **ratio,** with the larger **sample variance** serving as the numerator and the smaller serving as the denominator. This is the simplest use of the ***F* statistic** for testing the difference between the variances of two independent **normal populations.** The *F* test for two **population variances** may be used to compare two **distributions** for **homogeneity of variances** before proceeding to perform ***t* test.** See also *Ansari–Bradely test, Barton–David test, Conover test, F distribution, Klotz test, Mood test, Rosenbaum test, Siegel–Tukey test.*