

Useful Formulae

Complex Numbers

$$j = (-1)^{1/2} \quad j^2 = -1$$

$$\exp(j\theta) = \cos\theta + j\sin\theta$$

$$Z = a + jb = re^{j\theta} \quad r = (a^2 + b^2)^{1/2} \quad \tan\theta = b/a$$

$$Z^* = a - jb = re^{-j\theta} \quad \operatorname{Re}(Z) = a \quad \operatorname{Im}(Z) = b$$

$$\text{Magnitude}^2 = |Z|^2 = ZZ^* = a^2 + b^2 \quad \text{Argument} = \theta = \arctan(b/a)$$

$$\cos\theta = \frac{1}{2} [e^{j\theta} + e^{-j\theta}] \quad \sin\theta = \frac{1}{2j} [e^{j\theta} - e^{-j\theta}]$$

Expansions

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

$$\text{Small } x \quad (1+x)^n \approx 1 + nx \quad \sin x \approx x \quad \tan x \approx x \quad \cos x \approx 1$$

$$\text{Small } \Delta x \text{ in } x = x_o + \Delta x, \quad f(x) \approx f(x_o) + \Delta x \left[\frac{df}{dx} \right]_{x_o}$$

Trigonometry

$$\sin(\pi/2 \pm \theta) = \cos\theta \quad \sin^2\theta + \cos^2\theta = 1$$

$$\sin 2\theta = 2 \sin\theta \cos\theta \quad \cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin A + \sin B = 2\sin[1/2(A+B)]\cos[1/2(A-B)]$$

$$\cos A + \cos B = 2\cos[1/2(A+B)]\cos[1/2(A-B)]$$