Appendix J

Formulas for hypothesis testing

Test	Assumptions	Null hypothesis H_0	Alternative hypothesis H_A	Test statistics	Decision rules
1. Test concerning a population mean μ	Parent population is normal and standard deviation σ is known	$\mu = \mu_0$ $\mu \leq \mu_0$ $\mu \geq \mu_0$	$\mu \neq \mu_0$ $\mu > \mu_0$ $\mu < \mu_0$	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_{\alpha}$ Reject H_0, if $Z < -z_{\alpha}$
	Parent population is not necessarily normal and standard deviation σ is unknown ($n > 30$)	$\mu = \mu_0$ $\mu \leq \mu_0$ $\mu \geq \mu_0$	$\mu \neq \mu_0$ $\mu > \mu_0$ $\mu < \mu_0$	$Z = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_{\alpha}$ Reject H_0, if $Z < -z_{\alpha}$
	Parent population is normal and standard deviation σ is unknown	$\mu = \mu_0$ $\mu \leq \mu_0$ $\mu \geq \mu_0$	$\mu \neq \mu_0$ $\mu > \mu_0$ $\mu < \mu_0$	$t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$	Reject H_0, if $t > t_{\alpha/2,(n-1)}$ or $t < -t_{\alpha/2,(n-1)}$ Reject H_0, if $t > t_{\alpha,(n-1)}$ Reject H_0, if $t < -t_{\alpha,(n-1)}$
2. Test concerning a population variance σ^2	Parent population is normal and the mean μ is unknown	$\sigma^2 = \sigma_0^2$ $\sigma^2 \leq \sigma_0^2$ $\sigma^2 \geq \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$ $\sigma^2 > \sigma_0^2$ $\sigma^2 < \sigma_0^2$	$\chi^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$	Reject H_0, if $\chi^2 > \chi^2_{\alpha/2,(n-1)}$ or $\chi^2 < \chi^2_{1-\alpha/2,(n-1)}$ Reject H_0, if $\chi^2 > \chi^2_{\alpha,(n-1)}$ Reject H_0, if $\chi^2 < \chi^2_{1-\alpha,(n-1)}$
3. Test concerning a population proportion p	Parent population is Bernoulli ($n > 30$)	$p = p_0$ $p \leq p_0$ $p \geq p_0$	$p \neq p_0$ $p > p_0$ $p < p_0$	$Z = \dfrac{\bar{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_{\alpha}$ Reject H_0, if $Z < -z_{\alpha}$
4. Test concerning a population mean λ	Parent population is Poisson ($n > 30$)	$\lambda = \lambda_0$ $\lambda \leq \lambda_0$ $\lambda \geq \lambda_0$	$\lambda \neq \lambda_0$ $\lambda > \lambda_0$ $\lambda < \lambda_0$	$Z = \dfrac{\bar{X} - \lambda_0}{\sqrt{\lambda_0/n}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_{\alpha}$ Reject H_0, if $Z < -z_{\alpha}$

(*Continued*)

Test	Assumptions	Null hypothesis H_0	Alternative hypothesis H_A	Test statistics	Decision rules
5. Test concerning the difference between two population means $(\mu_1 - \mu_2)$	Parent populations are normal and standard deviations (σ_1 and σ_2) are known	$\mu_1 = \mu_2$ $\mu_1 \leq \mu_2$ $\mu_1 \geq \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	$Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_\alpha$ Reject H_0, if $Z < -z_\alpha$
	Parent populations are normal and standard deviations (σ_1 and σ_2) are unknown but assumed equal	$\mu_1 = \mu_2$ $\mu_1 \leq \mu_2$ $\mu_1 \geq \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	$t = \dfrac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{(1/n_1) + (1/n_2)}}$ where $S_p = \sqrt{\dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$	Reject H_0, if $t > t_{\alpha/2,\nu}$ or $t < -t_{\alpha/2,\nu}$ Reject H_0, if $t > t_{\alpha,\nu}$ Reject H_0, if $t < -t_{\alpha,\nu}$ where $\nu = n_1 + n_2 - 2$
	Parent populations are normal and standard deviations (σ_1 and σ_2) are unknown and unequal	$\mu_1 = \mu_2$ $\mu_1 \leq \mu_2$ $\mu_1 \geq \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	$t = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}$	Reject H_0, if $t > t_{\alpha/2,\nu}$ or $t < -t_{\alpha/2,\nu}$ Reject H_0, if $t > t_{\alpha,\nu}$ Reject H_0, if $t < -t_{\alpha,\nu}$ where $\nu = \dfrac{\left[(s_1^2/n_1) + (s_2^2/n_2)\right]^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$
	Parent populations are not necessarily normal and standard deviations (σ_1 and σ_2) are unknown $(n_1 > 30, n_2 > 30)$	$\mu_1 = \mu_2$ $\mu_1 \leq \mu_2$ $\mu_1 \geq \mu_2$	$\mu_1 \neq \mu_2$ $\mu_1 > \mu_2$ $\mu_1 < \mu_2$	$Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_\alpha$ Reject H_0, if $Z < -z_\alpha$

6. Test concerning the difference between population means of paired observations (μ_d)	Parent population is bivariate normal and standard deviation σ_d is known	$\mu_d = \mu_{d_0}$ $\mu_d \le \mu_{d_0}$ $\mu_d \ge \mu_{d_0}$	$\mu_d \ne \mu_{d_0}$ $\mu_d > \mu_{d_0}$ $\mu_d < \mu_{d_0}$	$Z = \dfrac{\bar{d} - \mu_{d_0}}{\sigma_d/\sqrt{n}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_\alpha$ Reject H_0, if $Z < -z_\alpha$
	Parent population is bivariate normal and standard deviation σ_d is unknown	$\mu_d = \mu_{d_0}$ $\mu_d \le \mu_{d_0}$ $\mu_d \ge \mu_{d_0}$	$\mu_d \ne \mu_{d_0}$ $\mu_d > \mu_{d_0}$ $\mu_d < \mu_{d_0}$	$t = \dfrac{\bar{d} - \mu_{d_0}}{S_d/\sqrt{n}}$ where $\bar{d}$ is the mean of the difference between sample paired observations and S_d is its standard deviation	Reject H_0, if $t > t_{\alpha/2,(n-1)}$ or $t < -t_{\alpha/2,(n-1)}$ Reject H_0, if $t > t_{\alpha,(n-1)}$ Reject H_0, if $t < -t_{\alpha,(n-1)}$
7. Test concerning the ratio between two population variances (σ_1^2/σ_2^2)	Parent populations are normal and their means (μ_1 and μ_2) are unknown	$\sigma_1^2 = \sigma_2^2$ $\sigma_1^2 \le \sigma_2^2$ $\sigma_1^2 \ge \sigma_2^2$	$\sigma_1^2 \ne \sigma_2^2$ $\sigma_1^2 > \sigma_2^2$ $\sigma_1^2 < \sigma_2^2$	$F = \dfrac{S_1^2}{S_2^2}$	Reject H_0, if $F > F_{\nu_1,\nu_2,\alpha/2}$ or $F < F_{\nu_1,\nu_2,1-\alpha/2}$ Reject H_0, if $F > F_{\nu_1,\nu_2,\alpha}$ Reject H_0, if $F < F_{\nu_1,\nu_2,1-\alpha}$ where $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$
8. Test concerning the difference between two population proportions ($p_1 - p_2$)	Parent populations are Bernoulli ($n_1 > 30$, $n_2 > 30$)	$p_1 = p_2$ $p_1 \le p_2$ $p_1 \ge p_2$	$p_1 \ne p_2$ $p_1 > p_2$ $p_1 < p_2$	$Z = \dfrac{\bar{p}_1 - \bar{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}}$ where $\hat{p} = \dfrac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_\alpha$ Reject H_0, if $Z < -z_\alpha$
9. Test concerning the difference between two population means ($\lambda_1 - \lambda_2$)	Parent populations are Poisson ($n_1 > 30$, $n_2 > 30$)	$\lambda_1 = \lambda_2$ $\lambda_1 \le \lambda_2$ $\lambda_1 \ge \lambda_2$	$\lambda_1 \ne \lambda_2$ $\lambda_1 > \lambda_2$ $\lambda_1 < \lambda_2$	$Z = \dfrac{\bar{X}_1 - \bar{X}_2}{S\sqrt{(1/n_1 + 1/n_2)}}$ where $S = \sqrt{\dfrac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}}$	Reject H_0, if $Z > z_{\alpha/2}$ or $Z < -z_{\alpha/2}$ Reject H_0, if $Z > z_\alpha$ Reject H_0, if $Z < -z_\alpha$

Appendix K

Formulas for confidence intervals

Confidence Interval for a Population Mean μ

When parent population is normal and σ^2 is known	$\bar{X} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$
When parent population is normal, σ^2 is unknown, and sample is small	$\bar{X} \pm t_{n-1,\alpha/2} \dfrac{S}{\sqrt{n}}$
When parent population is not necessarily normal, σ^2 is unknown, and sample is large	$\bar{X} \pm z_{\alpha/2} \dfrac{S}{\sqrt{n}}$
Sample size for the confidence interval of a population mean μ with estimation error e	$n = \dfrac{(z_{\alpha/2})^2\sigma^2}{e^2}$

Confidence Interval of a Population Proportion *p*

When parent population is Bernoulli and sample is large	$\bar{p} \pm z_{\alpha/2}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$
Sample size for the confidence interval of a population proportion p with estimation error e	$n = \dfrac{z_{\alpha/2}^2\bar{p}(1-\bar{p})}{e^2}$

Confidence Interval for a Population Variance σ^2

When parent population is normal and the mean μ is unknown	$\dfrac{(n-1)S^2}{\chi^2_{n-1,\alpha/2}} \le \sigma^2 \le \dfrac{(n-1)S^2}{\chi^2_{n-1,1-\alpha/2}}$

Confidence Interval for the Difference of Two Population Means $(\mu_1 - \mu_2)$

When parent populations are normal and σ_1^2 and σ_2^2 are known	$(\bar{X}_1 - \bar{X}_2) \pm z_{\alpha/2}\sqrt{\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}\right)}$
When parent populations are not necessarily normal, σ_1^2 and σ_2^2 are unknown, and samples are large	$(\bar{X}_1 - \bar{X}_2) \pm z_{\alpha/2}\sqrt{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)}$
When parent populations are normal, σ_1^2 and σ_2^2 are unknown, $\sigma_1^2 = \sigma_2^2$, and samples are small	$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2,\nu} S_p\sqrt{\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}$

where $S_p = \sqrt{\dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$

and $\nu = n_1 + n_2 - 2$

When parent populations are normal, σ_1^2 and σ_2^2 are unknown, $\sigma_1^2 \neq \sigma_2^2$ and samples are small

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2,\nu}\sqrt{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)}$$

$$\text{where } \nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1-1} + \frac{\left(s_2^2/n_2\right)^2}{n_2-1}}$$

Confidence intervals for the difference μ_d between Two Population Means of a Paired Sample of Size n ($\mu_d = \mu_1 - \mu_2$)

When parent population is bivariate normal and standard deviation σ_d is known

$$\bar{d} \pm z_{\alpha/2}\frac{\sigma_d}{\sqrt{n}}$$

When parent population is bivariate normal and standard deviation σ_d is unknown

$$\bar{d} \pm t_{\alpha/2,(n-1)}\frac{S_d}{\sqrt{n}}$$

Confidence Interval for the Difference between Two Population Proportions ($p_1 - p_2$)

When parent populations are Bernoulli and samples are large

$$(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

Confidence Interval for the Ratio of Two Population Variances (σ_1^2/σ_2^2)

When parent populations are normal and their means (μ_1 and μ_2) are unknown

$$\frac{S_1^2/S_2^2}{F_{\nu_1,\nu_2;\alpha/2}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2/S_2^2}{F_{\nu_1,\nu_2;1-\alpha/2}}$$

where $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$

Appendix L

Relations between moments and cumulants*

Expressions for Moments about Origin in Terms of Cumulants

The following expressions give the first 10 moments about the origin in terms of cumulants of a probability distribution:

$$\mu'_1 = \kappa_1$$

$$\mu'_2 = \kappa_2 + \kappa_1^2$$

$$\mu'_3 = \kappa_3 + 3\kappa_2\kappa_1 + \kappa_1^3$$

$$\mu'_4 = \kappa_4 + 4\kappa_3\kappa_1 + 3\kappa_2^2 + 6\kappa_2\kappa_1^2 + \kappa_1^4$$

$$\mu'_5 = \kappa_5 + 5\kappa_4\kappa_1 + 10\kappa_3\kappa_2 + 10\kappa_3\kappa_1^2 + 15\kappa_2^2\kappa_1 + 10\kappa_2\kappa_1^3 + \kappa_1^5$$

$$\begin{aligned}\mu'_6 = {} & \kappa_6 + 6\kappa_5\kappa_1 + 15\kappa_4\kappa_2 + 15\kappa_4\kappa_1^2 + 10\kappa_3^2 + 60\kappa_3\kappa_2\kappa_1 + 20\kappa_3\kappa_1^3 + 15\kappa_2^3 \\ & + 45\kappa_2^2\kappa_1^2 + 15\kappa_2\kappa_1^4 + \kappa_1^6,\end{aligned}$$

$$\begin{aligned}\mu'_7 = {} & \kappa_7 + 7\kappa_6\kappa_1 + 21\kappa_5\kappa_2 + 21\kappa_5\kappa_1^2 + 35\kappa_4\kappa_3 + 105\kappa_4\kappa_2\kappa_1 + 35\kappa_4\kappa_1^3 + 70\kappa_3^2\kappa_1 \\ & + 105\kappa_3\kappa_2^2 + 210\kappa_3\kappa_2\kappa_1^2 + 35\kappa_3\kappa_1^4 + 105\kappa_2^2\kappa_1^3 + 105\kappa_2^3\kappa_1 + 21\kappa_2\kappa_1^5 + \kappa_1^7\end{aligned}$$

$$\begin{aligned}\mu'_8 = {} & \kappa_8 + 8\kappa_7\kappa_1 + 28\kappa_6\kappa_2 + 28\kappa_6\kappa_1^2 + 56\kappa_5\kappa_3 + 168\kappa_5\kappa_2\kappa_1 + 56\kappa_5\kappa_1^3 + 35\kappa_4^2 \\ & + 280\kappa_4\kappa_3\kappa_1 + 210\kappa_4\kappa_2^2 + 420\kappa_4\kappa_2\kappa_1^2 + 70\kappa_4\kappa_1^4 + 280\kappa_3^2\kappa_2 + 280\kappa_3^2\kappa_1^2 \\ & + 840\kappa_3\kappa_2^2\kappa_1 + 560\kappa_3\kappa_2\kappa_1^3 + 56\kappa_3\kappa_1^5 + 105\kappa_2^4 + 420\kappa_2^3\kappa_1^2 + 210\kappa_2^2\kappa_1^4 \\ & + 28\kappa_2\kappa_1^6 + \kappa_1^8\end{aligned}$$

$$\begin{aligned}\mu'_9 = {} & \kappa_9 + 9\kappa_8\kappa_1 + 36\kappa_7\kappa_2 + 36\kappa_7\kappa_1^2 + 84\kappa_6\kappa_3 + 252\kappa_6\kappa_2\kappa_1 + 84\kappa_6\kappa_1^3 + 126\kappa_5\kappa_4 \\ & + 504\kappa_5\kappa_3\kappa_1 + 378\kappa_5\kappa_2^2 + 756\kappa_5\kappa_2\kappa_1^2 + 126\kappa_5\kappa_1^4 + 315\kappa_4^2\kappa_1 + 1260\kappa_4\kappa_3\kappa_2 \\ & + 1260\kappa_4\kappa_3\kappa_1^2 + 1890\kappa_4\kappa_2^2\kappa_1 + 1260\kappa_4\kappa_2\kappa_1^3 + 126\kappa_4\kappa_1^5 + 280\kappa_3^3 + 2520\kappa_3^2\kappa_2\kappa_1 \\ & + 840\kappa_3^2\kappa_1^3 + 1260\kappa_3\kappa_2^3 + 3780\kappa_3\kappa_2^2\kappa_1^2 + 1260\kappa_3\kappa_2\kappa_1^4 + 84\kappa_3\kappa_1^6 + 945\kappa_2^4\kappa_1 \\ & + 1260\kappa_2^3\kappa_1^3 + 378\kappa_2^2\kappa_1^5 + 36\kappa_2\kappa_1^7 + \kappa_1^9\end{aligned}$$

*The results given in this appendix are adapted from *Kendall's Advanced Theory of Statistics,* 6th ed., Vol. 1 (Chap. 3) by A. Stuart and K. Ord (Arnold, London, 1994).

$$
\begin{aligned}
\mu'_{10} = {} & \kappa_{10} + 10\kappa_9\kappa_1 + 45\kappa_8\kappa_2 + 45\kappa_8\kappa_1^2 + 120\kappa_7\kappa_3 + 360\kappa_7\kappa_2\kappa_1 + 120\kappa_7\kappa_1^3 + 210\kappa_6\kappa_4 \\
& + 840\kappa_6\kappa_3\kappa_1 + 630\kappa_6\kappa_2^2 + 1260\kappa_6\kappa_2\kappa_1^2 + 210\kappa_6\kappa_1^4 + 126\kappa_5^2 + 1260\kappa_5\kappa_4\kappa_1 \\
& + 2520\kappa_5\kappa_3\kappa_2 + 2520\kappa_5\kappa_3\kappa_1^2 + 3780\kappa_5\kappa_2^2\kappa_1 + 2520\kappa_5\kappa_2\kappa_1^3 + 252\kappa_5\kappa_1^5 \\
& + 1575\kappa_4^2\kappa_2 + 1575\kappa_4^2\kappa_1^2 + 2100\kappa_4\kappa_3^2 + 12{,}600\kappa_4\kappa_3\kappa_2\kappa_1 + 4200\kappa_4\kappa_3\kappa_1^3 \\
& + 3150\kappa_4\kappa_2^3 + 9450\kappa_4\kappa_2^2\kappa_1^2 + 3150\kappa_4\kappa_2\kappa_1^4 + 210\kappa_4\kappa_1^6 + 2800\kappa_3^3\kappa_1 + 6300\kappa_3^2\kappa_2^2 \\
& + 12{,}600\kappa_3^2\kappa_2\kappa_1^2 + 2100\kappa_3^2\kappa_1^4 + 12{,}600\kappa_3\kappa_2^3\kappa_1 + 12{,}600\kappa_3\kappa_2^2\kappa_1^3 + 2520\kappa_3\kappa_2\kappa_1^5 \\
& + 120\kappa_3\kappa_1^7 + 945\kappa_2^5 + 4725\kappa_2^4\kappa_1^2 + 3150\kappa_2^3\kappa_1^4 + 630\kappa_2^2\kappa_1^6 + 45\kappa_2\kappa_1^8 + \kappa_1^{10}
\end{aligned}
$$

Expressions for Central Moments in Terms of Cumulants

The following expressions give the first 10 central moments in terms of cumulants of a probability distribution:

$$
\begin{aligned}
\mu_2 &= \kappa_2 \\
\mu_3 &= \kappa_3 \\
\mu_4 &= \kappa_4 + 3\kappa_2^2 \\
\mu_5 &= \kappa_5 + 10\kappa_3\kappa_2 \\
\mu_6 &= \kappa_6 + 15\kappa_4\kappa_2 + 10\kappa_3^2 + 15\kappa_2^3 \\
\mu_7 &= \kappa_7 + 21\kappa_5\kappa_2 + 35\kappa_4\kappa_3 + 105\kappa_3\kappa_2^2 \\
\mu_8 &= \kappa_8 + 28\kappa_6\kappa_2 + 56\kappa_5\kappa_3 + 35\kappa_4^2 + 210\kappa_4\kappa_2^2 + 280\kappa_3^2\kappa_2 + 105\kappa_2^4 \\
\mu_9 &= \kappa_9 + 36\kappa_7\kappa_2 + 84\kappa_6\kappa_3 + 126\kappa_5\kappa_4 + 378\kappa_5\kappa_2^2 + 1260\kappa_4\kappa_3\kappa_2 + 280\kappa_3^3 + 1260\kappa_3\kappa_2^3 \\
\mu_{10} &= \kappa_{10} + 45\kappa_8\kappa_2 + 120\kappa_7\kappa_3 + 210\kappa_6\kappa_4 + 630\kappa_6\kappa_2^2 + 126\kappa_5^2 + 2520\kappa_5\kappa_3\kappa_2 \\
&\quad + 1575\kappa_4^2\kappa_2 + 2100\kappa_4\kappa_3^2 + 3150\kappa_4\kappa_2^3 + 6300\kappa_3^2\kappa_2^2 + 945\kappa_2^5
\end{aligned}
$$

Expressions for Cumulants in Terms of Moments about Origin

The following expressions give the first 10 cumulants in terms of moments about the origin of a probability distribution:

$$
\begin{aligned}
\kappa_1 &= \mu'_1 \\
\kappa_2 &= \mu'_2 - \mu'^2_1 \\
\kappa_3 &= \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1 \\
\kappa_4 &= \mu'_4 - 4\mu'_3\mu'_1 - 3\mu'^2_2 + 12\mu'_2\mu'^2_1 - 6\mu'^4_1 \\
\kappa_5 &= \mu'_5 - 5\mu'_4\mu'_1 - 10\mu'_3\mu'_2 + 20\mu'_3\mu'^2_1 + 30\mu'^2_2\mu'_1 - 60\mu'_2\mu'^3_1 + 24\mu'^5_1 \\
\kappa_6 &= \mu'_6 - 6\mu'_5\mu'_1 - 15\mu'_4\mu'_2 + 30\mu'_4\mu'^2_1 - 10\mu'^2_3 + 120\mu'_3\mu'_2\mu'_1 - 120\mu'_3\mu'^3_1 \\
&\quad + 30\mu'^3_2 - 270\mu'^2_2\mu'^2_1 + 360\mu'_2\mu'^4_1 - 120\mu'^6_1 \\
\kappa_7 &= \mu'_7 - 7\mu'_6\mu'_1 - 21\mu'_5\mu'_2 + 42\mu'_5\mu'^2_1 - 35\mu'_4\mu'_3 + 210\mu'_4\mu'_2\mu'_1 \\
&\quad - 210\mu'_4\mu'^3_1 + 140\mu'^2_3\mu'_1 + 210\mu'_3\mu'^2_2 - 1260\mu'_3\mu'_2\mu'^2_1 + 840\mu'_3\mu'^4_1 \\
&\quad - 630\mu'^3_2\mu'_1 + 2520\mu'^2_2\mu'^3_1 - 2520\mu'_2\mu'^5_1 + 720\mu'^7_1 \\
\kappa_8 &= \mu'_8 - 8\mu'_7\mu'_1 - 28\mu'_6\mu'_2 + 56\mu'_6\mu'^2_1 - 56\mu'_5\mu'_3 + 336\mu'_5\mu'_2\mu'_1 \\
&\quad - 336\mu'_5\mu'^3_1 - 35\mu'^2_4 + 560\mu'_4\mu'_3\mu'_1 + 420\mu'_4\mu'^2_2 - 2520\mu'_4\mu'_2\mu'^2_1 \\
&\quad + 1680\mu'_4\mu'^4_1 + 560\mu'^2_3\mu'_2 - 1680\mu'^2_3\mu'^2_1 - 5040\mu'_3\mu'^2_2\mu'_1 \\
&\quad + 13{,}440\mu'_3\mu'_2\mu'^3_1 - 6720\mu'_3\mu'^5_1 - 630\mu'^4_2 + 10{,}080\mu'^3_2\mu'^2_1 \\
&\quad - 25{,}200\mu'^2_2\mu'^4_1 + 20{,}160\mu'_2\mu'^6_1 - 5040\mu'^8_1
\end{aligned}
$$

$$
\begin{aligned}
\kappa_9 = {} & \mu_9' - 9\mu_8'\mu_1' - 36\mu_7'\mu_2' + 72\mu_7'\mu_1'^2 - 84\mu_6'\mu_3' + 504\mu_6'\mu_2'\mu_1' \\
& - 504\mu_6'\mu_1'^3 - 126\mu_5'\mu_4' + 1008\mu_5'\mu_3'\mu_1' + 756\mu_5'\mu_2'^2 - 4536\mu_5'\mu_2'\mu_1'^2 \\
& + 3024\mu_5'\mu_1'^4 + 630\mu_4'^2\mu_1' + 2520\mu_4'\mu_3'\mu_2' - 7560\mu_4'\mu_3'\mu_1'^2 \\
& - 11{,}340\mu_4'\mu_2'^2\mu_1' + 30{,}240\mu_4'\mu_2'\mu_1'^3 - 15{,}120\mu_4'\mu_1'^5 + 560\mu_3'^3 \\
& - 15{,}120\mu_3'^2\mu_2'\mu_1' + 20{,}160\mu_3'^2\mu_1'^3 - 7560\mu_3'\mu_2'^3 + 90{,}720\mu_3'\mu_2'^2\mu_1'^2 \\
& - 151{,}200\mu_3'\mu_2'\mu_1'^4 + 60{,}480\mu_3'\mu_1'^6 + 22{,}680\mu_2'^4\mu_1' - 151{,}200\mu_2'^3\mu_1'^3 \\
& + 272{,}160\mu_2'^2\mu_1'^5 - 181{,}440\mu_2'\mu_1'^7 + 40{,}320\mu_1'^9
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{10} = {} & \mu_{10}' - 10\mu_9'\mu_1' - 45\mu_8'\mu_2' + 90\mu_8'\mu_1'^2 - 120\mu_7'\mu_3' + 720\mu_7'\mu_2'\mu_1' \\
& - 720\mu_7'\mu_1'^3 - 210\mu_6'\mu_4' + 1680\mu_6'\mu_3'\mu_1' + 1260\mu_6'\mu_2'^2 \\
& - 7560\mu_6'\mu_2'\mu_1'^2 + 5040\mu_6'\mu_1'^4 - 126\mu_5'^2 + 2520\mu_5'\mu_4'\mu_1' \\
& + 5040\mu_5'\mu_3'\mu_2' - 15{,}120\mu_5'\mu_3'\mu_1'^2 - 22{,}680\mu_5'\mu_2'^2\mu_1' + 60{,}480\mu_5'\mu_2'\mu_1'^3 \\
& - 30{,}240\mu_5'\mu_1'^5 + 3150\mu_4'^2\mu_2' - 9450\mu_4'^2\mu_1'^2 + 4200\mu_4'\mu_3'^2 \\
& - 75{,}600\mu_4'\mu_3'\mu_2'\mu_1' + 100{,}800\mu_4'\mu_3'\mu_1'^3 - 18{,}900\mu_4'\mu_2'^3 \\
& + 226{,}800\mu_4'\mu_2'^2\mu_1'^2 - 378{,}000\mu_4'\mu_2'\mu_1'^4 + 151{,}200\mu_4'\mu_1'^6 - 16{,}800\mu_3'^3\mu_1' \\
& - 37{,}800\mu_3'^2\mu_2'^2 + 302{,}400\mu_3'^2\mu_2'\mu_1'^2 - 252{,}000\mu_3'^2\mu_1'^4 + 302{,}400\mu_3'\mu_2'^3\mu_1' \\
& - 1{,}512{,}000\mu_3'\mu_2'^2\mu_1'^3 + 1{,}814{,}400\mu_3'\mu_2'\mu_1'^5 - 604{,}800\mu_3'\mu_1'^7 \\
& + 22{,}680\mu_2'^5 - 567{,}000\mu_2'^4\mu_1'^2 + 2{,}268{,}000\mu_2'^3\mu_1'^4 - 3{,}175{,}200\mu_2'^2\mu_1'^6 \\
& + 1{,}814{,}400\mu_2'\mu_1'^8 - 362{,}880\mu_1'^{10}
\end{aligned}
$$

Expressions for Cumulants in Terms of Central Moments

The following expressions give the first 10 cumulants in terms of central moments of a probability distribution:

$$\kappa_2 = \mu_2$$

$$\kappa_3 = \mu_3$$

$$\kappa_4 = \mu_4 - 3\mu_2^2$$

$$\kappa_5 = \mu_5 - 10\mu_3\mu_2$$

$$\kappa_6 = \mu_6 - 15\mu_4\mu_2 - 10\mu_3^2 + 30\mu_2^3$$

$$\kappa_7 = \mu_7 - 21\mu_5\mu_2 - 35\mu_4\mu_3 + 210\mu_3\mu_2^2$$

$$\kappa_8 = \mu_8 - 28\mu_6\mu_2 - 56\mu_5\mu_3 - 35\mu_4^2 + 420\mu_4\mu_2^2 + 560\mu_3^2\mu_2 - 630\mu_2^4$$

$$
\begin{aligned}
\kappa_9 = {} & \mu_9 - 36\mu_7\mu_2 - 84\mu_6\mu_3 - 126\mu_5\mu_4 + 756\mu_5\mu_2^2 + 2520\mu_4\mu_3\mu_2 \\
& + 560\mu_3^3 - 7560\mu_3\mu_2^3
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{10} = {} & \mu_{10} - 45\mu_8\mu_2 - 120\mu_7\mu_3 - 210\mu_6\mu_4 + 1260\mu_6\mu_2^2 - 126\mu_5^2 \\
& + 5040\mu_5\mu_3\mu_2 + 3150\mu_4^2\mu_2 + 4200\mu_4\mu_3^2 - 18{,}900\mu_4\mu_2^3 \\
& - 37{,}800\mu_3^2\mu_2^2 + 22{,}680\mu_2^5
\end{aligned}
$$