Factoring by Grouping

OBJECTIVES

1. Factor a polynomial by grouping terms
2. Rewrite a polynomial so that it can be factored by the method of grouping terms

Some polynomials can be factored by grouping the terms and finding common factors within each group. Such a process is called factoring by grouping, and will be explored in this section.

Recall that in Section 4.1, we looked at the expression

\[3x(x + y) + 2(x + y)\]

and found that we could factor out the common binomial, \((x + y)\), giving us

\[(x + y)(3x + 2)\]

That technique will be used in the first example.

Example 1
Factoring by Grouping Terms

Suppose we want to factor the polynomial

\[ax - ay + bx - by\]

As you can see, the polynomial has no common factors. However, look at what happens if we separate the polynomial into two groups of two terms.

\[
\begin{align*}
ax - ay + bx - by &= ax - ay + bx - by \\
&= (ax - ay) + (bx - by) \\
&= a(x - y) + b(x - y)
\end{align*}
\]

Now each group has a common factor, and we can write the polynomial as

\[a(x - y) + b(x - y)\]

In this form, we can see that \(x - y\) is the GCF. Factoring out \(x - y\), we get

\[(x - y)(a + b)\]

CHECK YOURSELF 1

Use the factoring by grouping method.

\[x^2 - 2xy + 3x - 6y\]

Be particularly careful of your treatment of algebraic signs when applying the factoring by grouping method. Consider Example 2.

Example 2
Factoring by Grouping Terms

Factor \(2x^3 - 3x^2 - 6x + 9\).
We group the polynomial as follows.

\[2x^3 - 3x^2 - 6x + 9\]

Remove the common factor of \(-3\) from the second two terms.

\[= x^2(2x - 3) - 3(2x - 3)\]

\[= (2x - 3)(x^2 - 3)\]

**NOTE** Notice that \(9 = (-3)(-3)\).

**CHECK YOURSELF 2**

Factor by grouping.

\[3y^3 + 2y^2 - 6y - 4\]

It may also be necessary to change the order of the terms as they are grouped. Look at Example 3.

**Example 3**

**Factoring by Grouping Terms**

Factor \(x^2 - 6yz + 2xy - 3xz\).

Grouping the terms as before, we have

\[x^2 - 6yz + 2xy - 3xz\]

Do you see that we have accomplished nothing because there are no common factors in the first group?

We can, however, rearrange the terms to write the original polynomial as

\[x^2 + 2xy - 3xz - 6yz\]

\[= x(x + 2y) - 3z(x + 2y)\]

\[= (x + 2y)(x - 3z)\]

**Note:** It is often true that the grouping can be done in more than one way. The factored form will be the same.

**CHECK YOURSELF 3**

We can write the polynomial of Example 3 as

\[x^2 - 3xz + 2xy - 6yz\]

Factor, and verify that the factored form is the same in either case.

**CHECK YOURSELF ANSWERS**

1. \((x - 2y)(x + 3)\)
2. \((3y + 2)(y^2 - 2)\)
3. \((x - 3z)(x + 2y)\)
Factor each polynomial by grouping the first two terms and the last two terms.

1. \(x^3 - 4x^2 + 3x - 12\)  
2. \(x^3 - 6x^2 + 2x - 12\)

3. \(a^3 - 3a^2 + 5a - 15\)  
4. \(6x^3 - 2x^2 + 9x - 3\)

5. \(10x^3 + 5x^2 - 2x - 1\)  
6. \(x^5 + x^3 - 2x^2 - 2\)

7. \(x^4 - 2x^3 + 3x - 6\)  
8. \(x^3 - 4x^2 + 2x - 8\)

Factor each polynomial completely by removing any common factors, and then factor by grouping. Do not combine like terms.

9. \(3x - 6 + xy - 2y\)  
10. \(2x - 10 + xy - 5y\)

11. \(ab - ac + b^2 - bc\)  
12. \(ax + 2a + bx + 2b\)

13. \(3x^2 - 2xy + 3x - 2y\)  
14. \(xy - 5y^2 - x + 5y\)

15. \(5x^2 + 15st - 2st - 6t^2\)  
16. \(3a^3 + 3ab^2 + 2a^2b + 2b^3\)

17. \(3x^3 + 6x^2y - x^2y - 2xy^2\)  
18. \(2p^4 + 3p^3q - 2p^3q - 3p^2q^2\)

19. \(x^4 + 5x^3 - 2x^2 - 10x\)  
20. \(x^4y - 2x^3y + x^4 - 2x^3\)

21. \(2x^3 - 2x^2 + 3x^2 - 3x\)  
22. \(3b^4 - 3b^3c + 2b^3c - 2b^2c^2\)
Getting Ready for Section 4.6 [Section 3.4]  
Multiply.
(a) \((2x - 1)(2x + 3)\)  
(b) \((3a - 1)(a + 4)\)  
(c) \((x - 4)(2x - 3)\)  
(d) \((2w - 11)(w + 2)\)  
(e) \((y + 5)(2y + 9)\)  
(f) \((2a + 1)(x - 12)\)  
(g) \((p + 9)(2p + 5)\)  
(h) \((3a - 5)(2a + 4)\)  

Answers

1. \((x - 4)(x^2 + 3)\)  
3. \((a - 3)(a^2 + 5)\)  
5. \((5x^2 - 1)(2x + 1)\)  
13. \((3x - 2y)(x + 1)\)  
15. \((s + 3t)(5s - 2t)\)  
17. \(x(x + 2y)(3x - y)\)  
19. \(x(x + 5)(x^2 - 2)\)  
21. \(x(x - 1)(2x + 3)\)  

\textbf{a.} \(4x^2 + 4x - 3\)  
\textbf{b.} \(3a^2 + 11a - 4\)  
\textbf{c.} \(2x^2 - 11x + 12\)  
\textbf{d.} \(2w^2 - 7w - 22\)  
\textbf{e.} \(2y^2 + 19y + 45\)  
\textbf{f.} \(2x^2 - 23x - 12\)  
\textbf{g.} \(2p^2 + 23p + 45\)  
\textbf{h.} \(6a^2 + 2a - 20\)