The Evaluation of Travel Time

As discussed in Sec. 18.4, one of the important considerations for deciding how many service facilities to provide is the amount of time that customers must spend traveling to and from a facility. Therefore, the *expected round-trip travel time* $E(T)$ for a customer is one of the components of the objective function given in Sec. 18.4 for model 3, the decision model that is concerned with making the decision on the number of service facilities. We now shall elaborate on how to determine $E(T)$. In the process, we shall give the details of how the various values of $E(T)$ were obtained in Sec. 18.4 for Example 3.

$E(T)$ can be interpreted as the *average travel time* spent by customers in coming both to and from a given service facility. Therefore, the value of $E(T)$ depends very much upon the characteristics of the individual situation. However, we shall illustrate a rather general approach to evaluating $E(T)$ by developing a basic travel-time model and then calculating $E(T)$ for the more complicated situation involved in Example 3. In both cases it is assumed that the portion of the population assigned to the service facility under consideration is *distributed uniformly* throughout the assigned area, that each arrival returns to its *original location* after receiving service, and that the average speed of travel does *not* depend upon the distance traveled. Another basic assumption is that all travel is *rectilinear,* i.e., it progresses along a system of *orthogonal* paths (aisles, streets, highways, and so on) that are *parallel* to the main sides of the area under consideration.

A Basic Travel-Time Model

Description: Rectangular area and rectilinear travel, as shown in Fig. 18.8.

Definitions:
T = travel time (round trip) for an arrival.
v = average velocity (speed) of customers in traveling to and from facility.
a, b, c, d = respective distances from facility to boundary of area assigned to facility, as shown in Fig. 18.8.

Given: v, a, b, c, d.

To find: Expected value of T, $E(T)$.

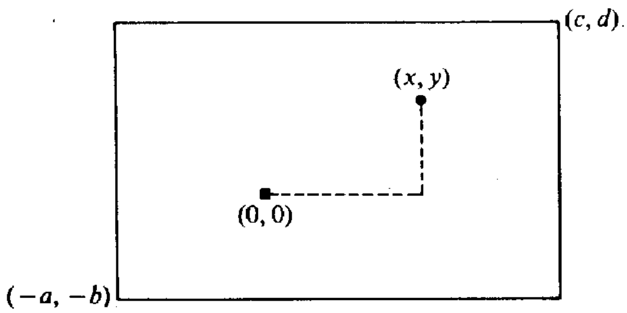


Figure 18.8 Graphical representation of a basic travel-time model, where the service facility is at (0, 0) and a random arrival comes from (and returns to) some location (x, y).

Using an orthogonal (x, y) coordinate system, Fig. 18.8 shows the coordinates (x, y) of the location of a *particular* customer. The x and y coordinates of the location from which a *random* arrival comes actually are *random variables* X and Y, where X ranges from $-a$ to c and Y ranges from $-b$ to d. Because the total round-trip distance traveled by the random arrival is

$$D = 2(|X| + |Y|)$$

and

$$T = \frac{D}{v},$$

it follows that

$$E(T) = \frac{2}{v}(E\{|X|\} + E\{|Y|\}).$$

Thus the problem is reduced to identifying the probability distributions of $|X|$ and $|Y|$ and then calculating their means.

First consider $|X|$. Its probability distribution can be obtained directly from the distribution of X. Because the customers are assumed to be distributed uniformly throughout the assigned area, and because the *height* of the rectangular area is the *same* for all possible values of $X = x$, X must have a *uniform distribution* between $-a$ and c, as shown in Fig. 18.9a. Because $|x| = |-x|$, adding the probability density function values at x and $-x$ then yields the probability distribution of $|X|$ shown in Fig. 18.9b

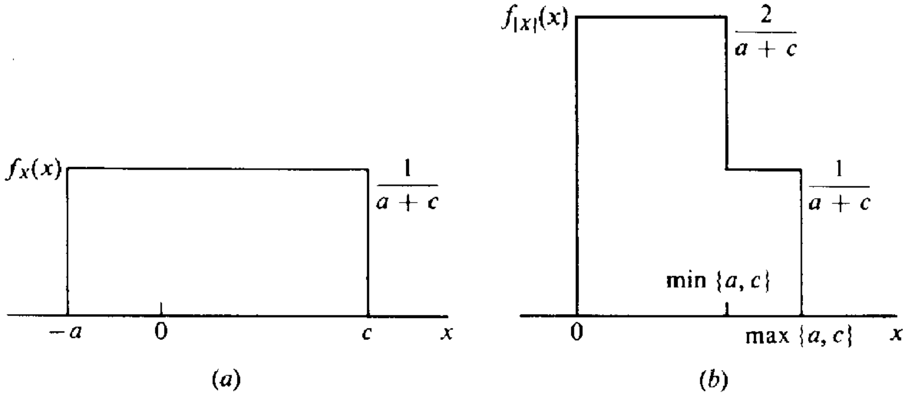


Figure 18.9 Probability density functions of (a) X; (b) $|X|$.

Therefore, noting that $|x| = x$ for $x \geq 0$,

$$E\{|X|\} = \int_0^{\max\{a,\,c\}} x f_{|X|}(x)\,dx$$

$$= \int_0^{\min\{a,\,c\}} \frac{2x}{a+c}\,dx + \int_{\min\{a,\,c\}}^{\max\{a,\,c\}} \frac{x}{a+c}\,dx$$

$$= \frac{1}{2}\frac{1}{a+c}\left[(\min\{a,\,c\})^2 + (\max\{a,\,c\})^2\right]$$

$$= \frac{a^2+c^2}{2(a+c)}.$$

The analysis for $|Y|$ is completely analogous, where the *width* of the rectangular area for possible values of $Y = y$ now determines the probability distribution of Y.

The result is that

$$E\{|Y|\} = \frac{b^2+d^2}{2(b+d)}.$$

Consequently, $$E(T) = \frac{1}{v}\left(\frac{a^2+c^2}{a+c} + \frac{b^2+d^2}{b+d}\right).$$

EXAMPLE 3—HOW MANY TOOL CRIBS? For the new plant being designed for the *Mechanical Company* (see Sec. 18.1), the layout of the portion of the factory area where the mechanics will work is shown in Fig. 18.7. The three *possible* locations for tool cribs are identified as Locations 1, 2, and 3, where access to these locations will be provided by a system of orthogonal aisles parallel to the sides of the indicated area. The coordinates are given in units of *feet*. The mechanics will be distributed quite uniformly throughout the area shown, and each mechanic will be assigned to the *nearest* tool crib. It is estimated that the mechanics will walk to and from a tool crib at an average speed of slightly less than 3 miles/hour, so v is set at $v = 15{,}000$ feet/hour.

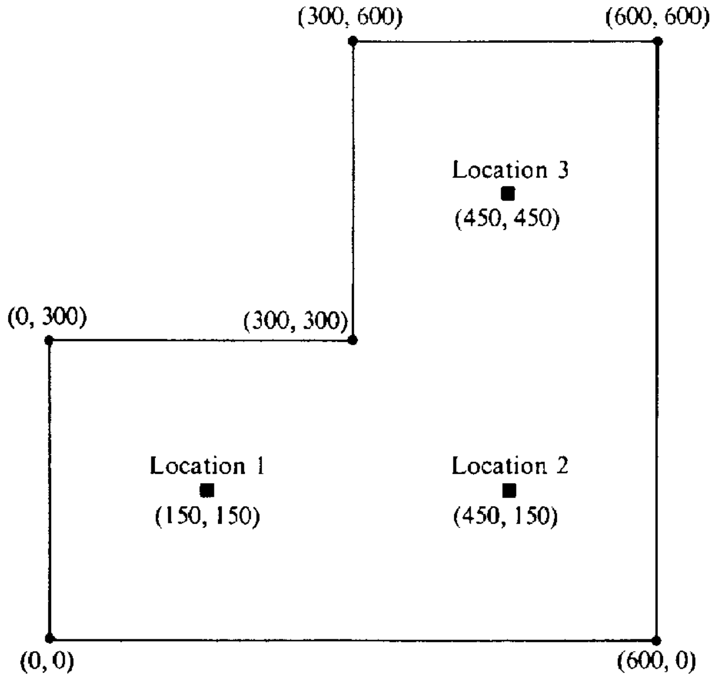


Figure 18.7 Layout for Example 3.

The three basic alternatives being considered are

Alternative 1: Have *three* tool cribs—use Locations 1, 2, and 3;
Alternative 2: Have *one* tool crib—use Location 2;
Alternative 3: Have *two* tool cribs—use Locations 1 and 3.

The calculation of $E(T)$ for each alternative is given next, followed by the use of model 3 to make the choice among them.

Alternative 1 ($n = 3$): If all three locations were used, *each* tool crib would service a 300×300 foot *square* area. Therefore, this case is just a special case of the basic travel-time model just presented, where $a = c = 150$ and $b = d = 150$. Consequently,

$$E(T) = \frac{1}{15{,}000 \text{ ft/hr}} \left(\frac{150^2 + 150^2}{150 + 150} + \frac{150^2 + 150^2}{150 + 150} \right) \text{ft}$$

$$= \frac{1}{15{,}000 \text{ ft/hr}} (300 \text{ ft})$$

$$= 0.02 \text{ hr}.$$

Alternative 2 ($n = 1$): With just *one* tool crib (in Location 2) to service the entire area shown in Fig. 18.7, the derivation of $E(T)$ is a little more complicated than it is for the basic travel-time model. The first step is to relabel Location 2 as the origin (0, 0) for an (x, y) coordinate system, so that 450 would be subtracted from the first coordinates shown and 150 would be subtracted from the second coordinates. The probability density function for X is then obtained by dividing the *height* for each possible value of $X = x$ by the total area (so that the area under the probability density function curve equals 1), as given in Fig. 18.10*a*. Combining the values for x and $-x$ then yields the probability distribution of $|X|$ shown in Fig. 18-10b.

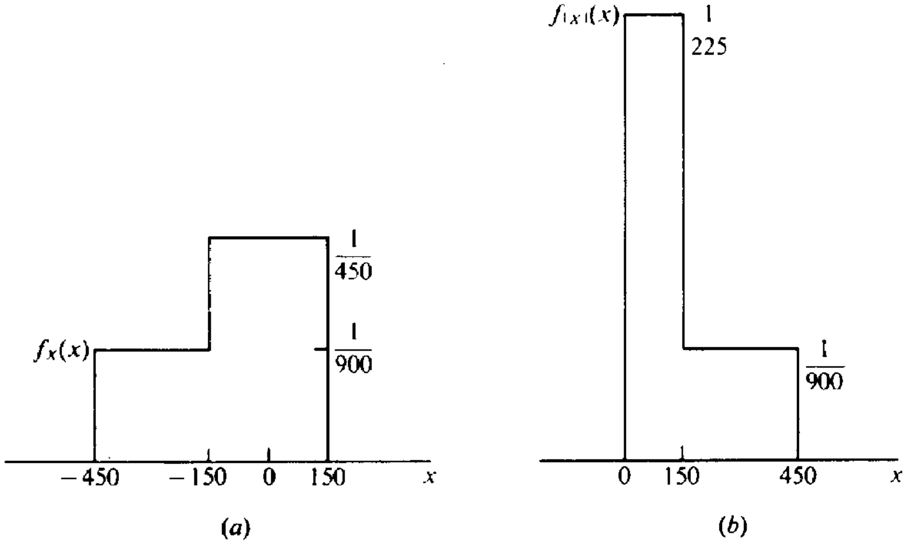


Figure 18.10 Probability density functions of (*a*) X and (*b*) $|X|$ for a tool crib at Location 2 of Fig. 18.7 under Alternative 2 (no other tool cribs).

Hence

$$E\{|X|\} = \int_0^{450} x f_{|X|}(x)\, dx$$

$$= \int_0^{150} x \left(\frac{1}{225}\right) dx + \int_{150}^{450} x \left(\frac{1}{900}\right) dx$$

$$= \frac{150^2}{450} + \frac{450^2 - 150^2}{1{,}800} = 150.$$

We suggest that you now try the same approach (using the *width* of the area rather than the height) to derive $E\{|Y|\}$. You will find that the probability distribution of $|Y|$ is *identical* to that for $|X|$, so $E\{|Y|\} = 150$. As a result,

$$E(T) = \frac{2}{15{,}000}(150 + 150)$$
$$= 0.04 \text{ hr.}$$

Alternative 3 ($n = 2$): With tool cribs in just Locations 1 and 3, the areas assigned to them would be divided by a line segment between (300, 300) and (600, 0) in Fig. 18.7. Notice that the two areas and their tool cribs are located symmetrically with respect to this line segment. Therefore, $E(T)$ is the same for both, so we shall derive it just for the tool crib in Location 1. (You might try it for the other tool crib for practice—see Prob. 18 S-1.)

Proceeding just as for Alternative 2, relabel Location 1 as the origin (0, 0) for an (x, y) coordinate system, so that 150 would be subtracted from all coordinates shown in Fig. 18.7. This relabeling leads directly to the probability density function of X, and then of $|X|$, shown in Fig. 18.11. As a result,

$$E\{|X|\} = \frac{1}{225}\int_0^{150} x\,dx + \frac{1}{300}\int_{150}^{450}\left(1 - \frac{x}{450}\right)x\,dx$$
$$= \frac{1}{225}\left[\frac{x^2}{2}\right]_0^{150} + \frac{1}{300}\left[\frac{x^2}{2} - \frac{x^3}{1{,}350}\right]_{150}^{450}$$
$$= \frac{1}{225}\frac{150^2}{2} + \frac{1}{300}\left(\frac{450^2}{2} - \frac{450^3}{1{,}350}\right) - \frac{1}{300}\left(\frac{150^2}{2} - \frac{150^3}{1{,}350}\right)$$
$$= 133\tfrac{1}{3}.$$

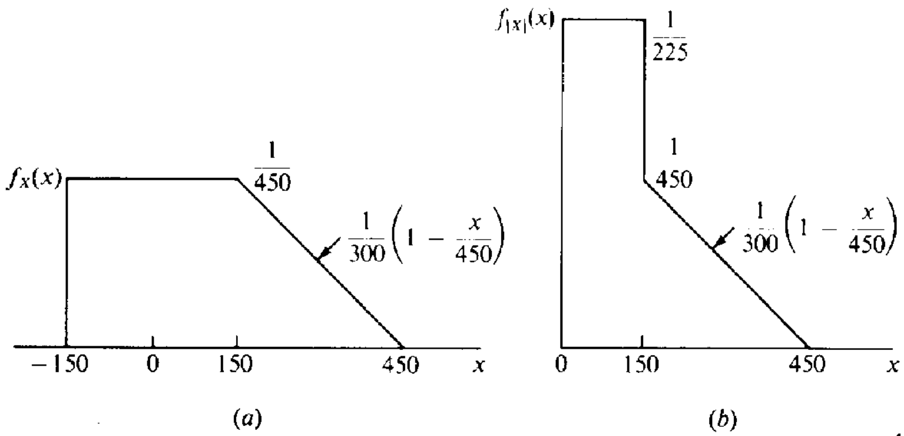


Figure 18.11 Probability density functions of (*a*) X and (*b*) $|X|$ for a tool crib at Location 1 of Fig. 18.7 under Alternative 3 (the only other tool crib is at Location 3).

Next, the probability density function of Y is obtained by using the *width* of the area assigned to the tool crib at Location 1 for each possible value of $Y = y$ and then dividing by the size of the area, as given in Fig. 18.12*a*. This result then yields the *uniform* distribution of $|Y|$ shown in Fig. 18.12*b*. Thus

$$E\{|Y|\} = \frac{1}{150}\int_0^{150} y\,dy$$

$$= 75.$$

Consequently,

$$E(T) = \frac{2}{15{,}000}\left(133\tfrac{1}{3} + 75\right)$$

$$= 0.0278 \text{ hr.}$$

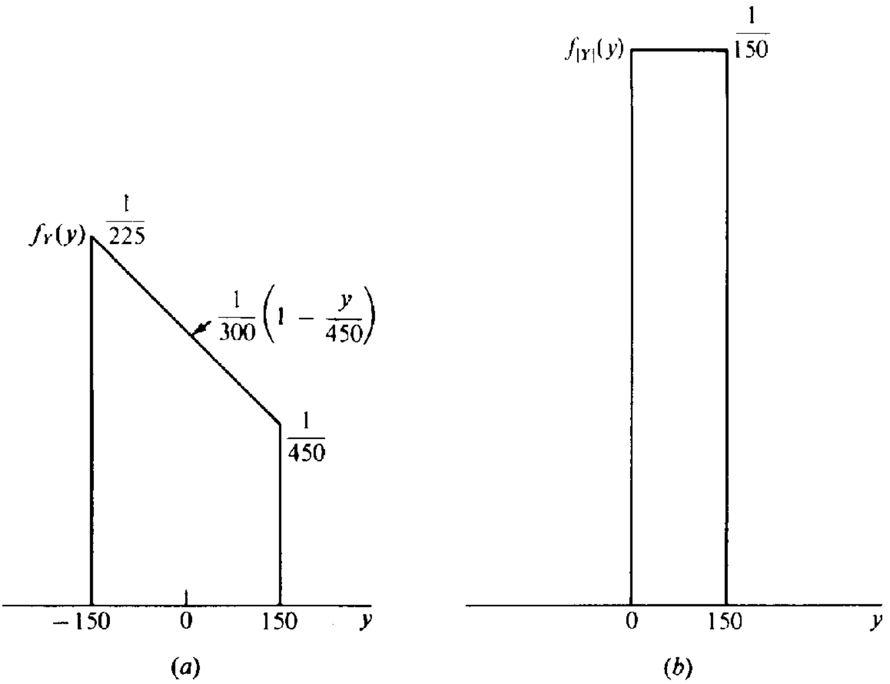


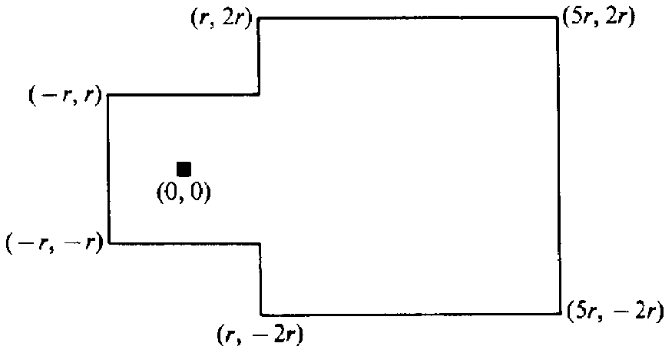
Figure 18.12 Probability density functions of (*a*) Y and (*b*) $|Y|$ for a tool crib at Location 1 of Fig. 18.7 under Alternative 3 (the only other tool crib is at Location 3).

Applying Model 3: Because $E(T)$ now has been evaluated for the three alternatives under consideration, the stage is set for using model 3 from Sec. 18.4 to choose among these alternatives. This is what was done at the end of Sec. 18.4.

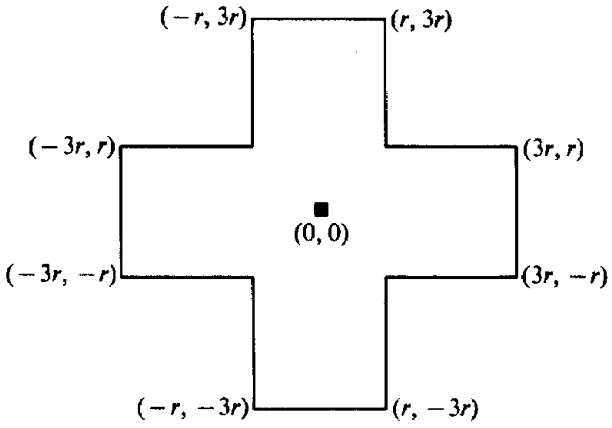
185-1. Consider *Alternative 3* (tool cribs in Locations 1 and 3) for the example illustrated in Fig. 18.7. Derive $E(T)$ for the tool crib in Location 3 by using the probability density functions of X and Y directly for this tool crib.

185-2. Suppose that the calling population for a particular service facility is uniformly distributed over *each* area shown, where the service facility is located at (0, 0). Making the same assumptions as heretofore, derive the expected round-trip travel time per arrival $E(T)$ in terms of the average velocity v and the distance r.

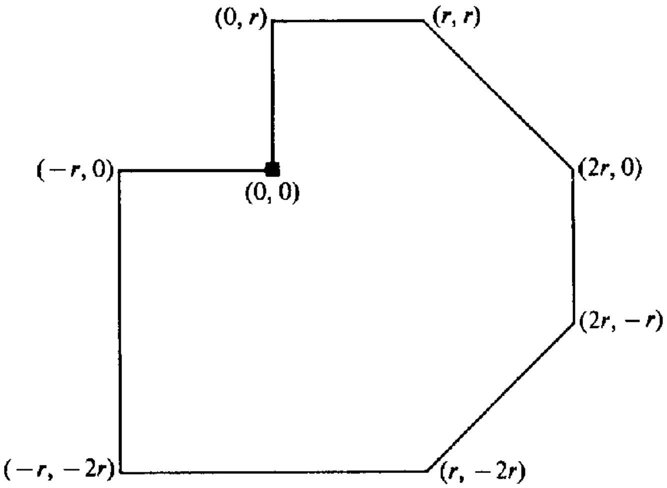
(*a*)



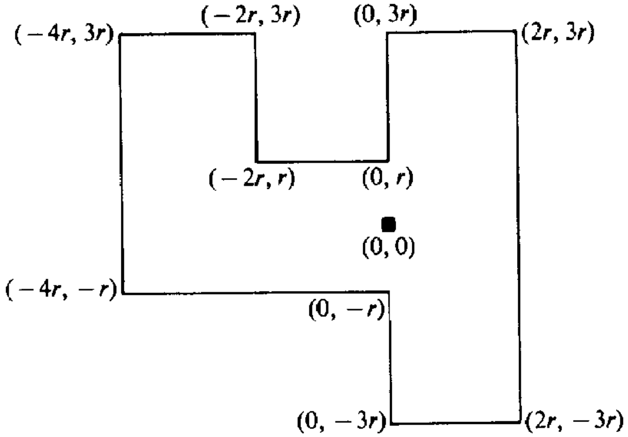
(*b*)



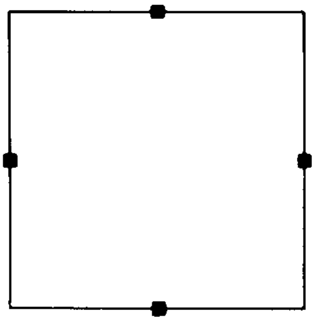
(c)


(0, r)
(r, r)
(−r, 0)
(0, 0)
(2r, 0)
(2r, −r)
(−r, −2r)
(r, −2r)

(d)


(−4r, 3r)
(−2r, 3r)
(0, 3r)
(2r, 3r)
(−2r, r)
(0, r)
(0, 0)
(−4r, −r)
(0, −r)
(0, −3r)
(2r, −3r)

18S-3. A job shop is being laid out in a square area with 600 feet on a side, and one of the decisions to be made is the *number* of facilities for the storage and shipping of final inventory. The capitalized cost associated with providing each facility would be \$10/hour. There are just four potential locations available for these facilities, one in the middle of each of the four sides of the square area as shown in the figure.



The loads to be moved to a storage and shipping facility would be distributed uniformly throughout the shop area, and they become available according to a *Poisson* process at a mean rate of 90 per hour. Each time a load becomes available, an appropriate materials-handling vehicle would be sent from the *nearest* facility to pick it up (with an expected loading time of 3 minutes) and bring it there, where the cost would be \$40/hour for time spent in traveling, loading, and waiting to be unloaded. The vehicles would travel at a speed of 20,000 feet per hour along a system of orthogonal aisles parallel to the sides of the shop area.

Another decision to be made is the number of men (m) to provide at each storage and shipping facility for unloading an arriving vehicle. These m men would work together on each vehicle, and the time required to unload it would have an *exponential* distribution, with a mean of $2/m$ minutes. The cost of providing each man is \$15/hour.

Determine the number of facilities and the value of m at each that will minimize expected total cost per hour.