where \( x \) is the number of years since 1980 and \( y \) is the amount of emission in thousands of metric tons (Energy Information Administration, www.eia.doe.gov).

a) In what years was the emission of CFC-12 gas 106 thousand metric tons?

1983, 1991

b) In what year will the emission of CFC-12 gas be zero?

1999

64. **Lottery tickets.** The formula \( R = -200x^2 + 5000x \) was used in Exercise 74 of Section 9.2 to predict the revenue when lottery tickets are sold for \( x \) dollars each. For what ticket price is the revenue $25,000?

In this section we will solve problems that involve quadratic equations.

**Geometric Applications**

Quadratic equations can be used to solve problems involving area.

**EXAMPLE 1**

The length of a rectangular flower bed is 2 feet longer than the width. If the area is 6 square feet, then what are the exact length and width? Also find the approximate dimensions of the rectangle to the nearest tenth of a foot.

**Solution**

Let \( x \) represent the width, and \( x + 2 \) represent the length as shown in Fig. 9.2. Write an equation using the formula for the area of a rectangle, \( A = LW \):

\[
x(x + 2) = 6 \quad \text{The area is 6 square feet.}
\]

\[
x^2 + 2x - 6 = 0
\]

We use the quadratic formula to solve the equation:

\[
x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)} = \frac{-2 \pm \sqrt{28}}{2}
\]

\[
= \frac{-2 \pm 2\sqrt{7}}{2} = \frac{2(-1 \pm \sqrt{7})}{2} = -1 \pm \sqrt{7}
\]

Because \(-1 - \sqrt{7}\) is negative, it cannot be the width of a rectangle. If \(x = -1 + \sqrt{7}\), then

\[
x + 2 = -1 + \sqrt{7} + 2 = 1 + \sqrt{7}.
\]

So the exact width is \(-1 + \sqrt{7}\) feet, and the exact length is \(1 + \sqrt{7}\) feet. We can check that these dimensions give an area of 6 square feet as follows:

\[
LW = (1 + \sqrt{7})(-1 + \sqrt{7}) = -1 - \sqrt{7} + \sqrt{7} + 7 = 6
\]

Use a calculator to find the approximate dimensions of 1.6 and 3.6 feet.

**Work Problems**

The work problems in this section are similar to the work problems that you solved in Chapter 5. However, you will need the quadratic formula to solve the work problems presented in this section.
EXAMPLE 2

Working together

Amy can mow the lawn by herself in 2 hours less time than Bob takes to mow the lawn by himself. When they work together, it takes them only 6 hours to mow the lawn. How long would it take each of them to mow the lawn working alone? Find the exact and approximate answers.

Solution

If \(x\) is the number of hours it takes Amy by herself to mow the lawn, then Amy mows at the rate of \(\frac{1}{x}\) lawn per hour. If \(x + 2\) is the number of hours it takes Bob to mow the lawn by himself, then Bob mows at the rate of \(\frac{1}{x + 2}\) lawn per hour. Make a table using the fact that the product of the rate and the time gives the amount of work completed (or the fraction of the lawn mowed).

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Amount of work</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{x}) lawn/hr</td>
<td>6 hr</td>
<td>(\frac{6}{x}) lawn</td>
</tr>
<tr>
<td>(\frac{1}{x + 2}) lawn/hr</td>
<td>6 hr</td>
<td>(\frac{6}{x + 2}) lawn</td>
</tr>
</tbody>
</table>

Because the total amount of work done is 1 lawn, we can write the following equation:

\[
\frac{6}{x} + \frac{6}{x + 2} = 1
\]

\[
x(x + 2)\frac{6}{x} + x(x + 2)\frac{6}{x + 2} = x(x + 2)1
\]

Multiply by the LCD.

\[
6x + 12 + 6x = x^2 + 2x
\]

\[
12x + 12 = x^2 + 2x
\]

\[
-x^2 + 10x + 12 = 0
\]

\[
x^2 - 10x - 12 = 0
\]

Multiply each side by \(-1\).

Use the quadratic formula with \(a = 1\), \(b = -10\), and \(c = -12\):

\[
x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-12)}}{2(1)}
\]

\[
x = \frac{10 \pm \sqrt{144}}{2} = \frac{10 \pm 2\sqrt{37}}{2} = 5 \pm \sqrt{37}
\]

Use a calculator to find that

\[x = 5 - \sqrt{37} \approx -1.08 \quad \text{and} \quad x = 5 + \sqrt{37} \approx 11.08.\]

Because \(x\) must be positive, Amy’s time alone is \(5 + \sqrt{37}\), or approximately 11.1 hours. Because Bob’s time alone is 2 hours more than Amy’s, Bob’s time is \(7 + \sqrt{37}\) or approximately 13.1 hours.

Vertical Motion

If an object is projected upward or downward with an initial velocity of \(v_0\) feet per second from an altitude of \(s_0\) feet, then its altitude \(s\) in feet after \(t\) seconds is
given by the formula
\[ s = -16t^2 + v_0t + s_0. \]
We use this formula in the next example.

**EXAMPLE 3**

**Vertical motion**

A soccer ball bounces straight up into the air off of the head of a soccer player from an altitude of 6 feet with an initial velocity of 40 feet per second. How long does it take the ball to reach the earth? Find the exact answer and an approximate answer.

**Solution**

The time that it takes the ball to reach the earth is the value of \( t \) for which \( s \) has a value of 0 in the formula
\[ s = -16t^2 + v_0t + s_0. \]
To find \( t \), we use \( s_0 = 6 \), \( v_0 = 40 \), and \( s_0 = 6 \):

\[
0 = -16t^2 + 40t + 6
\]

\[
16t^2 - 40t - 6 = 0
\]

\[
8t^2 - 20t - 3 = 0 \quad \text{Divide each side by 2.}
\]

\[
t = \frac{20 \pm \sqrt{(-20)^2 - 4(8)(-3)}}{2(8)}
\]

\[
t = \frac{20 \pm \sqrt{496}}{16} = \frac{20 \pm 4\sqrt{31}}{16}
\]

\[
t = \frac{5 \pm \sqrt{31}}{4}
\]

Because the time must be positive, we have

\[
t = \frac{5 + \sqrt{31}}{4} \approx 2.64 \text{ seconds.}
\]

It takes the ball \( \frac{5 + \sqrt{31}}{4} \) or 2.64 seconds to reach the earth.

---

**WARM-UPS**

**True or false? Explain your answer.**

1. Two numbers that have a sum of 10 are represented by \( x \) and \( x + 10 \).

2. The area of a right triangle is one-half the product of the lengths of the legs.

3. If the speed of a boat in still water is \( x \) mph and the current is 5 mph, then the speed of the boat with the current is \( 5x \) mph.

4. If Boudreaux eats a 50-pound bag of crawfish in \( x \) hours, then his eating rate is \( \frac{50}{x} \) bag/hr.

5. If the Concorde flew 1800 miles in \( x + 2 \) hours, then its average speed was \( \frac{1800}{x + 2} \) mph.

6. The quantity \( \frac{7 - \sqrt{50}}{2} \) is negative.

7. The quantity \( (-5 + \sqrt{27}) \) is positive.
(continued)

8. If the length of one side of a square is \( x + 9 \) meters, then the area of the square is \( x^2 + 81 \) square meters.

9. If Julia mows an entire lawn in \( x \) hours, then her mowing rate is \( \frac{1}{x} \) lawn/hr.

10. If John’s boat goes 20 miles per hour in still water, then against a 5-mph current it will go 15 miles per hour.

9.4 Exercises

Find the exact solution to each problem. See Example 1.

1. **Length and width.** The length of a rectangle is 2 meters longer than the width. If the area is 10 square meters, then what are the length and width?

2. **Unequal legs.** One leg of a right triangle is 4 centimeters longer than the other leg. If the area of the triangle is 8 square centimeters, then what are the lengths of the legs?

3. **Bracing a gate.** If the diagonal brace of the square gate shown in the figure is 8 feet long, then what is the length of a side of the square gate?

4. **Dimensions of a rectangle.** If one side of a rectangle is 2 meters shorter than the other side and the diagonal is 10 meters long, then what are the dimensions of the rectangle?

5. **Area of a parallelogram.** The base of a parallelogram is 6 inches longer than its height. If the area of the parallelogram is 10 square inches, then what are the base and height?

6. **Positive numbers.** Find two positive real numbers that have a sum of 8 and a product of 4.

Solve each problem. Give the exact answer and an approximate answer rounded to two decimal places. See Examples 2 and 3.

7. **In the berries.** On Monday, Alberta picked the strawberry patch, and Ernie sold the berries. On Tuesday, Ernie picked and Alberta sold, but it took him 2 hours longer to get the berries picked than it took Alberta. On Wednesday they worked together and got all of the berries picked in 2 hours. How long did it take Ernie to pick the berries by himself?

8. **Meter readers.** Claude and Melvin read the water meters for the city of Ponchatoula. When Claude reads all of the meters by himself, it takes him a full day longer than it takes Melvin to read all of the meters by himself. If they
can get the job done working together in 2 days, then how long does it take Claude by himself?

9. **Hanging wallpaper.** Working alone, Tasha can hang all of the paper in the McLendons’ new house in 8 hours less time than it takes Tena working alone. Working together, they completed the job in 20 hours. How long would it take Tasha working alone?

10. **Laying bricks.** Chau’s team of bricklayers can lay all of the bricks in the McLendons’ new house in 3 working days less than Hong’s team. To speed things up, the McLendons hire both teams and get the job done in 10 working days. How many working days do the McLendons save by using both teams rather than just the faster team?

11. **Hang time.** A punter kicks a football straight up from a height of 4 feet with an initial velocity of 60 feet per second. How long will it take the ball to reach the earth?

12. **Hunting accident.** Dwight accidentally fired his rifle straight into the air while sitting in his deer stand 30 feet off the ground. If the bullet left the barrel with a velocity of 200 feet per second, then how long did it take the bullet to fall to the earth?

13. **Going up.** A ball is tossed into the air at 20 feet per second from a height of 5 feet. How long (to the nearest tenth of a second) will it take the ball to reach the ground?

14. **Going down.** A comedian throws a watermelon downward at 30 feet per second from a height of 200 feet. How long (to the nearest tenth of a second) will it take the watermelon to reach the ground? (The initial velocity of the watermelon is negative.)

15. **Gone fishing.** Nancy traveled 6 miles upstream to do some fly fishing. It took her 20 minutes longer to get there than to return. If the current in the river is 2 miles per hour, then how fast will her boat go in still water?

16. **Commuting to work.** Gladys and Bonita commute to work daily. Bonita drives 40 miles and averages 9 miles per hour more than Gladys. Gladys drives 50 miles, and she is on the road one-half hour longer than Bonita. How fast does each of them drive?

17. **Expanding garden.** Olin’s garden is 5 feet wide and 8 feet long. He bought enough okra seed to plant 100 square feet in okra. If he wants to increase the width and the length by the same amount to plant all of his okra, then what should the increase be?

18. **Spring flowers.** Lillian has a 5-foot-square bed of tulips. She plans to surround this bed with a crocus bed of uniform width. If she has enough crocus bulbs to plant 100 square feet of crocus, then how wide should the crocus bed be?