Appendix A

The greek alphabet

Greek name	Capital letter	Small letter	Greek name	Capital letter	Small letter
alpha	A	α	**nu**	N	ν
beta	B	β	**xi**	Ξ	ξ
gamma	Γ	γ	**omicron**	O	o
delta	Δ	δ	**pi**	Π	π
epsilon	E	ε	**rho**	P	ρ
zeta	Z	ζ	**sigma**	Σ	σ
eta	H	η	**tau**	T	τ
theta	Θ	θ	**upsilon**	Υ	υ
iota	I	ι	**phi**	Φ	ϕ
kappa	K	κ	**chi**	X	χ
lambda	Λ	λ	**psi**	Ψ	ψ
mu	M	μ	**omega**	Ω	ω

Appendix B

Metric measures and their conversion: metric to British and British to metric

Metric measures

Length		Weight	
1000 micrometers	= 1 millimeter (mm)	1000 micrograms	= 1 milligram (mg)
10 millimeters	= 1 centimeter (cm)	10 milligrams	= 1 centigram (cg)
10 centimeters	= 1 decimeter (dm)	10 centigrams	= 1 decigram (dg)
10 decimeters	= 1 meter (m)	10 decigrams	= 1 gram (g)
10 meters	= 1 decameter (dam)	10 grams	= 1 decagram (dag)
10 decameters	= 1 hectometer (hm)	10 decagrams	= 1 hectogram (hg)
10 hectometers	= 1 kilometer (km)	10 hectograms	= 1 kilogram (kg)
		1000 kilograms	= 1 tonne

Volume and capacity		Area	
10 millimeters (mL)	= 1 cintiliter (cL)	1 are	= 100 square meters (m^2)
10 centiliters	= 1 deciliter (dL)	1 decare	= 10 ares
10 deciliters	= 1 liter (L)	1 hectare	= 100 ares
10 liters	= 1 decaliter (daL)	1 deciare	= 1/10 of an are
10 decaliters	= 1 hectoliter (hL)	1 centiare	= 1/100 of an are
10 hectoliters	= 1 kiloliter (kL)		= 1 square meter (m^2)

Conversion (Metric to British and British to Metric)

Length		Weight		Volume and Capacity	
1 cm	= 0.39370 in	1 g	= 0.03527 oz	1 cm^3	= 0.06102 cubic in (in^3)
1 m	= 39.37011 in	1 kg	= 2.20462 lb	1 m^3	= 1.30795 cubic yd (yd^3)
	= 3.28084 ft	1 tonne	= 0.98421 ton	1 liter	= 1.75980 pints
	= 1.09361 yd				= 0.21998 gallon
1 km	= 0.62137 mi	1 oz	= 28.34953 g	1 cubic in	= 16.38702 cubic cm (cm^3)
1 in	= 2.54000 cm	1 lb	= 0.45359 kg	1 cubic yd	= 0.76455 cubic m (m^3)
1 yd	= 0.91440 m	1 ton	= 1.01605 tonne	1 pint	= 0.56825 liter
1 mi	= 1.60934 km			1 gallon	= 4.54596 liters

Area

1 cm^2	= 0.15500 in^2	1 hectare (ha)	= 2.47106 acres	1 acre	= 0.40469 hectare (ha)
1 m^2	= 1.19599 yd^2	1 in^2	= 6.45159 cm^2	1 m^2	= 2.58998 km^2
1 km^2	= 0.38610 m^2	1 yd^2	= 0.83613 m^2		

Appendix C

Some important constants

Number	log	Number	log
$\pi \approx 3.14159265$	0.4971499		
$2\pi \approx 6.28318531$	0.7981799	$\pi^2 \approx 9.86960440$	0.9942997
$4\pi \approx 12.56637061$	1.0992099	$\frac{1}{\pi^2} \approx 0.10132118$	9.0057003 − 10
$\frac{\pi}{2} \approx 1.57079633$	0.1961199	$\sqrt{\pi} \approx 1.77245385$	0.2485749
$\frac{\pi}{3} \approx 1.04719755$	0.0200286	$\frac{1}{\sqrt{\pi}} \approx 0.56418958$	9.7514251 − 10
$\frac{4\pi}{3} \approx 4.18879020$	0.6220886	$\sqrt{\frac{3}{\pi}} \approx 0.97720502$	9.9899857 − 10
$\frac{\pi}{4} \approx 0.78539816$	9.8950899 − 10	$\sqrt{\frac{4}{\pi}} \approx 1.12837917$	0.0524551
$\frac{\pi}{6} \approx 0.52359878$	9.7189986 − 10	$\sqrt[3]{\pi} \approx 1.46459189$	0.1657166
$\frac{1}{\pi} \approx 0.31830989$	9.5028501 − 10	$\frac{1}{\sqrt[3]{\pi}} \approx 0.68278406$	9.8342834 − 10
$\frac{1}{2\pi} \approx 0.15915494$	9.2018201 − 10	$\sqrt[3]{\pi^2} \approx 2.14502940$	0.3314332
$\frac{3}{\pi} \approx 0.95492966$	9.9799714 − 10	$\sqrt[3]{\frac{3}{4\pi}} \approx 0.62035049$	9.7926371 − 10
$\frac{4}{\pi} \approx 1.27323954$	0.1049101	$\sqrt[3]{\frac{\pi}{6}} \approx 0.80599598$	9.9063329 − 10

(*Continued*)

Some important constants (*Continued*)

e = Euler constant	≈ 2.71828183	0.43429448
$M = \log_{10} e$	≈ 0.43429448	9.63778431 − 10
$1/M = \log_e 10$	≈ 2.30258509	0.36221569
$180/\pi$ = grades in radian	≈ 57.2957795	1.75812263
$\pi/180$ = radians in 1°	≈ 0.01745329	8.24187737 − 10
$\pi/10800$ = radians in 1'	≈ 0.0002908882	6.46372612 − 10
$\pi/648000$ = radians in 1"	$\approx 0.000004848136811095$	4.68557487 − 10
sin 1°	$\approx 0.000004848136811076$	4.68557487 − 10
tan 1°	$\approx 0.000004848136811133$	4.68557487 − 10
Centimeters in 1 foot	≈ 30.480	1.4840150
Feet in 1 centimeter	≈ 0.032808	8.5159850 − 10
Inches in 1 meter	≈ 39.37	1.5951654
Pounds in 1 kilogram	≈ 2.20462	0.3433340
Kilograms in 1 pound	≈ 0.453593	9.6566663 − 10

$$\pi \approx 3.14159\ 26535\ 89793\ 23846\ 26433\ 83280$$
$$e \approx 2.71828\ 18284\ 59045\ 23536\ 02874\ 71353$$
$$M \approx 0.43429\ 44819\ 03251\ 82765\ 11289\ 18917$$
$$1/M \approx 2.30258\ 50929\ 94045\ 68401\ 79914\ 54684$$
$$\log_{10} \pi \approx 0.49714\ 98726\ 94133\ 85435\ 12682\ 88291$$
$$\log_e \pi \approx 1.14472\ 98858\ 49400\ 17414\ 34273\ 51353$$

Appendix D

Some frequently used symbols and notations

Symbol/notation	Explanation
$=$	equal to
$\neq$	not equal to
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\equiv$	identical to
$\approx$	approximately equal to
$\cong$	congruent to
$\sim$	equivalent or similar
:	the ratio of, as the ratio of 4:7
$\lvert\;\rvert$	absolute value
*	over a Greek letter, an estimate (biased)
^	over a Greek letter, an estimate (unbiased)
$\sqrt{\ }$	square root
$\cup$	union
$\cap$	intersection
$\subset$	is a subset of
$\supset$	contains as a subset
$\emptyset$	empty set or null set
Ω	universal set
e, exp	Euler's constant, approximately equal to 2.71828
$\log_a$	logarithm to the base a
$\log_e$, ln	natural logarithm or logarithm to the base e
$P(A)$	probability of an event A
$P(A\mid B)$	probability of A given B
$P(X = x)$	probability that a discrete random variable X assumes the value x
$E(X)$	expected value of a random variable X
$M_x(t)$	moment generating function of a random variable X
$\phi_x(t)$	characteristic function of a random variable X
$\bar{X}$	arithmetic mean of a sample
μ	arithmetic mean of a population
s	standard deviation of a sample

Symbol/notation	Explanation
σ^2	variance of a population (second moment about mean)
μ_r	the rth moment about mean
μ'_r	the rth moment about origin
$\sqrt{\beta_1}$	coefficient of skewness
β_2	coefficient of kurtosis
$A:B$	A divided by B
$\propto$	is proportional to
$[a, b]$	closed interval from a to b
(a, b)	open interval from a to b
$\ni$	such that
$A \cup B, A + B$	union of two sets A and B
$A \cap B, AB$	intersection of two sets A and B
$A - B$	difference of two sets A and B
$A = B$	identity of two sets A and B
$A \neq B$	inequality of two sets A and B
$A \subset B$	A is a proper subset of B
$A \supset B$	A contains B as a proper subset
$A \not\subset B$	A is not conatined in B or A is not a subset of B
$A \subseteq B$	A is a subset of B
$A \supseteq B$	A contains B
$\bar{A}$	complement of the set A
$\bigcup_{i=1}^{m} A_i$	the union of the sets $A_1, A_2, \ldots, A_m$
$\bigcap_{i=1}^{m} A_i$	the intersection of the sets $A_1, A_2, \ldots, A_m$
$x!$	x factorial
$\binom{n}{r}, {}^nC_r$	combination of n things taken r at a time
${}^nP_r$	permutation of n things taken r at a time
$\lim_{x \to a}$	limit as x approaches a
$\int f(x)\,dx$	integral of $f(x)$ with respect to x
$\int_b^a f(x)\,dx$	definite integral from a to b of $f(x)$ with respect to x
$\int f(x, y)\,dx$	integral of $f(x, y)$ with respect to x holding y constant
$\iint f(x, y)\,dx\,dy$	double integral of $f(x, y)$ with respect to x and y
χ^2	chi-square
z	Fisher's z statistic
t	Student's t statistic
F	F ratio
df	degrees of freedom
Q_1	first quartile
Q_2	second quartile
Q_3	third quartile
ANOVA	analysis of variance
ANCOVA	analysis of covariance
r	sample correlation coefficient

(*Continued*)

Symbol/notation	Explanation
ρ	population correlation coefficient
CV	coefficient of variation
$r_{12.34\ldots n}$	partial correlation coefficient between variables 1 and 2 in a set of *n* variables
$R_{1.234\ldots n}$	multiple correlation coefficient between variable 1 and the remainder of a set of *n* variables
H_0	null hypothesis
H_A, H_1	alternative hypothesis
α	population regression intercept (in a regression equation), significance level or probability of Type I error (in hypotesis testing)
β	population regression coefficient or slope (in a regression equation), probability of Type II error (in hypothesis testing)
$1-\beta$	power

Appendix E

Some continuous probability distributions and their characteristics

Arc-Sine Distribution

Probability density function $f(x) = \dfrac{1}{\pi\sqrt{x(1-x)}}$ $\quad 0 < x < 1$

Mean: $\mu = 1/2$	**Variance:** $\sigma^2 = 1/8$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = 3/2$		

Beta Distribution

Probability density function $f(x) = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ $\quad 0 < x < 1, \alpha, \beta > 0$

Mean: $\mu = \dfrac{\alpha}{\alpha+\beta}$	**Variance:** $\sigma^2 = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Skewness: $\sqrt{\beta_1} = \dfrac{2(\beta-\alpha)(\alpha+\beta+1)^{1/2}}{(\alpha+\beta+2)(\alpha\beta)^{1/2}}$	**Kurtosis:** $\beta_2 = \dfrac{3(A+1)[2A^2+\alpha\beta(A-6)]}{\alpha\beta(A+2)(A+3)}$ where $A = \alpha + \beta$
Moment generating function: $M(\alpha, \alpha+\beta, t)$ where $M(p, q, x)$ denotes a confluent hypergeometric function	

Cauchy Distribution

Probability density function

$$f(x) = \frac{1}{b\pi}\left[1 + \left(\frac{x-a}{b}\right)^2\right]^{-1} \qquad -\infty < x < \infty, -\infty < a < \infty, b > 0$$

Mean: Does not exist	**Variance:** Does not exist	**Skewness:** Does not exist
Kurtosis: Does not exist	**Moment generating function:** Does not exist	**Characteristic function:** $\phi_X(t) = \exp\,[ait - b\,\lvert t\rvert]$

Chi Distribution

Probability density function $f(x) = \dfrac{(x)^{n-1}e^{-x^2/2}}{2^{(n/2)-1}\Gamma(n/2)}$ $\quad 0 < x < \infty,\ n = 1, 2, 3, \ldots$

Mean: $\mu = \dfrac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$	**Variance:** $\sigma^2 = \dfrac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} - \left[\dfrac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\right]^2$

Chi-Square Distribution

Probability density function $f(x) = \dfrac{(x)^{(\nu/2)-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}$ $\quad 0 < x < \infty,\ \nu = 1, 2, 3, \ldots$

Mean: $\mu = \nu$	**Variance:** $\sigma^2 = 2\nu$	**Skewness:** $\sqrt{\beta_1} = 2\sqrt{2}/\sqrt{\nu}$
Kurtosis: $\beta_2 = 3 + 12/\nu$	**Moment generating function:** $M_X(t) = (1-2t)^{-\nu/2}\ t < \dfrac{1}{2}$	**Characteristic function:** $\phi_X(t) = (1-2it)^{-\nu/2}$

Erlang Distribution

Probability density function $f(x) = \dfrac{1}{\beta^n\Gamma(n)}x^{n-1}e^{-x/\beta}$ $\quad x \geq 0, \beta > 0, n = 1, 2, 3, \ldots$

Mean: $\mu = n\beta$	**Variance:** $\sigma^2 = n\beta^2$	**Skewness:** $\sqrt{\beta_1} = 2/\sqrt{n}$
Kurtosis: $\beta_2 = 3 + 6/n$	**Moment generating function:** $M_X(t) = (1-\beta t)^{-n}$	**Characteristic function:** $\phi_X(t) = (1-\beta it)^{-n}$

Exponential Distribution

Probability density function $f(x) = \dfrac{1}{\beta}e^{-x/\beta}$ $\quad x \geq 0, \beta > 0$

Mean: $\mu = \beta$	**Variance:** $\sigma^2 = \beta^2$	**Skewness:** $\sqrt{\beta_1} = 2$
Kurtosis: $\beta_2 = 9$	**Moment generating function:** $M_X(t) = (1-\beta t)^{-1}$	**Characteristic function:** $\phi_X(t) = (1-\beta it)^{-1}$

Extreme-Value Distribution

Probability density function

$$f(x) = \exp\left(-e^{-(x-\alpha)/\beta}\right) \qquad -\infty < x < \infty, -\infty < \alpha < \infty, \beta > 0$$

Mean: $\mu = \alpha + \gamma\beta$ $\gamma = 0.5772\ldots$ is Euler's constant	**Variance:** $\sigma^2 = \dfrac{\pi^2\beta^2}{6}$	**Skewness:** $\sqrt{\beta_1} = 1.29857$
Kurtosis: $\beta_2 = 5.4$	**Moment generating function:** $M_X(t) = e^{\alpha t}\Gamma(1-\beta t) \qquad t < \dfrac{1}{\beta}$	**Characteristic function:** $\phi_X(t) = e^{\alpha it}\Gamma(1-\beta it)$

F Distribution

Probability density function

$$f(x) = \frac{\Gamma\left(\dfrac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\dfrac{\nu_1}{2}\right)\Gamma\left(\dfrac{\nu_2}{2}\right)}\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} x^{(\nu_1/2)-1}\left(1+\frac{\nu_1}{\nu_2}x\right)^{-(\nu_1+\nu_2)/2}$$

$$0 < x < \infty, \nu_1, \nu_2 = 1, 2, 3, \ldots$$

Mean: $\mu = \dfrac{\nu_2}{\nu_2 - 2} \qquad \nu_2 > 2$	**Variance:** $\sigma^2 = \dfrac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)} \qquad \nu_2 >$
Skewness: $\sqrt{\beta_1} = \dfrac{(2\nu_1+\nu_2-2)[8(\nu_2-4)]^{1/2}}{(\nu_2-6)[\nu_1(\nu_1+\nu_2-2)]^{1/2}} \quad \nu_2 > 6$	**Kurtosis:** $\beta_2 = 3 + \dfrac{12[a^2b + \nu_1(\nu_1+a)(5\nu_2-22)]}{\nu_1(\nu_2-6)(\nu_2-8)(\nu_1+a)}$ where $a = \nu_2 - 2, b = \nu_2 - 4, \nu_2 > 8$
Moment generating function: Does not exist	**Characteristic function:** $\phi_X(t) = F\left[\dfrac{1}{2}\nu_1, -\dfrac{1}{2}\nu_2, \dfrac{\nu_2}{\nu_1}it\right]$ Here $F[\alpha, \beta, x]$ is the confluent hypergeometric function defined as $F[\alpha,\beta,x] = 1 + \dfrac{\alpha}{\beta \cdot 1!}x + \dfrac{\alpha(\alpha+1)}{\beta(\beta+1)2!}x^2 + \cdots$

Gamma Distribution

Probability density function $\quad f(x) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta} \qquad 0 < x < \infty, \alpha > 0, \beta > 0$

Mean: $\mu = \alpha\beta$	**Variance:** $\sigma^2 = \alpha\beta^2$	**Skewness:** $\sqrt{\beta_1} = 2/\sqrt{\alpha}$
Kurtosis: $\beta_2 = 3 + 6/\alpha$	**Moment generating function:** $M_X(t) = (1-\beta t)^{-\alpha}$	**Characteristic function:** $\phi_X(t) = (1-\beta it)^{-\alpha}$

Half-Normal Distribution

Probability density function $f(x) = \frac{2\theta}{\pi} e^{-\theta^2 x^2/\pi}$ $x \geq 0, \theta > 0$

Mean: $\mu = \frac{1}{\theta}$	**Variance:** $\sigma^2 = \left(\frac{\pi - 2}{2}\right) \frac{1}{\theta^2}$	**Skewness:** $\sqrt{\beta_1} = \frac{4 - \pi}{\theta^3}$
Kurtosis: $\beta_2 = \frac{3\pi^2 - 4\pi - 12}{4\theta^4}$		

LaPlace (Double Exponential) Distribution

Probability density function

$$f(x) = \frac{1}{2\beta} e^{-|x-\eta|/\beta} \qquad -\infty < x < \infty, \beta > 0, -\infty < \eta < \infty$$

Mean: $\mu = \eta$	**Variance:** $\sigma^2 = 2\beta^2$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = 6$	**Moment generating function:** $M_X(t) = \frac{e^{\eta t}}{1 - \beta^2 t^2}$	**Characteristic function:** $\phi_X(t) = \frac{e^{\eta i t}}{1 + \beta^2 t^2}$

Logistic Distribution

Probability density function

$$f(x) = \frac{\exp[(x - \alpha)/\beta]}{\beta(1 + \exp[(x - \alpha)/\beta])^2} \qquad -\infty < x < \infty, -\infty < \alpha < \infty, \beta > 0$$

Mean: $\mu = \alpha$	**Variance:** $\sigma^2 = \frac{\beta^2 \pi^2}{3}$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = 4.2$	**Moment generating function:** $M_X(t) = e^{\alpha t} \pi \beta t \csc(\pi \beta t)$	**Characteristic function:** $\phi_X(t) = e^{\alpha i t} \pi \beta i t \csc(\pi \beta i t)$

Lognormal Distribution

Probability density function

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(1/2)[(\ln x - \mu)/\sigma]^2} \qquad 0 < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

Mean: $\mu = e^{\mu + (\sigma^2/2)}$	**Variance:** $\sigma^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$	**Skewness:** $\sqrt{\beta_1} = (e^{\sigma^2} + 2)(e^{\sigma^2} - 1)^{1/2}$
Kurtosis: $\beta_2 = e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3$	**Moment generating function:** Does not exist in closed form	

Noncentral Chi-Square Distribution

Probability density function

$$f(x) = \frac{\exp\left[-\frac{1}{2}(x+\lambda)\right]}{2^{\nu/2}} \sum_{j=0}^{\infty} \frac{x^{(\nu/2)+j-1}\lambda^j}{\Gamma\left(\frac{\nu}{2}+j\right)2^{2j}j!} \qquad x > 0, \lambda > 0, \nu = 1, 2, 3, \ldots$$

Mean: $\mu = \nu + \lambda$	**Variance:** $\sigma^2 = 2(\nu + 2\lambda)$	**Skewness:** $\sqrt{\beta_1} = \frac{\sqrt{8}(\nu + 3\lambda)}{(\nu + 2\lambda)^{3/2}}$
Kurtosis: $\beta_2 = 3 + \frac{12(\nu + 4\lambda)}{(\nu + 2\lambda)^2}$	**Moment generating function:** $M_X(t) = (1-2t)^{-\nu/2} \exp\left(\frac{\lambda t}{1-2t}\right)$	**Characteristic function:** $\phi_X(t) = (1-2it)^{-\nu/2} \exp\left(\frac{\lambda it}{1-2it}\right)$

Noncentral *F* Distribution

Probability density function

$$f(x) = \sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{2i+\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{(2i+\nu_1)/2} x^{(2i+\nu_1-2)/2} e^{-\lambda/2}\left(\frac{\lambda}{2}\right)}{\Gamma\left(\frac{\nu_2}{2}\right)\Gamma\left(\frac{2i+\nu_1}{2}\right)\nu_1!\left(1+\frac{\nu_1}{\nu_2}x\right)^{(2i+\nu_1+\nu_2)/2}} \qquad x > 0; \lambda > 0; \; \nu_1, \nu_2 = 1, 2, 3, \ldots$$

Mean: $\mu = \frac{(\nu_1 + \lambda)\nu_2}{(\nu_2 - 2)\nu_1} \qquad \nu_2 > 2$	**Variance:** $\sigma^2 = \frac{(\nu_1+\lambda)^2 + 2(\nu_1+\lambda)\nu_2^2}{(\nu_2-2)(\nu_2-4)\nu_1^2} - \frac{(\nu_1+\lambda)^2\nu_2^2}{(\nu_2-2)^2\nu_1^2} \qquad \nu_2 > 4$

Noncentral *t* Distribution

Probability density function

$$f(x) = \frac{\nu^{\nu/2}}{\Gamma\left(\frac{\nu}{2}\right)} \frac{e^{-\delta^2/2}}{\sqrt{\pi}(\nu + x^2)^{(\nu+1)/2}} \sum_{i=0}^{\infty} \Gamma\left(\frac{\nu+i+1}{2}\right)\left(\frac{\delta^i}{i!}\right)\left(\frac{2x^2}{\nu+x^2}\right)^{i/2}$$

$$-\infty < x < \infty, -\infty < \delta < \infty, \nu = 1, 2, 3, \ldots$$

***r*th moment about the origin:**

$$\mu_r' = c_r \frac{\Gamma\left(\frac{\nu-r}{2}\right)\nu^{r/2}}{2^{r/2}\Gamma\left(\frac{\nu}{2}\right)} \qquad \nu > r$$

where $$c_{2r-1} = \sum_{i=1}^{r} \frac{(2r-1)!\delta^{2r-1}}{(2i-1)!(r-i)!2^{r-i}}$$

$$c_{2r} = \sum_{r=0}^{r} \frac{(2r)!\delta^{2i}}{(2i)!(r-i)!2^{r-i}} \qquad r = 1, 2, 3, \ldots$$

Normal Distribution

Probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \qquad -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

Mean: $\mu = \mu$	**Variance:** $\sigma^2 = \sigma^2$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = 3$	**Moment generating function:** $M_X(t) = \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right]$	**Characteristic function:** $\phi_X(t) = \exp\left[\mu i t - \frac{\sigma^2 t^2}{2}\right]$

Pareto Distribution

Probability density function

$$f(x) = \frac{\alpha\beta^\alpha}{x^{\alpha+1}} \qquad x \geq \beta, \alpha > 0, \beta > 0$$

Mean: $\mu = \frac{\alpha\beta}{\alpha - 1} \qquad \alpha > 1$	**Variance:** $\sigma^2 = \frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)} \qquad \alpha > 2$
Skewness: $\sqrt{\beta_1} = \frac{2(\alpha+1)}{\alpha-3}\sqrt{\frac{\alpha-2}{\alpha}} \qquad \alpha > 3$	**Kurtosis:** $\beta_2 = \frac{3(\alpha-2)(3\alpha^2+\alpha+2)}{\alpha(\alpha-3)(\alpha-4)} \qquad \alpha > 4$
Moment generating function: Does not exist	

Rayleigh Distribution

Probability density function $f(x) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad x \geq 0,\ \sigma > 0$

Mean: $\mu = \sigma\sqrt{\pi/2}$	**Variance:** $\sigma^2 = 2\sigma^2\left(1 - \frac{\pi}{4}\right)$	**Skewness:** $\sqrt{\beta_1} = \frac{\sqrt{\pi}}{4}\frac{(\pi-3)}{\left(1-\frac{\pi}{4}\right)^{3/2}}$	**Kurtosis:** $\beta_2 = \frac{2 - \frac{3}{16}\pi^2}{\left(1-\frac{\pi}{4}\right)^2}$

t Distribution

Probability density function

$$f(x) = \frac{1}{\sqrt{\pi\nu}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2} \qquad -\infty < x < \infty, \nu = 1, 2, 3, \ldots$$

Mean: $\mu = 0 \qquad \nu > 1$	**Variance:** $\sigma^2 = \dfrac{\nu}{\nu - 2} \qquad \nu > 2$	**Skewness:** $\sqrt{\beta_1} = 0 \qquad \nu > 3$
Kurtosis: $\beta_2 = 3 + \dfrac{6}{\nu - 4} \qquad \nu > 4$	**Moment generating function:** Does not exist	**Characteristic function:** $\phi_X(t) = \dfrac{\sqrt{\pi}\,\Gamma(\nu/2)}{\Gamma[(\nu+1)/2]} \displaystyle\int_{-\infty}^{\infty} \frac{e^{itz\sqrt{\nu}}}{(1+z^2)^{(\nu+1)/2}}\,dz$

Triangular Distribution

Probability density function

$$f(x) = \begin{cases} 0 & x \le a \\ 4(x-a)/(b-a)^2 & a < x \le (a+b)/2 \\ 4(b-x)/(b-a)^2 & (a+b)/2 < x < b \\ 0 & x \ge b \end{cases}$$
$$-\infty < a < b < \infty$$

Mean: $\mu = \dfrac{a+b}{2}$	**Variance:** $\sigma^2 = \dfrac{(b-a)^2}{24}$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = \dfrac{12}{5}$	**Moment generating function:** $M_X(t) = \dfrac{4(e^{at/2} - e^{bt/2})^2}{t^2(b-a)^2}$	**Characteristic function:** $\phi_X(t) = \dfrac{4(e^{ait/2} - e^{bit/2})^2}{t^2(b-a)^2}$

Two-Parameter Exponential Distribution

Probability density function $\quad f(x) = \dfrac{1}{\beta} e^{-(x-\eta)/\beta} \qquad \eta < x < \infty, \beta > 0$

Mean: $\mu = \beta + \eta$	**Variance:** $\sigma^2 = \beta^2$	**Skewness:** $\sqrt{\beta_1} = 2.0$
Kurtosis: $\beta_2 = 9.0$	**Moment generating function:** $M_X(t) = \dfrac{e^{\eta t}}{1 - \beta t}$	**Characteristic function:** $\phi_X(t) = \dfrac{e^{\eta i t}}{(1 - \beta i t)}$

Uniform Distribution

Probability density function $\quad f(x) = \dfrac{1}{\beta - \alpha} \qquad \alpha \le x \le \beta,\ -\infty < \alpha < \beta < \infty$

Mean: $\mu = \dfrac{\alpha + \beta}{2}$	**Variance:** $\sigma^2 = \dfrac{(\beta - \alpha)^2}{12}$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = \dfrac{9}{5}$	**Moment generating function:** $M_X(t) = \dfrac{e^{\beta t} - e^{\alpha t}}{(\beta - \alpha)t}$	**Characteristic function:** $\phi_X(t) = \dfrac{(e^{\beta i t} - e^{\alpha i t})}{(\beta - \alpha) i t}$

Weibull Distribution

Probability density function $f(x) = \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} \exp[-(x/\beta)^{\alpha}] \quad 0 < x < \infty, \alpha,\ \beta > 0$

Mean: $\mu = \beta\Gamma\left(\frac{\alpha+1}{\alpha}\right)$	**Variance:** $\sigma^2 = \beta^2\left[\Gamma\left(\frac{\alpha+2}{\alpha}\right) - \left\{\Gamma\left(\frac{\alpha+1}{\alpha}\right)\right\}^2\right]$
Skewness: $\sqrt{\beta_1} = \frac{c - 3ab + 2a^3}{(b-a^2)^{3/2}}$	**Kurtosis:** $\beta_2 = \frac{d - 4ac + 6a^2b - 3a^4}{(b-a^2)^2}$ where $a = \Gamma\left(\frac{\alpha+1}{\alpha}\right)$, $b = \Gamma\left(\frac{\alpha+2}{\alpha}\right)$ $c = \Gamma\left(\frac{\alpha+3}{\alpha}\right)$, and $d = \Gamma\left(\frac{\alpha+4}{\alpha}\right)$
Moment generating function: $M_X(t) = \beta^t\Gamma(1 + t/\alpha)$	**Characteristic function:** $\phi_X(t) = \beta^{it}\Gamma(1 + it/\alpha)$

Appendix F

Some discrete probability distributions and their characteristics

Bernoulli Distribution

Probability function $p(x) = p^x q^{n-1}$ $x = 0, 1$, where $q = 1 - p$

Mean: $\mu = p$	**Variance:** $\sigma^2 = pq$	**Skewness:** $\sqrt{\beta_1} = \dfrac{1-2p}{\sqrt{pq}}$
Kurtosis: $\beta_2 = 3 + \dfrac{1-6pq}{\sqrt{pq}}$	**Moment generating function:** $M_X(t) = q + pe^t$	**Characteristic function:** $\phi_X(t) = q + pe^{it}$
Probability generating function: $\psi_X(t) = q + pt$		

Beta-Binomial Distribution

Probability function

$$p(x) = \frac{1}{(n+1)} \frac{B(a+x, b+n-x)}{B(x+1, n-x+1)B(a,b)}$$ $x = 0, 1, 2, \ldots, n;\ a, b > 0;$ $B(a, b)$ is the beta function

Mean: $\mu = \dfrac{na}{a+b}$	**Variance:** $\sigma^2 = \dfrac{nab(a+b+n)}{(a+b)^2(a+b+1)}$

Beta-Pascal Distribution

Probability function

$$p(x) = \frac{\Gamma(x)\Gamma(\nu)\Gamma(\rho+\nu)\Gamma[\nu+x-(\rho+r)]}{\Gamma(r)\Gamma(x-r+1)\Gamma(\rho)\Gamma(\nu-\rho)\Gamma(\nu+x)} \qquad x = r, r+1, \ldots; \nu > \rho > 0$$

Mean: $\mu = r\dfrac{\nu-1}{\rho-1} \qquad \rho > 1$	**Variance:** $\sigma^2 = r(r+\rho-1)\dfrac{(\nu-1)(\nu-\rho)}{(\rho-1)^2(\rho-2)} \qquad \rho > 2$

Binomial Distribution

Probability function $\quad p(x) = \binom{n}{x} p^x q^{(n-x)} \qquad x = 0, 1, 2, \ldots, n; 0 < p < 1; q = 1 - p$

Mean: $\mu = np$	**Variance:** $\sigma^2 = npq$	**Skewness:** $\sqrt{\beta_1} = \dfrac{1-2p}{\sqrt{npq}}$
Kurtosis: $\beta_2 = 3 + \dfrac{1-6pq}{npq}$	**Moment generating function:** $M_X(t) = (q + pe^t)^n$	**Characteristic function:** $\phi_X(t) = (q + pe^{it})^n$
Probability generating function: $\psi_X(t) = (q + pt)^n$		

Geometric Distribution

Probability function $\quad p(x) = pq^{x-1} \qquad x = 1, 2, 3, \ldots; 0 < p < 1; q = 1 - p$

Mean: $\mu = \dfrac{1}{p}$	**Variance:** $\sigma^2 = \dfrac{q}{p^2}$	**Skewness:** $\sqrt{\beta_1} = \dfrac{2-p}{\sqrt{q}}$
Kurtosis: $\beta_2 = 3 + \dfrac{p^2+6q}{q}$	**Moment generating function:** $M_X(t) = \dfrac{pe^t}{1-qe^t}$	**Characteristic function:** $\phi_X(t) = \dfrac{pe^t}{1-qe^{it}}$
Probability generating function: $\psi_X(t) = \dfrac{pt}{1-qt}$		

Hypergeometric Distribution

Probability function $$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \qquad x = 0, 1, 2, \ldots, n;$$

$$x \le M;\ n - x \le N - M;\ 1 \le n \le N;\ 1 \le M \le N;\ N = 1, 2, \ldots$$

Mean: $\mu = \frac{nM}{N}$	**Variance:** $\sigma^2 = \frac{nM}{N}\left[\frac{N-M}{N}\right]\left[\frac{N-n}{N-1}\right]$
Skewness: $\sqrt{\beta_1} = \frac{(N-2M)(N-2n)(\sqrt{N-1})}{(N-2)\sqrt{nM(N-M)(N-n)}}$	**Kurtosis:** $\beta_2 = \left[\frac{N^2(N-1)}{n(N-2)(N-3)(N-n)}\right](A+B)$ where $A = \frac{N(N+1) - 6n(N-n)}{M(N-M)}$ $B = \frac{3\left\{N^2(n-2) - Nn^2 + 6n(N-n)\right\}}{N^2}$
Moment generating function: $M_X(t) = \frac{(N-M)!(N-n)!}{N!} \times F(-n, -M, N-M-n+1, e^t)$	**Characteristic function:** $\phi_X(t) = \frac{(N-M)!(N-n)!}{N} \times F(-n, -M, N-M-n+1, e^{it})$
Probability generating function: $\psi_X(t) = \left[\frac{N-M}{N}\right]^n F(-n, -M, N-M-n+1, t)$ where $F(\alpha, \beta, \gamma, x)$ is the hypergeometric function defined as $F(\alpha, \beta, \gamma, x) = 1 + \frac{\alpha\beta}{\gamma}\frac{x}{1!} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)}\frac{x^2}{2!} + \cdots$	

Multinomial Distribution

Probability function $$p(x_1, \ldots, x_k) = \frac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k} \qquad x_i = 0, 1, 2, \ldots, n;$$

$$\sum_{i=1}^{k} x_i = n;\ \sum_{i=1}^{k} p_i = 1$$

Mean: $\mu = np_i,\ i = 1, 2, \ldots, k$	**Variance:** $\sigma^2 = np_i(1-p_i)$ $i = 1, 2, \ldots, k$	**Moment generating function:** $M_X(t) = (p_1 e^{t_1} + \cdots + p_k e^{t_k})^n$
Characteristic function: $\phi_X(t) = (p_1 e^{it_1} + \cdots + p_k e^{it_k})^n$	**Probability generating function** $\psi_X(t) = (p_1 t_1 + p_2 t_2 + \cdots + p_k t_k)^n$	

Negative Binomial Distribution

Probability function

$$p(x) = \binom{x-1}{r-1} p^r q^{x-r} \qquad x = r, r+1, \ldots; 0 < p < 1; q = 1 - p$$

Mean: $\mu = \frac{r}{p}$	**Variance:** $\sigma^2 = \frac{rq}{p^2}$	**Skewness:** $\sqrt{\beta_1} = \frac{2-p}{\sqrt{rq}}$
Kurtosis: $\beta_2 = 3 + \frac{p^2 + 6q}{rq}$	**Moment generating function:** $M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$	**Characteristic function:** $\phi_X(t) = \left(\frac{pe^{it}}{1 - qe^{it}}\right)^r$
Probability generating function: $\psi_X(t) = \left(\frac{pt}{1 - qt}\right)^r$		

Poisson Distribution

Probability function $\quad p(x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, \ldots; \lambda > 0$

Mean: $\mu = \lambda$	**Variance:** $\sigma^2 = \lambda$	**Skewness:** $\sqrt{\beta_1} = \frac{1}{\sqrt{\lambda}}$
Kurtosis: $\beta_2 = 3 + \frac{1}{\lambda}$	**Moment generating function:** $M_X(t) = e^{\lambda(e^t - 1)}$	**Characteristic function:** $\phi_X(t) = e^{\lambda(e^{it} - 1)}$
Probability generating function: $\phi_X(t) = e^{\lambda(t-1)}$		

Uniform Distribution

Probability function $\quad p(x) = \frac{1}{n} \qquad x = 1, 2, \ldots, n$

Mean: $\mu = \frac{n+1}{2}$	**Variance:** $\sigma^2 = \frac{n^2 - 1}{12}$	**Skewness:** $\sqrt{\beta_1} = 0$
Kurtosis: $\beta_2 = \frac{3}{5}\left(3 - \frac{4}{n^2 - 1}\right)$	**Moment generating function:** $M_X(t) = \frac{e^t(1 - e^{nt})}{n(1 - e^t)}$	**Characteristic function:** $\phi_X(t) = \frac{e^{it}(1 - e^{nit})}{n(1 - e^{it})}$
Probability generating function: $\psi_X(t) = \frac{t(1 - t^n)}{n(1 - t)}$		