SECTION 10.1

Zero and Negative Exponents and Scientific Notation

10.1 OBJECTIVES

1. *Define the zero exponent*
2. *Simplify expressions with negative exponents*
3. *Write a number in scientific notation*
4. *Solve an application of scientific notation*

In Section 5.1, we examined properties of exponents, but all of the exponents were positive integers. In this section, we look at zero and negative exponents. First we extend the quotient rule so that we can define an exponent of zero.

Recall that, in the quotient rule, to divide two expressions that have the same base, we keep the base and subtract the exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

Now, suppose that we allow *m to equal n*. We then have

$$\frac{a^m}{a^m} = a^{m-m} = a^0 \qquad \textbf{(1)}$$

But we know that it is also true that

$$\frac{a^m}{a^m} = 1 \qquad \textbf{(2)}$$

Comparing equations (1) and (2), we see that the following definition is reasonable.

We must have $a \neq 0$. The form 0^0 is called **indeterminate** and is considered in later mathematics classes.

The Zero Exponent

For any real number a where $a \neq 0$,

$$a^0 = 1$$

Example 1

The Zero Exponent

Use the above definition to simplify each expression.

(a) $17^0 = 1$ (b) $(a^3b^2)^0 = 1$

(c) $6x^0 = 6 \cdot 1 = 6$ (d) $-3y^0 = -3$

Note that in $6x^0$ the exponent 0 applied *only* to x.

✓ *CHECK YOURSELF 1*

Simplify each expression.

(a) 25^0 (b) $(m^4n^2)^0$ (c) $8s^0$ (d) $-7t^0$

Recall that, in the product rule, to multiply expressions with the same base, keep the base and add the exponents.

$$a^m \cdot a^n = a^{m+n}$$

Now, what if we allow one of the exponents to be negative and apply the product rule? Suppose, for instance, that $m = 3$ and $m = -3$. Then

$$a^m \cdot a^n = a^3 \cdot a^{-3} = a^{3+(-3)}$$
$$= a^0 = 1$$

so

$$a^3 \cdot a^{-3} = 1$$

John Wallis (1616–1702), an English mathematician, was the first to fully discuss the meaning of 0, negative, and rational exponents (which we discuss in Section 10.3).

Negative Integer Exponents

For any nonzero real number a and whole number n,

$$a^{-n} = \frac{1}{a^n}$$

and a^{-n} is the **multiplicative inverse** of a^n.

Example 2 illustrates this definition.

Example 2

Using Properties of Exponents

From this point on, to *simplify* will mean to write the expression with *positive exponents only.*

Also, we will restrict all variables so that they represent nonzero real numbers.

Simplify the following expressions.

(a) $y^{-5} = \dfrac{1}{y^5}$

(b) $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

(c) $(-3)^{-3} = \dfrac{1}{(-3)^3} = \dfrac{1}{-27} = -\dfrac{1}{27}$

(d) $\left(\dfrac{2}{3}\right)^{-3} = \dfrac{1}{\left(\dfrac{2}{3}\right)^3} = \dfrac{1}{\dfrac{8}{27}} = \dfrac{27}{8}$

✓ *CHECK YOURSELF 2*

Simplify each of the following expressions.

(a) a^{-10} (b) 2^{-4} (c) $(-4)^{-2}$ (d) $\left(\dfrac{5}{2}\right)^{-2}$

Example 3 illustrates the case where coefficients are involved in an expression with negative exponents. As will be clear, some caution must be used.

Example 3 Using Properties of Exponents

Simplify each of the following expressions.

(a) $2x^{-3} = 2 \cdot \dfrac{1}{x^3} = \dfrac{2}{x^3}$

The exponent -3 applies only to the variable x, and *not* to the coefficient 2.

Caution
The expressions

$4w^{-2}$ and $(4w)^{-2}$

are *not* the same. Do you see why?

(b) $4w^{-2} = 4 \cdot \dfrac{1}{w^2} = \dfrac{4}{w^2}$

(c) $(4w)^{-2} = \dfrac{1}{(4w)^2} = \dfrac{1}{16w^2}$

✓ *CHECK YOURSELF 3*

Simplify each of the following expressions.

(a) $3w^{-4}$ (b) $10x^{-5}$ (c) $(2y)^{-4}$ (d) $-5t^{-2}$

Suppose that a variable with a negative exponent appears in the denominator of an expression. Our previous definition can be used to write a complex fraction that can then be simplified. For instance,

$$\frac{1}{a^{-2}} = \frac{1}{\frac{1}{a^2}} = 1 \cdot \frac{a^2}{1} = a^2 \longleftarrow \text{Positive exponent in numerator}$$

Negative exponent in denominator

To divide, we invert and multiply.

To avoid the intermediate steps, we can write that, in general,

For any nonzero real number a and integer n,

$$\frac{1}{a^{-n}} = a^n$$

Example 4

Using Properties of Exponents

Simplify each of the following expressions.

(a) $\frac{1}{y^{-3}} = y^3$

(b) $\frac{1}{2^{-5}} = 2^5 = 32$

(c) $\frac{3}{4x^{-2}} = \frac{3x^2}{4}$ The exponent -2 applies only to x, not to 4.

(d) $\frac{a^{-3}}{b^{-4}} = \frac{b^4}{a^3}$

✓ *CHECK YOURSELF 4*

Simplify each of the following expressions.

(a) $\frac{1}{x^{-4}}$ (b) $\frac{1}{3^{-3}}$ (c) $\frac{2}{3a^{-2}}$ (d) $\frac{c^{-5}}{d^{-7}}$

To review these properties, return to Section 5.1.

The product and quotient rules for exponents apply to expressions that involve any integral exponent—positive, negative, or 0. Example 5 illustrates this concept.

Example 5

Using Properties of Exponents

Simplify each of the following expressions, and write the result, using positive exponents only.

(a) $x^3 \cdot x^{-7} = x^{3+(-7)}$

$= x^{-4} = \dfrac{1}{x^4}$

Add the exponents by the product rule.

(b) $\dfrac{m^{-5}}{m^{-3}} = m^{-5-(-3)} = m^{-5+3}$

$= m^{-2} = \dfrac{1}{m^2}$

Subtract the exponents by the quotient rule.

(c) $\dfrac{x^5x^{-3}}{x^{-7}} = \dfrac{x^{5+(-3)}}{x^{-7}} = \dfrac{x^2}{x^{-7}} = x^{2-(-7)} = x^9$

We apply first the product rule and then the quotient rule.

Note that m^{-5} in the numerator becomes m^5 in the denominator, and m^{-3} in the denominator becomes m^3 in the numerator. We then simplify as before.

In simplifying expressions involving negative exponents, there are often alternate approaches. For instance, in Example 5, part b, we could have made use of our earlier work to write

$$\frac{m^{-5}}{m^{-3}} = \frac{m^3}{m^5} = m^{3-5} = m^{-2} = \frac{1}{m^2}$$

✓ CHECK YOURSELF 5

Simplify each of the following expressions.

(a) $x^9 \cdot x^{-5}$ (b) $\dfrac{y^{-7}}{y^{-3}}$ (c) $\dfrac{a^{-3}a^2}{a^{-5}}$

The properties of exponents can be extended to include negative exponents. One of these properties, the *quotient-power rule,* is particularly useful when rational expressions are raised to a negative power. Let's look at the rule and apply it to negative exponents.

Quotient-Power Rule

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Raising Quotients to a Negative Power

$$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n \qquad a \neq 0,\ b \neq 0$$

Example 6

Extending the Properties of Exponents

Simplify each expression.

(a) $\left(\frac{s^3}{t^2}\right)^{-2} = \left(\frac{t^2}{s^3}\right)^2 = \frac{t^4}{s^6}$

(b) $\left(\frac{m^2}{n^{-2}}\right)^{-3} = \left(\frac{n^{-2}}{m^2}\right)^3 = \frac{n^{-6}}{m^6} = \frac{1}{m^6n^6}$

✓ *CHECK YOURSELF 6*

Simplify each expression.

(a) $\left(\frac{3t^2}{s^3}\right)^{-3}$ (b) $\left(\frac{x^5}{y^{-2}}\right)^{-3}$

Example 7

Using Properties of Exponents

Simplify each of the following expressions.

(a) $\left(\frac{3}{q^5}\right)^{-2} = \left(\frac{q^5}{3}\right)^2$

$= \frac{q^{10}}{9}$

(b) $\left(\frac{x^3}{y^4}\right)^{-3} = \left(\frac{y^4}{x^3}\right)^3$

$= \frac{(y^4)^3}{(x^3)^3} = \frac{y^{12}}{x^9}$

✓ *CHECK YOURSELF 7*

Simplify each of the following expressions.

(a) $\left(\dfrac{r^4}{5}\right)^{-2}$ (b) $\left(\dfrac{a^4}{b^3}\right)^{-3}$

As you might expect, more complicated expressions require the use of more than one of the properties, for simplification. Example 8 illustrates such cases.

Example 8

Using Properties of Exponents

Simplify each of the following expressions.

(a) $\dfrac{(a^2)^{-3}(a^3)^4}{(a^{-3})^3} = \dfrac{a^{-6} \cdot a^{12}}{a^{-9}}$ Apply the power rule to each factor.

$= \dfrac{a^{-6+12}}{a^{-9}} = \dfrac{a^6}{a^{-9}}$ Apply the product rule.

$= a^{6-(-9)} = a^{6+9} = a^{15}$ Apply the quotient rule.

It may help to separate the problem into three fractions, one for the coefficients and one for each of the variables.

(b) $\dfrac{8x^{-2}y^{-5}}{12x^{-4}y^3} = \dfrac{8}{12} \cdot \dfrac{x^{-2}}{x^{-4}} \cdot \dfrac{y^{-5}}{y^3}$

$= \dfrac{2}{3} \cdot x^{-2-(-4)} \cdot y^{-5-3}$

$= \dfrac{2}{3} \cdot x^2 \cdot y^{-8} = \dfrac{2x^2}{3y^8}$

(c) $\left(\dfrac{pr^3s^{-5}}{p^3r^{-3}s^{-2}}\right)^{-2} = (p^{1-3}r^{3-(-3)}s^{-5-(-2)})^{-2}$

$= (p^{-2}r^6s^{-3})^{-2}$ Apply the quotient rule inside the parentheses.

$= (p^{-2})^{-2}(r^6)^{-2}(s^{-3})^{-2}$ Apply the rule for a product to a power.

$= p^4r^{-12}s^6 = \dfrac{p^4s^6}{r^{12}}$ Apply the power rule.

Caution

Be Careful! Another possible first step (and generally an efficient one) is to rewrite an expression by using our earlier definitions.

$$a^{-n} = \frac{1}{a^n} \qquad \text{and} \qquad \frac{1}{a^{-n}} = a^n$$

For instance, in Example 8, part b, we would *correctly* write

$$\frac{8x^{-2}y^{-5}}{12x^{-4}y^3} = \frac{8x^4}{12x^2y^3y^5}$$

Caution

A *common error* is to write

$$\frac{8x^{-2}y^{-5}}{12x^{-4}y^3} = \frac{12x^4}{8x^2y^3y^5} \qquad \text{This is } \textit{not} \text{ correct.}$$

The coefficients should not have been moved along with the factors in x. Keep in mind that the negative exponents apply *only* to the variables. The coefficients remain *where they were* in the original expression when the expression is rewritten using this approach.

✓ CHECK YOURSELF 8

Simplify each of the following expressions.

(a) $\dfrac{(x^5)^{-2}(x^2)^3}{(x^{-4})^3}$ (b) $\dfrac{12a^{-3}b^{-2}}{16a^{-2}b^3}$ (c) $\left(\dfrac{xy^{-3}z^{-5}}{x^{-4}y^{-2}z^3}\right)^{-3}$

Let us now take a look at an important use of exponents, scientific notation.

We begin the discussion with a calculator exercise. On most calculators, if you multiply 2.3 times 1000, the display will read

2300

Multiply by 1000 a second time. Now you will see

2300000.

Multiplying by 1000 a third time will result in the display

2.3 09 or 2.3 E09

This must equal 2,300,000,000.

And multiplying by 1000 again yields

2.3 12 or 2.3 E12

Consider the following table:

$2.3 = 2.3 \times 10^0$
$23 = 2.3 \times 10^1$
$230 = 2.3 \times 10^2$
$2300 = 2.3 \times 10^3$
$23{,}000 = 2.3 \times 10^4$
$230{,}000 = 2.3 \times 10^5$

Can you see what is happening? This is the way calculators display very large numbers. The number on the left is always between 1 and 10, and the number on the right indicates the number of places the decimal point must be moved to the right to put the answer in standard (or decimal) form.

This notation is used frequently in science. It is not uncommon in scientific applications of algebra to find yourself working with very large or very small numbers. Even in the time of Archimedes (287–212 B.C.), the study of such numbers was not unusual. Archimedes estimated that the universe was 23,000,000,000,000,000 m in diameter, which is the approximate distance light travels in $2\frac{1}{2}$ years. By comparison, Polaris (the North Star) is 680 light-years from the earth. Example 10 will discuss the idea of light-years.

In scientific notation, his estimate for the diameter of the universe would be

$$2.3 \times 10^{16} \text{ m}$$

In general, we can define scientific notation as follows.

Scientific Notation

Any number written in the form

$$a \times 10^n$$

where $1 \le a < 10$ and n is an integer, is written in scientific notation.

Example 9

Using Scientific Notation

Note the pattern for writing a number in scientific notation.

Write each of the following numbers in scientific notation.

(a) $120{,}000. = 1.2 \times 10^5$
5 places — The power is 5.

(b) $88{,}000{,}000. = 8.8 \times 10^7$
7 places — The power is 7.

The exponent on 10 shows the *number of places* we must move the decimal point so that the multiplier will be a number between 1 and 10. A positive exponent tells us to move right, while a negative exponent indicates to move left.

(c) $520{,}000{,}000. = 5.2 \times 10^8$
8 places

(d) $4{,}000{,}000{,}000. = 4 \times 10^9$
9 places

(e) $0.0005 = 5 \text{ x } 10^{-4}$
4 places — If the decimal point is to be moved to the left, the exponent will be negative.

Note: To convert back to standard or decimal form, the process is simply reversed.

(f) $0.0000000081 = 8.1 \times 10^{-9}$
9 places

✓ *CHECK YOURSELF 9*

Write in scientific notation.

(a) 212,000,000,000,000,000 (b) 0.00079

(c) 5,600,000 (d) 0.0000007

Example 10 An Application of Scientific Notation

(a) Light travels at a speed of 3.05×10^8 meters per second (m/s). There are approximately 3.15×10^7 s in a year. How far does light travel in a year?

We multiply the distance traveled in 1 s by the number of seconds in a year. This yields

$$(3.05 \times 10^8)(3.15 \times 10^7) = (3.05 \cdot 3.15)(10^8 \cdot 10^7)$$

Multiply the coefficients, and add the exponents.

$$= 9.6075 \times 10^{15}$$

Note that $9.6075 \times 10^{15} \approx 10 \times 10^{15} = 10^{16}$

For our purposes we round the distance light travels in 1 year to 10^{16} m. This unit is called a **light-year,** and it is used to measure astronomical distances.

(b) The distance from earth to the star Spica (in Virgo) is 2.2×10^{18} m. How many light-years is Spica from earth?

We divide the distance (in meters) by the number of meters in 1 light-year.

$$\frac{2.2 \times 10^{18}}{10^{16}} = 2.2 \times 10^{18-16}$$

$$= 2.2 \times 10^2 = 220 \text{ light-years}$$

✓ CHECK YOURSELF 10

The farthest object that can be seen with the unaided eye is the Andromeda galaxy. This galaxy is 2.3×10^{22} m from earth. What is this distance in light-years?

✓ CHECK YOURSELF ANSWERS

1. (a) 1; (b) 1; (c) 8; (d) -7. **2.** (a) $\frac{1}{a^{10}}$; (b) $\frac{1}{16}$; (c) $\frac{1}{16}$; (d) $\frac{4}{25}$. **3.** (a) $\frac{3}{w^4}$; (b) $\frac{10}{x^5}$; (c) $\frac{1}{16y^4}$; (d) $-\frac{5}{t^2}$. **4.** (a) x^4; (b) 27; (c) $\frac{2a^2}{3}$; (d) $\frac{d^7}{c^5}$. **5.** (a) x^4; (b) $\frac{1}{y^4}$; (c) a^4. **6.** (a) $\frac{s^9}{27t^6}$; (b) $\frac{1}{x^{15}y^6}$. **7.** (a) $\frac{25}{r^8}$; (b) $\frac{b^9}{a^{12}}$. **8.** (a) x^8; (b) $\frac{3}{4ab^5}$; (c) $\frac{y^3z^{24}}{x^{15}}$. **9.** (a) 2.12×10^{17}; (b) 7.9×10^{-4}; (c) 5.6×10^6; (d) 7×10^{-7}. **10.** 2,300,000 light-years.

Exercises ■ 10.1

In Exercises 1 to 22, simplify each expression.

1. x^{-5}

2. 3^{-3}

3. 5^{-2}

4. x^{-8}

5. $(-5)^{-2}$

6. $(-3)^{-3}$

7. $(-2)^{-3}$

8. $(-2)^{-4}$

9. $\left(\frac{2}{3}\right)^{-3}$

10. $\left(\frac{3}{4}\right)^{-2}$

11. $3x^{-2}$

12. $4x^{-3}$

13. $-5x^{-4}$

14. $(-2x)^{-4}$

15. $(-3x)^{-2}$

16. $-5x^{-2}$

17. $\frac{1}{x^{-3}}$

18. $\frac{1}{x^{-5}}$

19. $\frac{2}{5x^{-3}}$

20. $\frac{3}{4x^{-4}}$

21. $\frac{x^{-3}}{y^{-4}}$

22. $\frac{x^{-5}}{y^{-3}}$

In Exercises 23 to 32, use the properties of exponents to simplify expressions.

23. $x^5 \cdot x^{-3}$

24. $y^{-4} \cdot y^5$

25. $a^{-9} \cdot a^6$

26. $w^{-5} \cdot w^3$

27. $z^{-2} \cdot z^{-8}$

28. $b^{-7} \cdot b^{-1}$

29. $a^{-5} \cdot a^5$

30. $x^{-4} \cdot x^4$

31. $\frac{x^{-5}}{x^{-2}}$

32. $\frac{x^{-3}}{x^{-6}}$

In Exercises 33 to 58, use the properties of exponents to simplify the following.

33. $(x^5)^3$ **34.** $(w^4)^6$ **35.** $(2x^{-3})(x^2)^4$

36. $(p^4)(3p^3)^2$ **37.** $(3a^{-4})(a^3)(a^2)$ **38.** $(5y^{-2})(2y)(y^5)$

39. $(x^4y)(x^2)^3(y^3)^0$ **40.** $(r^4)^2(r^2s)(s^3)^2$ **41.** $(ab^2c)(a^4)^4(b^2)^3(c^3)^4$

42. $(p^2qr^2)(p^2)(q^3)^2(r^2)^0$ **43.** $(x^5)^{-3}$ **44.** $(x^{-2})^{-3}$

45. $(b^{-4})^{-2}$ **46.** $(a^0b^{-4})^3$ **47.** $(x^5y^{-3})^2$

48. $(p^{-3}q^2)^{-2}$ **49.** $(x^{-4}y^{-2})^{-3}$ **50.** $(3x^{-2}y^{-2})^3$

51. $(2x^{-3}y^0)^{-5}$ **52.** $\dfrac{a^{-6}}{b^{-4}}$ **53.** $\dfrac{x^{-2}}{y^{-4}}$

54. $\left(\dfrac{x^{-3}}{y^2}\right)^{-3}$ **55.** $\dfrac{x^{-4}}{y^{-2}}$ **56.** $\dfrac{(3x^{-4})^2(2x^2)}{x^6}$

57. $(4x^{-2})^2(3x^{-4})$ **58.** $(5x^{-4})^{-4}(2x^3)^{-5}$

In Exercises 59 to 90, simplify each expression.

59. $(2x^5)^4(x^3)^2$ **60.** $(3x^2)^3(x^2)^4(x^2)$ **61.** $(2x^{-3})^3(3x^3)^2$

62. $(x^2y^3)^4(xy^3)^0$ **63.** $(xy^5z)^4(xyz^2)^8(x^6yz)^5$ **64.** $(x^2y^2z^2)^0(xy^2z)^2(x^3yz^2)$

65. $(3x^{-2})(5x^2)^2$ **66.** $(2a^3)^2(a^0)^5$ **67.** $(2w^3)^4(3w^{-5})^2$

68. $(3x^3)^2(2x^4)^5$ **69.** $\dfrac{3x^6}{2y^9} \cdot \dfrac{y^5}{x^3}$ **70.** $\dfrac{x^8}{y^6} \cdot \dfrac{2y^9}{x^3}$

71. $(-7x^2y)(-3x^5y^6)^4$ **72.** $\left(\dfrac{2w^5z^3}{3x^3y^9}\right)\left(\dfrac{x^5y^4}{w^4z^0}\right)^2$ **73.** $(2x^2y^{-3})(3x^{-4}y^{-2})$

74. $(-5a^{-2}b^{-4})(2a^5b^0)$ **75.** $\dfrac{(x^{-3})(y^2)}{y^{-3}}$ **76.** $\dfrac{6x^3y^{-4}}{24x^{-2}y^{-2}}$

77. $\dfrac{15x^{-3}y^2z^{-4}}{20x^{-4}y^{-3}z^2}$ **78.** $\dfrac{24x^{-5}y^{-3}z^2}{36x^{-2}y^3z^{-2}}$ **79.** $\dfrac{x^{-5}y^{-7}}{x^0y^{-4}}$

80. $\left(\dfrac{xy^3z^{-4}}{x^{-3}y^{-2}z^2}\right)^{-2}$

81. $\dfrac{x^{-2}y^2}{x^3y^{-2}} \cdot \dfrac{x^{-4}y^2}{x^{-2}y^{-2}}$

82. $\left(\dfrac{x^{-3}y^3}{x^{-4}y^2}\right)^3 \cdot \left(\dfrac{x^{-2}y^{-2}}{xy^4}\right)^{-1}$

83. $x^{2n} \cdot x^{3n}$

84. $x^{n+1} \cdot x^{3n}$

85. $\dfrac{x^{n+3}}{x^{n+1}}$

86. $\dfrac{x^{n-4}}{x^{n-1}}$

87. $(y^n)^{3n}$

88. $(x^{n+1})^n$

89. $\dfrac{x^{2n} \cdot x^{n+2}}{x^{3n}}$

90. $\dfrac{x^n \cdot x^{3n+5}}{x^{4n}}$

In Exercises 91 to 94, express each number in scientific notation.

91. The distance from the earth to the sun: 93,000,000 mi.

92. The diameter of a grain of sand: 0.000021 m.

93. The diameter of the sun: 130,000,000,000 cm.

94. The number of molecules in 22.4 L of a gas: 602,000,000,000,000,000,000,000 (Avogadro's number)

95. The mass of the sun is approximately 1.98×10^{30} kg. If this were written in standard or decimal form, how many 0s would follow the digit 8?

96. Archimedes estimated the universe to be 2.3×10^{19} millimeters (mm) in diameter. If this number were written in standard or decimal form, how many 0s would follow the digit 3?

In Exercises 97 to 100, write each expression in standard notation.

97. 8×10^{-3} **98.** 7.5×10^{-6} **99.** 2.8×10^{-5} **100.** 5.21×10^{-4}

In Exercises 101 to 104, write each of the following in scientific notation.

101. 0.0005 **102.** 0.000003 **103.** 0.00037 **104.** 0.000051

In Exercises 105 to 108, compute the expressions using scientific notation, and write your answer in that form.

105. $(4 \times 10^{-3})(2 \times 10^{-5})$

106. $(1.5 \times 10^{-6})(4 \times 10^2)$

107. $\dfrac{9 \times 10^3}{3 \times 10^{-2}}$

108. $\dfrac{7.5 \times 10^{-4}}{1.5 \times 10^2}$

In Exercises 109 to 114, perform the indicated calculations. Write your result in scientific notation.

109. $(2 \times 10^5)(4 \times 10^4)$

110. $(2.5 \times 10^7)(3 \times 10^5)$

111. $\dfrac{6 \times 10^9}{3 \times 10^7}$

112. $\dfrac{4.5 \times 10^{12}}{1.5 \times 10^7}$

113. $\dfrac{(3.3 \times 10^{15})(6 \times 10^{15})}{(1.1 \times 10^8)(3 \times 10^6)}$

114. $\dfrac{(6 \times 10^{12})(3.2 \times 10^8)}{(1.6 \times 10^7)(3 \times 10^2)}$

115. Megrez, the nearest of the Big Dipper stars, is 6.6×10^{17} m from earth. Approximately how long does it take light, traveling at 10^{16} m/year, to travel from Megrez to earth?

116. Alkaid, the most distant star in the Big Dipper, is 2.1×10^{18} m from earth. Approximately how long does it take light to travel from Alkaid to earth?

117. The number of liters of water on earth is 15,500 followed by 19 zeros. Write this number in scientific notation. Then use the number of liters of water on earth to find out how much water is available for each person on earth. The population of earth is 5.3 billion.

118. If there are 5.3×10^9 people on earth and there is enough freshwater to provide each person with 8.79×10^5 L, how much freshwater is on earth?

119. The United States uses an average of 2.6×10^6 L of water per person each year. The United States has 2.5×10^8 people. How many liters of water does the United States use each year?

120. Can $(a + b)^{-1}$ be written as $\dfrac{1}{a} + \dfrac{1}{b}$ by using the properties of exponents? If not, why not? Explain.

121. Write a short description of the difference between $(-4)^{-2}$, -4^{-3}, $(-4)^3$, and -4^3. Are any of these equal?

122. If $n > 0$, which of the following expressions are negative?

$$-n^{-3},\ n^{-3},\ (-n)^{-3},\ (-n)^3,\ -n^3$$

If $n < 0$, which of these expressions are negative? Explain what effect a negative in the exponent has on the sign of the result when an exponential expression is simplified.

123. **Take the best offer.** You are offered a 28-day job in which you have a choice of two different pay arrangements. Plan 1 offers a flat \$4,000,000 at the end of the 28th day on the job. Plan 2 offers 1¢ the first day, 2¢ the second day, 4¢ the third day, and so on, with the amount doubling each day. Make a table to decide which offer is the best. Write a formula for the amount you make on the nth day and a formula for the total after n days. Which pay arrangement should you take? Why?