92. **Master’s degrees.** In 1985, 15.9% of all degrees awarded in U.S. higher education were master’s degrees (National Center for Education Statistics). If the formulas \( M = 7.79n + 287.87 \) and \( T = 30.95n + 1,808.22 \) give the number of master’s degrees and the total number of higher education degrees awarded in thousands, respectively, in the year \( 1985 + n \), then what is the first year in which more than 20% of all degrees awarded will be master’s degrees?

93. **Weighted average.** Professor Jorgenson gives only a midterm exam and a final exam. The semester average is computed by taking \( \frac{2}{3} \) of the midterm exam score plus \( \frac{1}{3} \) of the final exam score. The grade is determined from the grading scale given in the table. If Stanley scored only 56 on the midterm, then for what range of scores on the final exam would he get a C or better in the course?

<table>
<thead>
<tr>
<th>Grading</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>90–100</td>
<td>A</td>
</tr>
<tr>
<td>80–89</td>
<td>B</td>
</tr>
<tr>
<td>70–79</td>
<td>C</td>
</tr>
<tr>
<td>60–69</td>
<td>D</td>
</tr>
</tbody>
</table>

**TABLE FOR EXERCISES 93 AND 94**

94. **C or better.** Professor Brown counts her midterm as \( \frac{2}{3} \) of the grade and her final as \( \frac{1}{3} \) of the grade. Wilbert scored only 56 on the midterm. If Professor Brown also uses the grading scale given in the table, then what range of scores on the final exam would give Wilbert a C or better in the course?

95. **Designer jeans.** A pair of ordinary jeans at A-Mart costs \$50 less than a pair of designer jeans at Enrico’s. In fact, you can buy four pairs of A-Mart jeans for less than one pair of Enrico’s jeans. What is the price range for a pair of A-Mart jeans?

96. **United Express.** Al and Rita both drive parcel delivery trucks for United Express. Al averages 20 mph less than Rita. In fact, Al is so slow that in 5 hours he covered fewer miles than Rita did in 3 hours. What are the possible values for Al’s rate of speed?

**GETTING MORE INVOLVED**

97. **Discussion.** If 3 is added to every number in \((4, \infty)\), the resulting set is \((7, \infty)\). In each of the following cases, write the resulting set of numbers in interval notation. Explain your results.

- a) The number \(-6\) is subtracted from every number in \([2, \infty)\).
- b) Every number in \((-\infty, -3)\) is multiplied by 2.
- c) Every number in \((8, \infty)\) is divided by 4.
- d) Every number in \((6, \infty)\) is multiplied by \(-2\).
- e) Every number in \((-\infty, -10)\) is divided by \(-5\).

98. **Writing.** Explain why saying that \(x\) is at least 9 is equivalent to saying that \(x\) is greater than or equal to 9. Explain why saying that \(x\) is at most 5 is equivalent to saying that \(x\) is less than or equal to 5.

---

**2.5 COMPOUND INEQUALITIES**

In this section we will use the ideas of union and intersection from Chapter 1 along with our knowledge of inequalities from Section 2.4 to work with compound inequalities.

**Basics**

The inequalities that we studied in Section 2.4 are referred to as **simple inequalities.** If we join two simple inequalities with the connective “and” or the connective “or,” we get a **compound inequality.** A compound inequality using the connective “and” is true if and only if both simple inequalities are true.
EXAMPLE 1

Compound inequalities using the connective “and”

Determine whether each compound inequality is true.

a) \(3 > 2\) and \(3 < 5\)

b) \(6 > 2\) and \(6 < 5\)

Solution

a) The compound inequality is true because \(3 > 2\) is true and \(3 < 5\) is true.

b) The compound inequality is false because \(6 < 5\) is false.

A compound inequality using the connective “or” is true if one or the other or both of the simple inequalities are true. It is false only if both simple inequalities are false.

EXAMPLE 2

Compound inequalities using the connective “or”

Determine whether each compound inequality is true.

a) \(2 < 3\) or \(2 > 7\)

b) \(4 < 3\) or \(4 \geq 7\)

Solution

a) The compound inequality is true because \(2 < 3\) is true.

b) The compound inequality is false because both \(4 < 3\) and \(4 \geq 7\) are false.

If a compound inequality involves a variable, then we are interested in the solution set to the inequality. The solution set to an “and” inequality consists of all numbers that satisfy both simple inequalities, whereas the solution set to an “or” inequality consists of all numbers that satisfy at least one of the simple inequalities.

EXAMPLE 3

Solutions of compound inequalities

Determine whether \(5\) satisfies each compound inequality.

a) \(x < 6\) and \(x < 9\)

b) \(2x - 9 \leq 5\) or \(-4x \geq -12\)

Solution

a) Because \(5 < 6\) and \(5 < 9\) are both true, \(5\) satisfies the compound inequality.

b) Because \(2 \cdot 5 - 9 \leq 5\) is true, it does not matter that \(-4 \cdot 5 \geq -12\) is false. So \(5\) satisfies the compound inequality.

Graphing the Solution Set

The solution set to a compound inequality such as

\[
x > 2 \quad \text{and} \quad x < 5
\]

consists of all numbers that are in the solution sets to both simple inequalities. So the solution set to this compound inequality is the intersection of those two solution sets. In symbols,

\[
\{x \mid x > 2 \text{ and } x < 5\} = \{x \mid x > 2\} \cap \{x \mid x < 5\}.
\]
**Example 4**

**Graphing compound inequalities**

Graph the solution set to the compound inequality \( x > 2 \) and \( x < 5 \).

**Solution**

We first sketch the graph of \( x > 2 \) and then the graph of \( x < 5 \), as shown in the top two number lines in Fig. 2.15. The intersection of these two solution sets is the portion of the number line that is shaded on both graphs, just the part between 2 and 5, not including the endpoints. The graph of \( \{ x \mid x > 2 \text{ and } x < 5 \} \) is shown at the bottom of Fig. 2.15. We write this set in interval notation as \((2, 5)\).

![Figure 2.15](image)

The solution set to a compound inequality such as
\[
x > 4 \quad \text{or} \quad x < -1
\]
consists of all numbers that satisfy one or the other or both of the simple inequalities. So the solution set to the compound inequality is the union of the solution sets to the simple inequalities. In symbols,
\[
\{ x \mid x > 4 \text{ or } x < -1 \} = \{ x \mid x > 4 \} \cup \{ x \mid x < -1 \}.
\]

**Example 5**

**Graphing compound inequalities**

Graph the solution set to the compound inequality \( x > 4 \) or \( x < -1 \).

**Solution**

To find the union of the solution sets to the simple inequalities, we sketch their graphs as shown at the top of Fig. 2.16. We graph the union of these two sets by putting both shaded regions together on the same line as shown in the bottom graph in Fig. 2.16. This set is written in interval notation as \((-\infty, -1) \cup (4, \infty)\).

![Figure 2.16](image)
2.5 Compound Inequalities

When graphing the intersection of two simple inequalities, do not draw too much. For the intersection, graph only numbers that satisfy both inequalities. Omit numbers that satisfy one but not the other inequality. Graphing a union is usually easier because we can simply draw both solution sets on the same number line.

It is not always necessary to graph the solution set to each simple inequality before graphing the solution set to the compound inequality. We can save time and work if we learn to think of the two preliminary graphs but draw only the final one.

**Example 6** Overlapping intervals

Sketch the graph and write the solution set in interval notation to each compound inequality.

**a)** $x < 3$ and $x < 5$

**b)** $x > 4$ or $x > 0$

**Solution**

**a)** To graph $x < 3$ and $x < 5$, we shade only the numbers that are both less than 3 and less than 5. So numbers between 3 and 5 are not shaded in Fig. 2.17. The compound inequality $x < 3$ and $x < 5$ is equivalent to the simple inequality $x < 3$. The solution set can be written as $(-\infty, 3)$.

**b)** To graph $x > 4$ or $x > 0$, we shade both regions on the same number line as shown in Fig. 2.18. The compound inequality $x > 4$ or $x > 0$ is equivalent to the simple inequality $x > 0$. The solution set is $(0, \infty)$.

**Example 7** All or nothing

Sketch the graph and write the solution set in interval notation to each compound inequality.

**a)** $x < 2$ and $x > 6$

**b)** $x < 3$ or $x > 1$

**Solution**

**a)** A number satisfies $x < 2$ and $x > 6$ if it is both less than 2 and greater than 6. There are no such numbers. The solution set is the empty set, $\emptyset$.

**b)** To graph $x < 3$ or $x > 1$, we shade both regions on the same number line as shown in Fig. 2.19. Since the two regions cover the entire line, the solution set is the set of all real numbers $(-\infty, \infty)$. 
If we start with a more complicated compound inequality, we first simplify each part of the compound inequality and then find the union or intersection.

**Example 8**

**Intersection**

Solve \( x + 2 > 3 \) and \( x - 6 < 7 \). Graph the solution set.

**Solution**

First simplify each simple inequality:

\[
\begin{align*}
5/1002 &> 3/1002 & \quad \text{and} & \quad x - 6 + 6 < 7 + 6 \\
5/1002 &> 1 & \quad \text{and} & \quad x < 13
\end{align*}
\]

The intersection of these two solution sets is the set of numbers between (but not including) 1 and 13. Its graph is shown in Fig. 2.20. The solution set is written in interval notation as \((1, 13)\).

**Example 9**

**Union**

Graph the solution set to the inequality

\[
5 - 7x \geq 12 \quad \text{or} \quad 3x - 2 < 7.
\]

**Solution**

First solve each of the simple inequalities:

\[
\begin{align*}
5 - 7x - 5 &\geq 12 - 5 \quad \text{or} \quad 3x - 2 + 2 < 7 + 2 \\
-7x &\geq 7 \quad \text{or} \quad 3x < 9 \\
x &\leq -1 \quad \text{or} \quad \quad \quad \quad \quad \quad \quad \quad x < 3
\end{align*}
\]

The union of the two solution intervals is \((-\infty, 3)\). The graph is shown in Fig. 2.21.

An inequality may be read from left to right or from right to left. Consider the inequality \( 1 < x \). If we read it in the usual way, we say, “1 is less than \( x \).” The meaning is clearer if we read the variable first. Reading from right to left, we say, “\( x \) is greater than 1.”

Another notation is commonly used for the compound inequality

\[
x > 1 \quad \text{and} \quad x < 13.
\]

This compound inequality can also be written as

\[
1 < x < 13.
\]
Reading from left to right, we read $1 < x < 13$ as “1 is less than $x$ is less than 13.” The meaning of this inequality is clearer if we read the variable first and read the first inequality symbol from right to left. Reading the variable first, $1 < x < 13$ is read as “$x$ is greater than 1 and less than 13.” So $x$ is between 1 and 13, and reading $x$ first makes it clear.

**CAUTION** We write $a < x < b$ only if $a < b$, and we write $a > x > b$ only if $a > b$. Similar rules hold for $\leq$ and $\geq$. So $4 < x < 9$ and $-6 \geq x \geq -8$ are correct uses of this notation, but $5 < x < 2$ is not correct. Also, the inequalities should not point in opposite directions as in $5 < x > 7$.

**Example 10**

Another notation

Solve the inequality and graph the solution set:

$$-2 \leq 2x - 3 < 7$$

**Solution**

This inequality could be written as the compound inequality

$$2x - 3 \geq -2 \quad \text{and} \quad 2x - 3 < 7.$$  

However, there is no need to rewrite the inequality because we can solve it in its original form.

$$\begin{align*}
-2 + 3 & \leq 2x - 3 + 3 < 7 + 3 \\
1 & \leq 2x \leq 10 \\
\frac{1}{2} & \leq x < 5
\end{align*}$$  

Add 3 to each part.

$$\begin{align*}
\frac{1}{2} & \leq 2x < \frac{10}{2} \\
\frac{1}{2} & \leq x < 5
\end{align*}$$  

Divide each part by 2.

The solution set is $\left[\frac{1}{2}, 5\right)$, and its graph is shown in Fig. 2.22.

**Example 11**

Solving a compound inequality

Solve the inequality $-1 < 3 - 2x < 9$ and graph the solution set.

**Solution**

$$\begin{align*}
-1 - 3 & < 3 - 2x < 9 - 3 \\
-4 & < -2x < 6 \\
2 & > x > -3 \\
-3 & < x < 2
\end{align*}$$  

Subtract 3 from each part of the inequality.

Divide each part by $-2$ and reverse both inequality symbols.

Rewrite the inequality with the smallest number on the left.

The solution set is $(-3, 2)$, and its graph is shown in Fig. 2.23.
Applications

When final exams are approaching, students are often interested in finding the final exam score that would give them a certain grade for a course.

EXAMPLE 12 Final exam scores

Fiana made a score of 76 on her midterm exam. For her to get a B in the course, the average of her midterm exam and final exam must be between 80 and 89 inclusive. What possible scores on the final exam would give Fiana a B in the course?

Solution

Let \( x \) represent her final exam score. Between 80 and 89 inclusive means that an average between 80 and 89 as well as an average of exactly 80 or 89 will get a B. So the average of the two scores must be greater than or equal to 80 and less than or equal to 89.

\[
80 \leq \frac{x + 76}{2} \leq 89
\]

\[
160 \leq x + 76 \leq 178 \quad \text{Multiply by 2.}
\]

\[
160 - 76 \leq x \leq 178 - 76 \quad \text{Subtract 76.}
\]

\[
84 \leq x \leq 102
\]

If Fiana scores between 84 and 102 inclusive, she will get a B in the course.

True or false? Explain your answer.

1. \( 3 < 5 \) and \( 3 \leq 10 \)
2. \( 3 < 5 \) or \( 3 < 10 \)
3. \( 3 > 5 \) and \( 3 < 10 \)
4. \( 3 \geq 5 \) or \( 3 \leq 10 \)
5. \( 4 < 8 \) and \( 4 > 2 \)
6. \( 4 < 8 \) or \( 4 > 2 \)
7. \( -3 < 0 < -2 \)
8. \( (3, \infty) \cup (8, \infty) = (8, \infty) \)
9. \( (3, \infty) \cup [8, \infty) = [8, \infty) \)
10. \( (-2, \infty) \cap (-\infty, 9) = (-2, 9) \)
2.5 **EXERCISES**

**Reading and Writing**  After reading this section, write out the answers to these questions. Use complete sentences.

1. What is a compound inequality?

2. When is a compound inequality using “and” true?

3. When is a compound inequality using “or” true?

4. How do we solve compound inequalities?

5. What is the meaning of $a < b < c$?

6. What is the meaning of $5 < x > 7$?

**Exercises**

**Determine whether each compound inequality is true.** See Examples 1 and 2.

7. $-6 < 5$ and $-6 > -3$

8. $3 < 5$ or $0 < -3$

9. $4 \leq 4$ and $-4 \leq 0$

10. $1 < 5$ and $1 > -3$

11. $6 < 5$ or $-4 > -3$

12. $4 \leq -4$ or $0 \leq 0$

**Determine whether $-4$ satisfies each compound inequality.** See Example 3.

13. $x < 5$ and $x > -3$

14. $x < 5$ or $x > -3$

15. $x - 3 \geq -7$ or $x + 1 > 1$

16. $2x \leq -8$ and $5x \leq 0$

17. $2x - 1 \leq -7$ or $2x > 18$

18. $-3x > 0$ and $3x - 4 < 11$

**Graph the solution set to each compound inequality.** See Examples 4–7.

19. $x > -1$ and $x < 4$

20. $x \leq 3$ and $x \leq 0$

21. $x \geq 2$ or $x \leq 5$

22. $x < -1$ or $x < 3$

23. $x \leq 6$ or $x > -2$

24. $x > -2$ and $x \leq 4$

25. $x \leq 6$ and $x > 9$

26. $x < 7$ or $x > 0$

27. $x \leq 6$ or $x > 9$

28. $x \geq 4$ and $x \leq -4$

29. $x \geq 6$ and $x \leq 1$

30. $x > 3$ or $x < -3$

31. $x - 3 > 7$ or $3 - x > 2$

32. $x - 5 > 6$ or $2 - x > 4$

33. $3 < x$ and $1 + x > 10$

34. $-0.3x < 9$ and $0.2x > 2$

35. $\frac{1}{2} > 5$ or $-\frac{1}{3} < 2$

36. $5 < x$ or $3 - \frac{1}{2}x < 7$

37. $2x - 3 \leq 5$ and $x - 1 > 0$

38. $\frac{3}{4}x < 9$ and $-\frac{1}{3}x \leq -15$

39. $\frac{1}{2}x - \frac{1}{3} \geq -\frac{1}{6} \text{ or } \frac{2}{3}x \leq \frac{1}{10}$

40. $\frac{1}{4}x - \frac{1}{3} > -\frac{1}{5} \text{ and } \frac{1}{2}x < 2$

41. $0.5x < 2$ and $-0.6x < -3$

42. $0.3x < 0.6$ or $0.05x > -4$
Solve each compound inequality. Write the solution set in interval notation and graph it. See Examples 10 and 11.

43. \(5 < 2x - 3 < 11\)

44. \(-2 < 3x + 1 < 10\)

45. \(-1 < 5 - 3x \leq 14\)

46. \(-1 \leq 3 - 2x < 11\)

47. \(-3 < \frac{3m + 1}{2} \leq 5\)

48. \(0 \leq \frac{3 - 2x}{2} < 5\)

49. \(-2 < \frac{1 - 3x}{-2} < 7\)

50. \(-3 < \frac{2x - 1}{3} < 7\)

51. \(3 \leq 3 - 5(x - 3) \leq 8\)

52. \(2 \leq 4 - \frac{1}{2}(x - 8) \leq 10\)

Write each union or intersection of intervals as a single interval if possible.

53. \((2, \infty) \cup (4, \infty)\)

54. \((-3, \infty) \cup (-6, \infty)\)

55. \((-\infty, 5) \cap (-\infty, 9)\)

56. \((-\infty, -2) \cap (-\infty, 1)\)

57. \((-\infty, 4] \cap [2, \infty)\)

58. \((-\infty, 8) \cap [3, \infty)\)

59. \((-\infty, 5) \cup [-3, \infty)\)

60. \((-\infty, -2] \cup (2, \infty)\)

61. \((3, \infty) \cap (-\infty, 3]\)

62. \([-4, \infty) \cap (-\infty, -6]\)

63. \((3, 5) \cap [4, 8)\)

64. \([-2, 4] \cap (0, 9]\)

65. \([1, 4) \cup (2, 6]\)

66. \([1, 3) \cup (0, 5]\)

Write either a simple or a compound inequality that has the given graph as its solution set.

67.

68. \([-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8]\)

69. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5]\)

70. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

71. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

72. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

73. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

74. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

75. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

76. \([-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6]\)

Solve each compound inequality and write the solution set using interval notation.

77. \(2 < x < 7 \text{ and } 2x > 10\)

78. \(3 < 5 - x < 8 \text{ or } -3x < 0\)

79. \(-1 < 3x + 2 \leq 5 \text{ or } \frac{3 - x}{2} - 6 > 9\)

80. \(0 < 5 - 2x \leq 10 \text{ and } -6 < 4 - x < 0\)

81. \(-3 < \frac{x - 1}{2} < 5 \text{ and } -1 < \frac{1 - x}{2} < 2\)

82. \(-3 < \frac{3x - 1}{5} < \frac{1}{2} \text{ and } \frac{1}{3} < \frac{3 - 2x}{6} < \frac{9}{2}\)

Solve each problem by using a compound inequality. See Example 12.

83. Aiming for a C. Professor Johnson gives only a midterm exam and a final exam. The semester average is computed by taking \(\frac{1}{3}\) of the midterm exam score plus \(\frac{2}{3}\) of the final exam score. To get a C, Beth must have a semester average between 70 and 79 inclusive. If Beth scored only 64 on the midterm, then for what range of scores on the final exam would Beth get a C?
84. **Two tests only.** Professor Davis counts his midterm as \( \frac{2}{3} \) of the grade, and his final as \( \frac{1}{3} \) of the grade. Jason scored only 64 on the midterm. What range of scores on the final exam would put Jason’s average between 70 and 79 inclusive?

85. **Keep on truckin’.** Abdul is shopping for a new truck in a city with an 8% sales tax. There is also an $84 title and license fee to pay. He wants to get a good truck and plans to spend at least $12,000 but no more than $15,000. What is the price range for the truck?

86. **Selling-price range.** Renee wants to sell her car through a broker who charges a commission of 10% of the selling price. The book value of the car is $14,900, but Renee still owes $13,104 on it. Although the car is in only fair condition and will not sell for more than the book value, Renee must get enough to at least pay off the loan. What is the price range for the truck?

87. **Hazardous to her health.** Trying to break her smoking habit, Jane calculates that she smokes only three full cigarettes a day, one after each meal. The rest of the time she smokes on the run and smokes only half of the cigarette. She estimates that she smokes the equivalent of 5 to 12 cigarettes per day. How many times a day does she light up on the run?

88. **Possible width.** The length of a rectangle is 20 meters longer than the width. The perimeter must be between 80 and 100 meters. What are the possible values for the width of the rectangle?

89. **Higher education.** The formulas

\[
B = 16.45n + 980.20
\]

and

\[
M = 7.79n + 287.87
\]

can be used to approximate the number of bachelor’s and master’s degrees in thousands, respectively, awarded in the year 1985 + \( n \) (National Center for Education Statistics, www.nces.ed.gov).

a) How many bachelor’s degrees were awarded in 1995?

b) In what year will the number of bachelor’s degrees that are awarded reach 1.26 million?

c) What is the first year in which both \( B \) is greater than 1.3 million and \( M \) is greater than 0.5 million?

d) What is the first year in which either \( B \) is greater than 1.3 million or \( M \) is greater than 0.5 million?

90. **Senior citizens.** The number of senior citizens (65 years old and over) in the United States in millions in the year 1970 + \( n \) can be estimated by using the formula

\[
S = 0.48n + 19.71
\]

(U.S. Bureau of the Census, www.census.gov). The percentage of senior citizens living below the poverty level in the year 1970 + \( n \) can be estimated by using the formula

\[
p = -0.72n + 24.2.
\]

a) How many senior citizens were there in 1998?

b) In what year did the percentage of seniors living below the poverty level reach 2.6%?

c) What is the first year in which we can expect both the number of seniors to be greater than 36 million and fewer than 2.6% living below the poverty level?