

Factoring a Quadratic Trinomial with Leading Coefficient Greater than 1

• Example 1

Factor the quadratic expression

$$6x^2 + x - 2$$

Suppose that, for some integers a , b , m , and n ,

$$6x^2 + x - 2 = (ax + m)(bx + n) = abx^2 + (an + bm)x + mn.$$

We have thus

$$ab = 6, \quad an + bm = 1, \quad \text{and} \quad mn = -2$$

Which gives also, using the first and the last of the above equations

$$an \cdot bm = -12$$

Let us find integers an and bm satisfying both $an + bm = 1$ and $an \cdot bm = -12$.

The pairs of integers whose product is -12 are

$$-1, 12, \quad 1, -12$$

$$-2, 6, \quad 2, -6$$

$$-3, 4, \quad 3, -4$$

The only possibility is $an = 4$ and $bm = -3$. We obtain in succession,

$$\begin{aligned} 6x^2 + x - 2 &= 6x^2 + (4 - 3)x - 2 && \text{(because } an = 4 \text{ and } bm = -3) \\ &= 6x^2 + 4x - 3x - 2 \\ &= 2x(3x + 2) - (3x + 2) && \text{(gathering terms and factoring out } 2x) \\ &= (2x - 1)(3x + 2) && \text{(factoring out } 3x + 2) \end{aligned}$$

Warning. THIS METHOD DOES NOT ALWAYS WORK, AND IS PRACTICAL ONLY IF a , b , m , AND n ARE INTEGERS.

• • • CHECK YOURSELF 1

Factor.

$$4x^2 + 29x + 7$$

• • • CHECK YOURSELF ANSWER

1. $(x + 7)(4x + 1)$.

5.14 Exercises

Name _____

Section _____

Date _____

A N S W E R S

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

Factor.

1. $6x^2 + 19x + 10$

2. $6x^2 - 7x - 3$

3. $15x^2 + x - 6$

4. $12w^2 + 19w + 4$

5. $6m^2 + 25m - 25$

6. $8x^2 - 6x - 9$

7. $9x^2 - 12x + 4$

8. $20x^2 - 23x + 6$

9. $12x^2 - 8x - 15$

10. $16a^2 + 40a + 25$

11. $3y^2 + 7y - 6$

12. $12x^2 + 11x - 15$

13. $8x^2 - 27x - 20$

14. $24v^2 + 5v - 36$