

# 4.15

## Range of a Real Function

### • Example 1

Graph the range of the function

$$s(t) = 3t^2 - 8t + 4$$

on the real number line.

The function  $s$  is a quadratic function.

To find the minimum value of  $s$ , we use the fact that a quadratic function in the form

$$f(x) = ax^2 + bx + c, a, b, c \in \mathbb{R}, a \neq 0 \text{ attains a minimum at } x = -\frac{b}{2a} \text{ if } a > 0.$$

If  $a < 0$ ,  $f(x)$  attains a maximum at  $x = -\frac{b}{2a}$ .

Thus, at  $t = -\frac{-8}{2(3)} = \frac{4}{3}$ ,  $s$  attains a minimum value of

$$\begin{aligned} s\left(\frac{4}{3}\right) &= 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4 \\ &= \frac{16}{3} - \frac{32}{3} + 4 \\ &= -\frac{4}{3} \end{aligned}$$

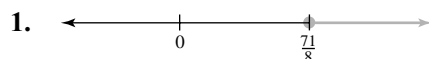
The range of the function is the set of all the real numbers greater than or equal to  $-\frac{4}{3}$ . The graph is given below.



### • • • CHECK YOURSELF

Graph the range of the function  $f(x) = 2x^2 - 3x + 10$  on the real number line.

### • • • CHECK YOURSELF ANSWER



# 4.15 Exercises

Name \_\_\_\_\_

Section \_\_\_\_\_

Date \_\_\_\_\_

## A N S W E R S

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

7. \_\_\_\_\_

8. \_\_\_\_\_

9. \_\_\_\_\_

10. \_\_\_\_\_

Graph the range of each of the functions below on the real number line.

1.  $h(t) = t^2 - 2t + 9$

2.  $k(y) = y^2 + 6y - 5$

3.  $w(h) = 2h^2 - 8h - 3$

4.  $f(x) = 3x^2 - 7x - 2$

5.  $g(s) = s^2 - 4s + 10$

6.  $p(q) = 4q^2 - 4q - 1$

7.  $s(t) = -16t^2 + 32t$

8.  $h(v) = -2v^2 - v + 1$

9.  $l(p) = -p^2 - p + 7$

10.  $m(y) = -3y^2 - 8y - 2$