

Range of a Real Function

• Example 1

Graph the range of the function

$$s(t) = 3t^2 - 8t + 4$$

on the real number line.

The function s is a quadratic function.

To find the minimum value of s , we use the fact that a quadratic function in the form

$f(x) = ax^2 + bx + c$, $a, b, c \in \mathbb{R}$, $a \neq 0$ attains a minimum at $x = -\frac{b}{2a}$ if $a > 0$.

If $a < 0$, $f(x)$ attains a maximum at $x = -\frac{b}{2a}$.

Thus, at $t = -\frac{-8}{2(3)} = \frac{4}{3}$, s attains a minimum value of

$$\begin{aligned} s\left(\frac{4}{3}\right) &= 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4 \\ &= \frac{16}{3} - \frac{32}{3} + 4 \\ &= -\frac{4}{3} \end{aligned}$$

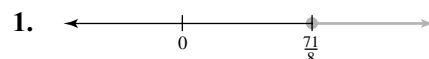
The range of the function is the set of all the real numbers greater than or equal to $-\frac{4}{3}$. The graph is given below.



• • • CHECK YOURSELF

Graph the range of the function $f(x) = 2x^2 - 3x + 10$ on the real number line.

• • • CHECK YOURSELF ANSWER



4.15 Exercises

Name _____

Section _____

Date _____

A N S W E R S

1. _____

2. _____

3. _____

4. _____

5. _____

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8. _____

9. _____

10. _____

Graph the range of each of the functions below on the real number line.

1. $h(t) = t^2 - 2t + 9$

2. $k(y) = y^2 + 6y - 5$

3. $w(h) = 2h^2 - 8h - 3$

4. $f(x) = 3x^2 - 7x - 2$

5. $g(s) = s^2 - 4s + 10$

6. $p(q) = 4q^2 - 4q - 1$

7. $s(t) = -16t^2 + 32t$

8. $h(v) = -2v^2 - v + 1$

9. $l(p) = -p^2 - p + 7$

10. $m(y) = -3y^2 - 8y - 2$