1.5 Evaluating Algebraic Expressions

OBJECTIVES

1. Evaluate algebraic expressions given any signed number value for the variables
2. Use a calculator to evaluate algebraic expressions
3. Find the sum of a set of signed numbers
4. Interpret summation notation

In applying algebra to problem solving, you will often want to find the value of an algebraic expression when you know certain values for the letters (or variables) in the expression. Finding the value of an expression is called evaluating the expression and uses the following steps.

Step by Step: To Evaluate an Algebraic Expression

Step 1 Replace each variable by the given number value.
Step 2 Do the necessary arithmetic operations, following the rules for order of operations.

Example 1

Evaluating Algebraic Expressions

Suppose that \( a = 5 \) and \( b = 7 \).

(a) To evaluate \( a + b \), we replace \( a \) with 5 and \( b \) with 7.

\[
 a + b = 5 + 7 = 12
\]

(b) To evaluate \( 3ab \), we again replace \( a \) with 5 and \( b \) with 7.

\[
 3ab = 3 \cdot 5 \cdot 7 = 105
\]

CHECK YOURSELF 1

If \( x = 6 \) and \( y = 7 \), evaluate.

(a) \( y - x \)  
(b) \( 5xy \)

We are now ready to evaluate algebraic expressions that require following the rules for the order of operations.
CHAPTER 1 THE LANGUAGE OF ALGEBRA

Example 2
Evaluating Algebraic Expressions
Evaluate the following expressions if \( a = 2 \), \( b = 3 \), \( c = 4 \), and \( d = 5 \).

(a) \( 5a + 7b = 5 \cdot 2 + 7 \cdot 3 \quad \text{Multiply first.} \)
\[
= 10 + 21 = 31 \quad \text{Then add.}
\]
(b) \( 3c^2 = 3 \cdot 4^2 \quad \text{Evaluate the power.} \)
\[
= 3 \cdot 16 = 48 \quad \text{Then multiply.}
\]
(c) \( 7(c + d) = 7(4 + 5) \quad \text{Add inside the parentheses.} \)
\[
= 7 \cdot 9 = 63
\]
(d) \( 5a^3 - 2d^2 = 5 \cdot 2^3 - 2 \cdot 5^2 \quad \text{Evaluate the powers.} \)
\[
= 5 \cdot 8 - 2 \cdot 25 \quad \text{Multiply.}
\]
\[
= 40 - 50 = 30 \quad \text{Subtract.}
\]

CHECK YOURSELF 2
If \( x = 3 \), \( y = 2 \), \( z = 4 \), and \( w = 5 \), evaluate the following expressions.

(a) \( 4x^2 + 2 \)
(b) \( 5(z + w) \)
(c) \( 7(z^2 - y^2) \)

To evaluate algebraic expressions when a fraction bar is used, do the following: Start by doing all the work in the numerator, then do the work in the denominator. Divide the numerator by the denominator as the last step.

Example 3
Evaluating Algebraic Expressions
If \( p = 2 \), \( q = 3 \), and \( r = 4 \), evaluate:

(a) \( \frac{8p}{r} \)
Replace \( p \) with 2 and \( r \) with 4.
\[
\frac{8p}{r} = \frac{8 \cdot 2}{4} = \frac{16}{4} = 4 \quad \text{Divide as the last step.}
\]
(b) \( \frac{7q + r}{p + q} \) \quad \text{Now evaluate the top and bottom separately.}
\[
= \frac{7 \cdot 3 + 4}{2 + 3} = \frac{21 + 4}{5} = \frac{25}{5} = 5
\]

CHECK YOURSELF 3
Evaluate the following if \( c = 5 \), \( d = 8 \), and \( e = 3 \).

(a) \( \frac{6c}{e} \)
(b) \( \frac{4d + e}{c} \)
(c) \( \frac{10d - e}{d + e} \)
Example 4 shows how a scientific calculator can be used to evaluate algebraic expressions.

**Example 4**

*Using a Calculator to Evaluate Expressions*

Use a scientific calculator to evaluate the following expressions.

**(a)** \( \frac{4x + y}{z} \) if \( x = 2, y = 1, \) and \( z = 3 \)

Replace \( x \) with 2, \( y \) with 1, and \( z \) with 3:

\[
\frac{4x + y}{z} = \frac{4 \cdot 2 + 1}{3}
\]

Now, use the following keystrokes:

\[
[4 \times 2 + 1] \div 3
\]

The display will read 3.

**(b)** \( \frac{7x - y}{3z - x} \) if \( x = 2, y = 6, \) and \( z = 2 \)

\[
\frac{7x - y}{3z - x} = \frac{7 \cdot 2 - 6}{3 \cdot 2 - 2}
\]

Use the following keystrokes:

\[
[7 \times 2 - 6] \div [3 \times 2 - 2]
\]

The display will read 2.

**CHECK YOURSELF 4**

*Use a scientific calculator to evaluate the following if \( x = 2, y = 6, \) and \( z = 5.***

**(a)** \( \frac{2x + y}{z} \)

**(b)** \( \frac{4y - 2z}{x} \)

**Example 5**

*Evaluating Expressions*

Evaluate \( 5a + 4b \) if \( a = -2 \) and \( b = 3.***

Replace \( a \) with \(-2\) and \( b \) with 3.

\[
5a + 4b = 5(-2) + 4(3)
\]

\[
= -10 + 12
\]

\[
= 2
\]

**CHECK YOURSELF 5**

*Evaluate \( 3x + 5y \) if \( x = -2 \) and \( y = -5.***
We follow the same rules no matter how many variables are in the expression.

**Example 6**

**Evaluating Expressions**

Evaluate the following expressions if $a = -4$, $b = 2$, $c = -5$, and $d = 6$.

(a) $7a - 4c = 7(-4) - 4(-5)$

This becomes $-(-20)$, or $+20$.

$= -28 + 20$

$= -8$

(b) $7c^2 = 7(-5)^2 = 7 \cdot 25$

$= 175$

(c) $b^2 - 4ac = 2^2 - 4(-4)(-5)$

$= 4 - 4(-4)(-5)$

$= 4 - 80$

$= -76$

(d) $b(a + d) = 2(-4 + 6)$

This becomes $(20)$, or $20$.

$= 2(2)$

$= 4$

**CHECK YOURSELF 6**

Evaluate if $p = -4$, $q = 3$, and $r = -2$.

(a) $5p - 3r$

(b) $2p^2 + q$

(c) $p(q + r)$

(d) $-q^2$

(e) $(-q)^2$

If an expression involves a fraction, remember that the fraction bar is a grouping symbol. This means that you should do the required operations first in the numerator and then the denominator. Divide as the last step.

**Example 7**

**Evaluating Expressions**

Evaluate the following expressions if $x = 4$, $y = -5$, $z = 2$, and $w = -3$.

(a) $\frac{z - 2y}{x} = \frac{2 - 2(-5)}{4} = \frac{2 + 10}{4}$

$= \frac{12}{4} = 3$
When an expression is evaluated by a calculator, the same order of operations that we introduced in Section 0.3 is followed.

### Algebraic Calculator Notation vs. Calculator Notation

<table>
<thead>
<tr>
<th>Operation</th>
<th>Algebraic Notation</th>
<th>Calculator Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>$6 + 2$</td>
<td>$6 + 2$</td>
</tr>
<tr>
<td>Subtraction</td>
<td>$4 - 8$</td>
<td>$4 - 8$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>$(3)(-5)$</td>
<td>$3 \times (-5)$ or $3 \times 5 \div 5$</td>
</tr>
<tr>
<td>Division</td>
<td>$\frac{8}{6}$</td>
<td>$8 \div 6$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$3^4$</td>
<td>$3^4$ or $\sqrt[4]{3}$</td>
</tr>
</tbody>
</table>

In many applications, you will need to find the sum of a set of numbers that you are working with. In mathematics, the shorthand symbol for “sum of” is the Greek letter Σ (capital sigma, the “S” of the Greek alphabet). The expression $\Sigma x$, in which $x$ refers to all the numbers in a given set, means the sum of all the numbers in that set.

### Example 8

**Summing a Set**

Find $\Sigma x$ for the following set of numbers:

$-2, -6, 3, 5, -4$

\[\Sigma x = -2 + (-6) + 3 + 5 + (-4)\]
\[= (-8) + 3 + 5 + (-4)\]
\[= (-8) + 8 + (-4)\]
\[= -4\]
Find $\Sigma x$ for each set of numbers.

(a) $-3, 4, -7, -9, 8$  
(b) $-2, 6, -5, -3, 4, 7$

**CHECK YOURSELF ANSWERS**

1. (a) 1; (b) 10  
2. (a) 38; (b) 45; (c) 84  
3. (a) 10; (b) 7; (c) 7  
4. (a) 2; (b) 7  
5. $-31$  
6. (a) $-14$; (b) 35; (c) $-4$; (d) $-9$; (e) 9  
7. (a) $-2$; (b) $-2$  
8. (a) $-7$; (b) 7
Evaluate each of the expressions if \( a = -2, \ b = 5, \ c = -4, \) and \( d = 6. \)

1. \( 3c - 2b \)

2. \( 4e - 2b \)

3. \( 8b + 2c \)

4. \( 7a - 2c \)

5. \( -b^2 + b \)

6. \( (-b)^2 + b \)

7. \( 3a^2 \)

8. \( 6c^2 \)

9. \( c^2 - 2d \)

10. \( 3a^2 + 4c \)

11. \( 2a^2 + 3b^2 \)

12. \( 4b^2 - 2c^2 \)

13. \( 2(a + b) \)

14. \( 5(b - c) \)

15. \( 4(2a - d) \)

16. \( 6(3c - d) \)

17. \( a(b + 3c) \)

18. \( c(3a - d) \)

19. \( \frac{6d}{c} \)

20. \( \frac{8b}{5c} \)

21. \( \frac{3d + 2c}{b} \)

22. \( \frac{2b + 3d}{2a} \)
23. \( \frac{2b - 3a}{c + 2d} \)  
24. \( \frac{3d - 2b}{5a + d} \)  
25. \( d^2 - b^2 \)  
26. \( c^2 - a^2 \)  
27. \( (d - b)^2 \)  
28. \( (c - a)^3 \)  
29. \( (d - b)(d + b) \)  
30. \( (c - a)(c + a) \)  
31. \( a^3 - b^3 \)  
32. \( c^3 + a^3 \)  
33. \( (d - b)^3 \)  
34. \( (c + a)^3 \)  
35. \( (d - b)(d^2 + db + b^2) \)  
36. \( (c + a)(c^2 - ac + a^2) \)  
37. \( b^2 + a^2 \)  
38. \( d^2 - a^2 \)  
39. \( (b + a)^3 \)  
40. \( (d - a)^2 \)  
41. \( a^2 + 2ad + d^2 \)  
42. \( b^2 - 2bc + c^2 \)  

Use your calculator to evaluate each expression if \( x = -2.34, y = -3.14, \) and \( z = 4.12. \) Round your answer to the nearest tenth.

43. \( x + yz \)  
44. \( y - 2z \)  
45. \( x^2 - z^2 \)  
46. \( x^2 + y^2 \)
47. \( \frac{xy}{z - x} \)

48. \( \frac{y^2}{zy} \)

49. \( \frac{2x + y}{2x + z} \)

50. \( \frac{x^2y^2}{xz} \)

For the following data sets, evaluate \( \Sigma x \).

51. 1, 2, 3, 7, 8, 9, 11

52. 2, 4, 5, 6, 10, 11, 12

53. -5, -3, -1, 2, 3, 4, 8

54. -4, -2, -1, 5, 7, 8, 10

55. 3, 2, -1, -4, -3, 8, 6

56. 3, -4, 2, -1, 2, -7, 9

57. \(-\frac{1}{2}, -\frac{3}{4}, 2, 3, \frac{1}{4}, \frac{3}{2}, -1\)

58. \(-\frac{1}{3}, -\frac{5}{3}, -1, 1, 3, \frac{2}{3}, \frac{5}{3}\)

59. -2.5, -3.2, 2.6, -1, 2, 4, -3

60. -2.4, -3.1, -1.7, 3, 1, 2, 5

In each of the following problems, decide if the given values make the statement true or false.

61. \( x - 7 = 2y + 5; x = 22, y = 5 \)

62. \( 3(x - y) = 6; x = 5, y = -3 \)

63. \( 2(x + y) = 2x + y; x = -4, y = -2 \)

64. \( x^2 - y^2 = x - y; x = 4, y = -3 \)

65. Electrical resistance. The formula for the total resistance in a parallel circuit is given by the formula \( R_T = \frac{R_1R_2}{R_1 + R_2} \). Find the total resistance if \( R_1 = 6 \) ohms (\( \Omega \)) and \( R_2 = 10 \Omega \).

66. Area. The formula for the area of a triangle is given by \( A = \frac{1}{2}ab \). Find the area of a triangle if \( a = 4 \) centimeters (cm) and \( b = 8 \) cm.
67. **Perimeter.** The perimeter of a rectangle of length \( L \) and width \( W \) is given by the formula \( P = 2L + 2W \). Find the perimeter when \( L = 10 \) inches (in.) and \( W = 5 \) in.

68. **Simple interest.** The simple interest \( I \) on a principal of \( P \) dollars at interest rate \( r \) for time \( t \), in years, is given by \( I = Prt \). Find the simple interest on a principal of $6000 at 8 percent for 3 years. (Note: 8% = 0.08)

69. **Simple interest.** Use the simple interest formula to find the principal if the total interest earned was $150 and the rate of interest was 4% for 2 years.

70. **Simple interest.** Use the simple interest formula to find the rate of interest if $10,000 earns $1500 interest in 3 years.

71. **Temperature conversion.** The formula that relates Celsius and Fahrenheit temperature is \( F = \frac{9}{5}C + 32 \). If the temperature of the day is \(-10^\circ C\), what is the Fahrenheit temperature?

72. **Geometry.** If the area of a circle whose radius is \( r \) is given by \( A = \pi r^2 \), where \( \pi = 3.14 \), find the area when \( r = 3 \) meters (m).

73. Write an English interpretation of each of the following algebraic expressions.

(a) \((2x^2 - y)^3\)  
(b) \(3n - \frac{n - 1}{2}\)  
(c) \((2n + 3)(n - 4)\)
74. Is \( a^n + b^n = (a + b)^n \)? Try a few numbers and decide if you think this is true for all numbers, for some numbers, or never true. Write an explanation of your findings and give examples.

75. Enjoyment of patterns in art, music, and language is common to all cultures, and many cultures also delight in and draw spiritual significance from patterns in numbers. One such set of patterns is that of the “magic” square. One of these squares appears in a famous etching by Albrecht Dürer, who lived from 1471 to 1528 in Europe. He was one of the first artists in Europe to use geometry to give perspective, a feeling of three dimensions, in his work.

The magic square in his work is this one:

<table>
<thead>
<tr>
<th>16</th>
<th>3</th>
<th>2</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>15</td>
<td>1</td>
</tr>
</tbody>
</table>

Why is this square “magic”? It is magic because every row, every column, and both diagonals add to the same number. In this square there are sixteen spaces for the numbers 1 through 16.

**Part 1:** What number does each row and column add to?

Write the square that you obtain by adding \(-17\) to each number. Is this still a magic square? If so, what number does each column and row add to? If you add 5 to each number in the original magic square, do you still have a magic square? You have been studying the operations of addition, multiplication, subtraction, and division with integers and with rational numbers. What operations can you perform on this magic square and still have a magic square? Try to find something that will not work. Use algebra to help you decide what will work and what won’t. Write a description of your work and explain your conclusions.

**Part 2:** Here is the oldest published magic square. It is from China, about 250 B.C.E. Legend has it that it was brought from the River Lo by a turtle to the Emperor Yu, who was a hydraulic engineer.

<table>
<thead>
<tr>
<th>4</th>
<th>9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Check to make sure that this is a magic square. Work together to decide what operation might be done to every number in the magic square to make the sum of each row, column, and diagonal the opposite of what it is now. What would you do to every number to cause the sum of each row, column, and diagonal to equal zero?
Getting Ready for Section 1.6 [Sections 1.3 and 1.4]

(a) \((8 + 9) - 5\)  
(b) \(15 - 4 - 11\)  
(c) \(5(4 + 3) - 9\)  
(d) \(-3(5 - 7) + 11\)  
(e) \(-6(-9 + 7) - 4\)  
(f) \(8 - 7(-2 - 6)\)

Answers

1. -22  3. 32  5. -20  7. 12  9. 4  11. 83  13. 6  
15. -40  17. 14  19. -9  21. 2  23. 2  25. 11  27. 1  
29. 11  31. 91  33. 1  35. 91  37. 29  39. 9  41. 16  
43. -15.3  45. -11.5  47. 1.1  49. 14.0  51. 41  53. 8  
55. 11  57. \(\frac{9}{2}\)  59. -1.1  61. True  63. False  65. 3.75 \(\Omega\)  
67. 30 in.  69. $1875  71. 14\degree F  73.  
75. 

a. 12  b. 0  c. 26  d. 17  e. 8  f. 64