

4.21

Composition of Two Functions

• Example 1

Let $f(x) = \log x$ and $h(t) = t^3$.

(a) Find $(h \circ f)(x)$.

By definition, we have

$$\begin{aligned}(h \circ f)(x) &= h(f(x)) \\ &= h(\log x)^3 \\ &= \log^3 x\end{aligned}$$

(b) Find the domain and range of $(h \circ f)(x)$

The range of f is the set \mathbb{R} of real numbers, which is also the domain of h . Thus the domain of $h \circ f$ is the domain of f and the range of $h \circ f$ is the range of h . The domain of $h \circ f$ is $\{x \in \mathbb{R} \mid x > 0\}$ and the range of $h \circ f$ is \mathbb{R} .

(c) Evaluate $h(f(\sqrt{10}))$.

$$h(f(\sqrt{10})) = h(\log \sqrt{10}) = h\left(\frac{1}{2} \log 10\right) = h\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

• • • CHECK YOURSELF 1

Let $f(x) = \log_5(x+1)$ and $g(t) = t^2 + 1$.

- Find $(g \circ f)(x) = g(f(x))$.
- Find the domain and range of $g \circ f$.
- Evaluate $g(f(4))$

• • • CHECK YOURSELF ANSWER

- (a) $g(f(x)) = \log_5^2(x+1) + 1$; (b) Domain: $x > -1$, Range: $-\infty < x < \infty$;
(c) $g(f(4)) = 2$.

4.21 Exercises

Name _____

Section _____

Date _____

A N S W E R S

1. _____

2. _____

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4. _____

5. _____

6. _____

For each of the following pairs of functions, find (a) the given composition, (b) the domain and range of the composition, and (c) evaluate the composition at the given number.

1. $g(x) = \sqrt{x}$, $f(t) = t + 1$
Find $g(f(t))$ and evaluate $g(f(8))$.

2. $s(t) = 2^t$, $h(y) = y^2$
Find $s(h(y))$ and evaluate $s(h(2))$.

3. $u(v) = \log_3(v - 1)$, $w(x) = \sqrt[3]{x}$
Find $w(u(v))$ and evaluate $w(u(10))$.

4. $f(x) = x^3$, $h(z) = \log_3 \sqrt{z}$
Find $h(f(x))$ and evaluate $h(f(3))$.

5. $v(y) = \log_2 y$, $w(z) = \frac{z}{z + 1}$
Find $v(w(z))$ and evaluate $v(w(1))$.

6. $n(s) = 3^s$, $m(t) = \sqrt{t} + 1$
Find $n(m(t))$ and evaluate $n(m(1))$.