1.1 OBJECTIVES

1. Represent addition, subtraction, multiplication, and division by using the symbols of algebra
2. Identify algebraic expressions

Overcoming Math Anxiety

Throughout this text, we will present you with a series of class-tested techniques that are designed to improve your performance in this math class.

Hint #2 Become familiar with your syllabus.

In the first class meeting, your instructor probably handed out a class syllabus. If you haven't done so already, you need to incorporate important information into your calendar and address book.

1. Write all important dates in your calendar. This includes homework due dates, quiz dates, test dates, and the date and time of the final exam. Never allow yourself to be surprised by any deadline!
2. Write your instructor's name, contact number, and office number in your address book. Also include the office hours. Make it a point to see your instructor early in the term. Although this is not the only person who can help clear up your confusion, it is the most important person.
3. Make note of other resources that are made available to you. This includes CDs, video tapes, web pages, and tutoring.

Given all of these resources, it is important that you never let confusion or frustration mount. If you can't "get it" from the text, try another resource. All of the resources are there specifically for you, so take advantage of them!

In arithmetic, you learned how to do calculations with numbers by using the basic operations of addition, subtraction, multiplication, and division.

In algebra, you will still use numbers and the same four operations. However, you will also use letters to represent numbers. Letters such as $x, y, L,$ or $W$ are called variables when they represent numerical values.

Here we see two rectangles whose lengths and widths are labeled with numbers.

If we need to represent the length and width of any rectangle, we can use the variables $L$ and $W$. 

![Diagram of rectangles with variables L and W]
You are familiar with the four symbols (+, -, ×, ÷) used to indicate the fundamental operations of arithmetic.

Let’s look at how these operations are indicated in algebra. We begin by looking at addition.

**Definitions: Addition**

\[ x + y \text{ means the sum of } x \text{ and } y \text{ or } x \text{ plus } y. \]

**Example 1**

**Writing Expressions That Indicate Addition**

(a) The sum of \( a \) and 3 is written as \( a + 3 \).
(b) \( L \) plus \( W \) is written as \( L + W \).
(c) 5 more than \( m \) is written as \( m + 5 \).
(d) \( x \) increased by 7 is written as \( x + 7 \).

**CHECK YOURSELF 1**

Write, using symbols.

(a) The sum of \( y \) and 4  
(b) \( a \) plus \( b \)  
(c) 3 more than \( x \)  
(d) \( n \) increased by 6

Let’s look at how subtraction is indicated in algebra.

**Definitions: Subtraction**

\[ x - y \text{ means the difference of } x \text{ and } y \text{ or } x \text{ minus } y. \]

**Example 2**

**Writing Expressions That Indicate Subtraction**

(a) \( r \) minus \( s \) is written as \( r - s \).
(b) The difference of \( m \) and 5 is written as \( m - 5 \).
(c) \( x \) decreased by 8 is written as \( x - 8 \).
(d) 4 less than \( a \) is written as \( a - 4 \).

**CHECK YOURSELF 2**

Write, using symbols.

(a) \( w \) minus \( z \)  
(b) The difference of \( a \) and 7  
(c) \( y \) decreased by 3  
(d) 5 less than \( b \)
You have seen that the operations of addition and subtraction are written exactly the same way in algebra as in arithmetic. This is not true in multiplication because the sign $\times$ looks like the letter $x$. So in algebra we use other symbols to show multiplication to avoid any confusion. Here are some ways to write multiplication.

**Definitions: Multiplication**

- A centered dot $x \cdot y$
- Parentheses $(x)(y)$
- Writing the letters next to each other $xy$

These all indicate the product of $x$ and $y$ or $x$ times $y$.

**Example 3**

**Writing Expressions That Indicate Multiplication**

(a) The product of 5 and $a$ is written as $5 \cdot a$, $(5)(a)$, or $5a$. The last expression, $5a$, is the shortest and the most common way of writing the product.

(b) 3 times 7 can be written as $3 \cdot 7$ or $(3)(7)$.

(c) Twice $z$ is written as $2z$.

(d) The product of 2, $s$, and $t$ is written as $2st$.

(e) 4 more than the product of 6 and $x$ is written as $6x + 4$.

**CHECK YOURSELF 3**

Write, using symbols.

(a) $m$ times $n$  
(b) The product of $h$ and $b$

(c) The product of 8 and 9  
(d) The product of 5, $w$, and $y$

(e) 3 more than the product of 8 and $a$

Before we move on to division, let's look at how we can combine the symbols we have learned so far.

**Definitions: Expression**

An expression is a meaningful collection of numbers, variables, and signs of operation.

**Example 4**

**Identifying Expressions**

(a) $2m + 3$ is an expression. It means that we multiply 2 and $m$, then add 3.

(b) $x + \cdot + 3$ is not an expression. The three operations in a row have no meaning.

(c) $y = 2x - 1$ is not an expression. The equals sign is not an operation sign.

(d) $3a + 5b - 4c$ is an expression. Its meaning is clear.
CHECK YOURSELF 4

Identify which are expressions and which are not.

(a) \(7 \div x\)  
(b) \(6 + y = 9\)  
(c) \(a + b - c\)  
(d) \(3x - 5yz\)

To write more complicated products in algebra, we need some “punctuation marks.” Parentheses ( ) mean that an expression is to be thought of as a single quantity. Brackets [ ] are used in exactly the same way as parentheses in algebra. Look at the following example showing the use of these signs of grouping.

Example 5

Expressions with More Than One Operation

(a) 3 times the sum of \(a\) and \(b\) is written as \(3(a + b)\)

NOTE This can be read as “3 times the quantity \(a\) plus \(b\).”

(b) The sum of 3 times \(a\) and \(b\) is written as \(3a + b\).

NOTE No parentheses are needed here because the 3 multiplies only the \(a\).

(c) 2 times the difference of \(m\) and \(n\) is written as \(2(m - n)\).

(d) The product of \(s\) plus \(t\) and \(s\) minus \(t\) is written as \((s + t)(s - t)\).

(e) The product of \(b\) and 3 less than \(b\) is written as \(b(b - 3)\).

CHECK YOURSELF 5

Write, using symbols.

(a) Twice the sum of \(p\) and \(q\)

(b) The sum of twice \(p\) and \(q\)

(c) The product of \(a\) and the quantity \(b - c\)

(d) The product of \(x\) plus 2 and \(x\) minus 2

(e) The product of \(x\) and 4 more than \(x\)

NOTE In algebra the fraction form is usually used.

Now let’s look at the operation of division. In arithmetic, you use the division sign \(\div\), the long division symbol \(\underline{\text{\ }}\), and the fraction notation. For example, to indicate the quotient when 9 is divided by 3, you could write

\[9 \div 3 \quad \text{or} \quad 3\sqrt{9} \quad \text{or} \quad \frac{9}{3}\]
Definitions: Division

\[
\frac{x}{y} \text{ means } x \text{ divided by } y \text{ or the quotient of } x \text{ and } y.
\]

Example 6
Writing Expressions That Indicate Division

(a) \( \frac{m}{3} \) means \( m \) divided by 3.

(b) \( \frac{a + b}{5} \) means the quotient of \( a + b \) divided by 5.

(c) \( \frac{p + q}{p - q} \) means the sum \( p + q \) divided by the difference \( p - q \).

Check Yourself 6
Write, using symbols.

(a) \( \frac{r}{s} \) divided by \( s \)
(b) The quotient when \( x \) minus \( y \) is divided by 7
(c) The difference \( a \) minus 2 divided by the sum \( a \) plus 2

Notice that we can use many different letters to represent variables. In Example 6 the letters \( m, a, b, p, \) and \( q \) represented different variables. We often choose a letter that reminds us of what it represents, for example, \( L \) for \( \text{length} \) or \( W \) for \( \text{width} \).

Example 7
Writing Geometric Expressions

(a) Length times width is written \( L \cdot W \).

(b) One-half of altitude times base is written \( \frac{1}{2} a \cdot b \).

(c) Length times width times height is written \( L \cdot W \cdot H \).

(d) Pi (\( \pi \)) times diameter is written \( \pi d \).

Check Yourself 7
Write each geometric expression, using symbols.

(a) Two times length plus two times width
(b) Two times pi (\( \pi \)) times radius
CHECK YOURSELF ANSWERS

1. (a) \( y + 4 \); (b) \( a + b \); (c) \( x + 3 \); (d) \( n + 6 \) 
2. (a) \( w - z \); (b) \( a - 7 \); (c) \( y - 3 \); (d) \( b - 5 \) 
3. (a) \( mn \); (b) \( hh \); (c) \( 8 \cdot 9 \) or \( (8)(9) \); (d) \( 5wy \); (e) \( 8a + 3 \) 
4. (a) Not an expression; (b) not an expression; (c) an expression; (d) an expression 
5. (a) \( 2(p + q) \); (b) \( 2p + q \); (c) \( a(b - c) \); (d) \( (x + 2)(x - 2) \); (e) \( x(x + 4) \) 
6. (a) \( \frac{r}{s} \); (b) \( \frac{x - y}{7} \); (c) \( \frac{a - 2}{a + 2} \) 
7. (a) \( 2L + 2W \); (b) \( 2\pi r \)
Write each of the following phrases, using symbols.

1. The sum of $c$ and $d$
2. $a$ plus 7
3. $w$ plus $z$
4. The sum of $m$ and $n$
5. $x$ increased by 2
6. 3 more than $b$
7. 10 more than $y$
8. $m$ increased by 4
9. $a$ minus $b$
10. 5 less than $s$
11. $b$ decreased by 7
12. $r$ minus 3
13. 6 less than $r$
14. $x$ decreased by 3
15. $w$ times $z$
16. The product of 3 and $c$
17. The product of 5 and $t$
18. 8 times $a$
19. The product of 8, $m$, and $n$
20. The product of 7, $r$, and $s$
21. The product of 3 and the quantity $p$ plus $q$
22. The product of 5 and the sum of $a$ and $b$
23. Twice the sum of $x$ and $y$
24. 3 times the sum of $m$ and $n$
25. The sum of twice $x$ and $y
26. The sum of 3 times $m$ and $n$
27. Twice the difference of $x$ and $y$
28. 3 times the difference of $c$ and $d$
29. The quantity $a$ plus $b$ times the quantity $a$ minus $b$
30. The product of $x$ plus $y$ and $x$ minus $y$
31. The product of $m$ and 3 less than $m$
32. The product of $a$ and 7 more than $a$
33. $x$ divided by 5
34. The quotient when $b$ is divided by 8
35. The quotient of $a$ plus $b$, divided by 7
36. The difference $x$ minus $y$, divided by 9
37. The difference of $p$ and $q$, divided by 4
38. The sum of $a$ and 5, divided by 9
39. The sum of $a$ and 3, divided by the difference of $a$ and 3
40. The difference of $m$ and $n$, divided by the sum of $m$ and $n$

Write each of the following phrases, using symbols. Use the variable $x$ to represent the number in each case.

41. 5 more than a number
42. A number increased by 8
43. 7 less than a number
44. A number decreased by 10
45. 9 times a number
46. Twice a number
47. 6 more than 3 times a number
48. 5 times a number, decreased by 10
49. Twice the sum of a number and 5
50. 3 times the difference of a number and 4
51. The product of 2 more than a number and 2 less than that same number
52. The product of 5 less than a number and 5 more than that same number

53. The quotient of a number and 7

54. A number divided by 3

55. The sum of a number and 5, divided by 8

56. The quotient when 7 less than a number is divided by 3

57. 6 more than a number divided by 6 less than that same number

58. The quotient when 3 less than a number is divided by 3 more than that same number

Write each of the following geometric expressions using symbols.

59. Four times the length of a side (s)

60. \( \frac{4}{3} \pi \) times the cube of the radius (r)

61. The radius (r) squared times the height (h) times \( \pi \)

62. Twice the length (L) plus twice the width (W)

63. One-half the product of the height (h) and the sum of two unequal sides (\( b_1 \) and \( b_2 \))

64. Six times the length of a side (s) squared

Identify which are expressions and which are not.

65. \( 2(x + 5) \)

66. \( 4 + (x - 3) \)

67. \( 4 + \div m \)

68. \( 6 + a = 7 \)

69. \( 2b = 6 \)

70. \( x(y + 3) \)

71. \( 2a + 5b \)

72. \( 4x + \cdot 7 \)

73. Population growth. The Earth’s population has doubled in the last 40 years. If we let \( x \) represent the Earth’s population 40 years ago, what is the population today?

74. Species extinction. It is estimated that the Earth is losing 4000 species of plants and animals every year. If \( S \) represents the number of species living last year, how many species are on Earth this year?

75. Interest. The simple interest \( I \) earned when a principal \( P \) is invested at a rate \( r \) for a time \( t \) is calculated by multiplying the principal times the rate times the time. Write a formula for the interest earned.
76. Kinetic energy. The kinetic energy of a particle of mass \( m \) is found by taking one-half of the product of the mass and the square of the velocity \( (v) \). Write a formula for the kinetic energy of a particle.

\[
\text{Kinetic energy} = \frac{1}{2}mv^2
\]

77. Rewrite the following algebraic expressions in English phrases. Exchange papers with another student to edit your writing. Be sure the meaning in English is the same as in algebra. These expressions are not complete sentences, so your English does not have to be in complete sentences. Here is an example.

Algebra: \( 2(x - 1) \)

English: We could write “One less than a number is doubled.” Or we might write “A number is diminished by one and then multiplied by two.”

(a) \( n + 3 \)  
(b) \( \frac{x + 2}{5} \)  
(c) \( 3(5 + a) \)  
(d) \( 3 - 4n \)  
(e) \( \frac{x + 6}{x - 1} \)

*Getting Ready for Section 1.2 [Section 0.3]*

Evaluate the following:

(a) \( 8 - (5 + 2) \)  
(b) \( (8 - 5) + 2 \)  
(c) \( 16 ÷ 4 \cdot 2 \)  
(d) \( 16 ÷ (4 \cdot 2) \)  
(e) \( 6 \cdot 2 \)  
(f) \( 2 \cdot 6 \)

**Answers**

1. \( c + d \)  
3. \( w + z \)  
5. \( x + 2 \)  
7. \( y + 10 \)  
9. \( a - b \)  
11. \( b - 7 \)  
13. \( r - 6 \)  
15. \( wz \)  
17. \( 5t \)  
19. \( 8mn \)  
21. \( 3(p + q) \)  
23. \( 2(x + y) \)  
25. \( 2x + y \)  
27. \( 2(x - y) \)  
29. \( (a + b)(a - b) \)  
31. \( m(m - 3) \)  
33. \( \frac{x}{5} \)  
35. \( \frac{a + b}{7} \)  
37. \( \frac{p - q}{4} \)  
39. \( \frac{a + 3}{a - 3} \)  
41. \( x + 5 \)  
43. \( x - 7 \)  
45. \( 9x \)  
47. \( 3x + 6 \)  
49. \( 2(x + 5) \)  
51. \( (x + 2)(x - 2) \)  
53. \( \frac{x}{7} \)  
55. \( \frac{x + 5}{8} \)  
57. \( \frac{x + 6}{x - 6} \)  
59. \( 4s \)  
61. \( \pi r^2h \)  
63. \( \frac{1}{2}h(b_1 + b_2) \)  
65. Expression

67. Not an expression  
69. Not an expression  
71. Expression

73. \( 2x \)  
75. \( I = Prt \)  
77. \( a \)  
1  
5  
8

d. \( 2 \)  
\( e. 12 \)  
\( f. 12 \)

*Exercises headed “Getting Ready for . . .” are designed to help you prepare for material in the next section of the text. If you have any difficulty with these exercises, please review the section referred to in brackets.*