Direct Variation

OBJECTIVES

1. Write an equation for a direct variation relationship
2. Graph the equation of a direct variation relationship

Pedro makes $25 an hour as an electrician. If he works 1 hour, he makes $25; if he works 2 hours, he makes $50; and so on. We say his total pay varies directly with the number of hours worked.

Definitions: Direct Variation

If \( y \) is a constant multiple of \( x \), we write

\[ y = kx \]

in which \( k \) is a constant

We say that \( y \) varies directly as \( x \), or that \( y \) is directly proportional to \( x \). The constant \( k \) is called the constant of variation.

Example 1

Writing an Equation for Direct Variation

Marina earns $9 an hour as a tutor. Write the equation that describes the relationship between the number of hours she works and her pay.

Her pay \( (P) \) is equal to the rate of pay \( (r) \) times the number of hours worked \( (h) \), so

\[ P = r \cdot h \quad \text{or} \quad P = 9h \]

CHECK YOURSELF 1

Sorina is driving at a constant rate of 50 m/h. Write the equation that shows the distance she travels \( (d) \) in \( h \) hours.

NOTE Remember that \( k \) is the constant of variation.

Example 2

Finding the Constant of Variation

If \( y \) varies directly with \( x \), and \( y = 30 \) when \( x = 6 \), find \( k \).

Because \( y \) varies directly with \( x \), we know from the definition that

\[ y = kx \]
We need to find $k$. We do this by substituting $30$ for $y$ and $6$ for $x$.

$$30 = k(6) \quad \text{or} \quad k = 5$$

**CHECK YOURSELF 2**

*If $y$ varies directly with $x$ and $y = 100$ when $x = 25$, find the constant of variation.*

The graph for a linear equation of direct variation will always pass through the origin. The next example will illustrate.

**Example 3**

**Graphing an Equation of Direct Variation**

Let $y$ vary directly as $x$, with a constant of variation $k = 3.5$. Graph the equation of variation.

The equation of variation is $y = 3.5x$, so the graph will have a slope of $3.5$.

**CHECK YOURSELF 3**

*Let $y$ vary directly as $x$, with a constant of variation $k = \frac{7}{3}$ Graph the equation of variation.*

With many applications it is necessary to adjust the scale on the $x$ or $y$ axis to present a reasonable graph. The next example will help prepare us for applications of this nature.

**Example 4**

**Graphing an Equation of Direct Variation**

Let $y$ vary directly as $x$, with a constant of variation $k = 200$. Graph the equation of variation.

The equation is $y = 200x$. What happens if we try to sketch this graph on a grid with the same scale on the $x$ and $y$ axes? The slope indicates that $y$ increases by $200$ when $x$ increases by $1$. Can you see that it would be impossible to produce a meaningful graph on this grid?
Let $y$ vary directly as $x$, with a constant of variation $k = -1500$. Graph the equation of variation.

Now we will examine an application that requires us to adjust the scale on the axes.

Example 5

Graphing a Direct Variation Equation

Tim Duncan earns approximately $5000 per minute for playing basketball. Sketch the graph that represents the equation of direct variation between minutes played and money earned.

We know that the equation will be $y = 5000x$.

For this graph, we will use only the first quadrant. Do you see why? All of the other quadrants represent negative time, negative money, or both.

We will also use a different scale on each axis. The $x$ axis, which represents minutes played, is marked every 200 minutes, to 2000 minutes. The $y$ axis, representing money made, is marked every 1 million dollars for $10,000,000$.

As we do with any direct variation graph, we start at the origin. The slope of 5000 can be looked at in many ways. It is usually easiest to move one mark along the $x$ axis and see how much change we have in the $y$ direction.
The average secretary makes about $0.24 for each minute worked. Sketch the graph of the equation of direct variation.

One mark in the x direction is 200 minutes. Because \( y = 5000(200) = 1,000,000 \), we find our second point at (200, $1,000,000). Connecting that point to the origin, we get the graph shown.

**CHECK YOURSELF 5**

The average secretary makes about $0.24 for each minute worked. Sketch the graph of the equation of direct variation.

**CHECK YOURSELF ANSWERS**

1. \( d = 50h \)  
2. \( k = 4 \)  
3.  
4.  
5.  

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1. **Salary.** Robin earns $12 per hour. Write an equation that shows how much she makes \((S)\) in \(h\) hours.

2. **Salary.** Kwang earns $11.50 per hour. Write an equation that shows how much he earns \((S)\) in \(h\) hours.

3. **Distance.** Lee is traveling at a constant rate of 55 miles per hour (mi/h). Write an equation that shows how far he travels \((D)\) in \(h\) hours.

4. **Distance.** An airplane is traveling at a constant rate of 450 mi/h. Write an equation that shows how far the plane travels \((D)\) in \(h\) hours.

In exercises 5 to 10, find the constant of variation \(k\).

5. \(y\) varies directly with \(x\); \(y = 54\) when \(x = 6\).

6. \(m\) varies directly with \(n\); \(m = 144\) when \(n = 8\).

7. \(V\) varies directly with \(h\); \(V = 189\) when \(h = 9\).

8. \(d\) varies directly with \(t\); \(d = 750\) when \(t = 15\).

9. \(y\) varies directly with \(x\); \(y = 2100\) when \(x = 600\).

10. \(y\) varies directly with \(x\); \(y = 400\) when \(x = 1000\).

In exercises 11 to 18, \(y\) varies directly with \(x\) and the value of \(k\) is given. Graph the equation of variation.

11. \(k = 2\)

12. \(k = 4\)
13. $k = 2.5$

14. $k = \frac{11}{5}$

15. $k = 100$

16. $k = 300$

17. $k = 50$

18. $k = 400$
19. At a factory that makes grinding wheels, Kalila makes $0.20 for each wheel completed. Sketch the equation of direct variation.

20. Palmer makes $1.25 per page for each page that he types. Sketch the equation of direct variation.

21. Cesar makes $2.50 for each tire he details. Sketch the equation of direct variation.
22. Tanesha makes $0.15 for each problem she checks in a math text. Sketch the equation of direct variation.

23. **Salary.** Josephine works part-time in a local video store. Her salary varies directly as the number of hours worked. Last week she earned $43.20 for working 8 hours. This week she earned $118.80. How many hours did she work this week?

24. **Revenue.** The revenue for a sandwich shop is directly proportional to its advertising budget. When the owner spent $2000 a month on advertising, the revenue was $120,000. If the revenue is now $180,000, how much is the owner spending on advertising?

**Answers**

1. \( S = 12h \)
2. 
3. \( D = 55h \)
4. 9
5. 21
6. 3.5
7. 22 hours