

7.1

The Slope-Intercept Form

7.1 OBJECTIVES

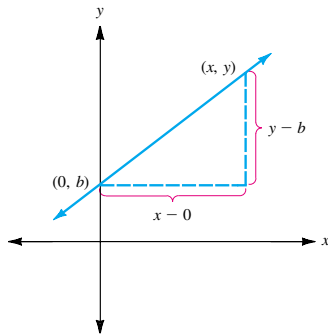
1. Find the slope and y intercept from the equation of a line
2. Given the slope and y intercept, write the equation of a line
3. Use the slope and y intercept to graph a line

In Chapter 6, we used two points to find the slope of a line. In this chapter we will use the slope to find the graph of an equation.

First, we want to consider finding the equation of a line when its slope and y intercept are known.

Suppose that the y intercept of a line is $(0, b)$. Then the point at which the line crosses the y axis has coordinates $(0, b)$. Look at the sketch at left.

Now, using any other point (x, y) on the line and using our definition of slope, we can write



$$m = \frac{y - b}{x - 0} \quad (1)$$

Change in y .

Change in x .

or

$$m = \frac{y - b}{x} \quad (2)$$

Multiplying both sides of equation (2) by x , we have

$$mx = y - b \quad (3)$$

Finally, adding b to both sides of equation (3) gives

$$mx + b = y$$

or

$$y = mx + b \quad (4)$$

We can summarize the above discussion as follows:

Definitions: The Slope-Intercept Form for a Line

An equation of the line with slope m and y intercept $(0, b)$ is

$$y = mx + b$$

NOTE In this form, the equation is *solved for y* . The coefficient of x will give you the slope of the line, and the constant term gives the y intercept.

Finding the Slope and y Intercept

Find the slope and y intercept for the graph of the equation

$$y = -\frac{2}{3}x - 5$$

↑ ↑
 m b

The slope of the line is $-\frac{2}{3}$; the y intercept is $(0, -5)$.

**CHECK YOURSELF 1**

Find the slope and y intercept for the graph of each of the following equations.

(a) $y = -3x - 7$

(b) $y = \frac{3}{4}x + 5$

As Example 2 illustrates, we may have to solve for y as the first step in determining the slope and the y intercept for the graph of an equation.

Example 2**Finding the Slope and y Intercept**

Find the slope and y intercept for the graph of the equation

$$3x + 2y = 6$$

First, we must solve the equation for y .

$$3x + 2y = 6$$

$$2y = -3x + 6 \quad \text{Add } (-3x) \text{ to both sides.}$$

$$y = -\frac{3}{2}x + 3 \quad \text{Divide each term by 2.}$$

The equation is now in slope-intercept form. The slope is $-\frac{3}{2}$, and the y intercept is $(0, 3)$.

NOTE If we write the equation as

$$y = \frac{-3x + 6}{2}$$

it is more difficult to identify the slope and the intercept.

**CHECK YOURSELF 2**

Find the slope and y intercept for the graph of the equation

$$2x - 5y = 10$$

As we mentioned earlier, knowing certain properties of a line (namely, its slope and y intercept) will also allow us to write the equation of the line by using the slope-intercept form. Example 3 illustrates this approach.

Example 3**Writing the Equation of a Line**

Write the equation of a line with slope $-\frac{3}{4}$ and y intercept $(0, -3)$.

We know that $m = -\frac{3}{4}$ and $b = -3$. In this case,

$$y = -\frac{3}{4}x + (-3)$$

or

$$y = -\frac{3}{4}x - 3$$

which is the desired equation.

**CHECK YOURSELF 3**

Write the equation of a line with the following:

(a) slope -2 and y intercept $(0, 7)$ (b) slope $\frac{2}{3}$ and y intercept $(0, -3)$

We can also use the slope and y intercept of a line in drawing its graph. Consider Example 4.

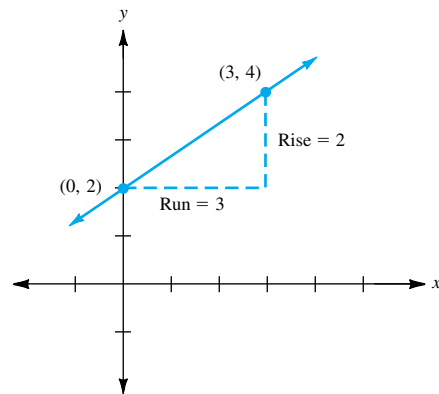
Example 4**Graphing a Line**Graph the line with slope $\frac{2}{3}$ and y intercept $(0, 2)$.

Because the y intercept is $(0, 2)$, we begin by plotting the point $(0, 2)$. Because the horizontal change (or run) is 3, we move 3 units to the right *from that y intercept*. Then because the vertical change (or rise) is 2, we move 2 units up to locate another point on the desired graph. Note that we will have located that second point at $(3, 4)$. The final step is to simply draw a line through that point and the y intercept.

NOTE

$$m = \frac{2}{3} = \frac{\text{rise}}{\text{run}}$$

The line rises from left to right because the slope is positive.



The equation of this line is $y = \frac{2}{3}x + 2$.

**CHECK YOURSELF 4**Graph the equation of a line with slope $\frac{3}{5}$ and y intercept $(0, -2)$.**Step by Step: Graphing by Using the Slope-Intercept Form**

- Step 1** Write the original equation of the line in slope-intercept form.
- Step 2** Determine the slope m and the y intercept $(0, b)$.
- Step 3** Plot the y intercept at $(0, b)$.
- Step 4** Use m (the change in y over the change in x) to determine a second point on the desired line.
- Step 5** Draw a line through the two points determined above to complete the graph.

You have now seen two methods for graphing lines: the slope-intercept method (Section 7.1) and the intercept method (Section 6.3). When you graph a linear equation, you should first decide which is the appropriate method.

Example 5

Selecting an Appropriate Graphing Method

Decide which of the two methods for graphing lines—the intercept method or the slope-intercept method—is more appropriate for graphing equations (a), (b), and (c).

(a) $2x - 5y = 10$

Because both intercepts are easy to find, you should choose the intercept method to graph this equation.

(b) $2x + y = 6$

This equation can be quickly graphed by either method. As it is written, you might choose the intercept method. It can, however, be rewritten as $y = -2x + 6$. In that case the slope-intercept method is more appropriate.

(c) $y = \frac{1}{4}x - 4$

Because the equation is in slope-intercept form, that is the more appropriate method to choose.



CHECK YOURSELF 5

Which would be more appropriate for graphing each equation, the intercept method or the slope-intercept method?

(a) $x + y = -2$

(b) $3x - 2y = 12$

(c) $y = -\frac{1}{2}x - 6$



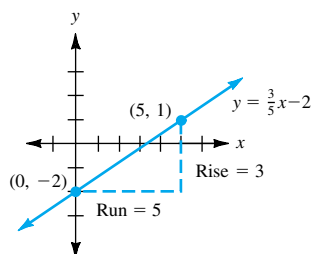
CHECK YOURSELF ANSWERS

1. (a) Slope is -3 , y intercept is $(0, -7)$; (b) Slope is $\frac{3}{4}$, y intercept is $(0, 5)$

2. $y = \frac{2}{5}x - 2$; the slope is $\frac{2}{5}$; the y intercept is $(0, -2)$

3. (a) $y = -2x + 7$; (b) $y = \frac{2}{3}x - 3$

4. 5. (a) Either; (b) intercept; (c) slope-intercept



7.1

Exercises

Find the slope and y intercept of the line represented by each of the following equations.

1. $y = 3x + 5$

2. $y = -7x + 3$

3. $y = -2x - 5$

4. $y = 5x - 2$

5. $y = \frac{3}{4}x + 1$

6. $y = -4x$

7. $y = \frac{2}{3}x$

8. $y = -\frac{3}{5}x - 2$

9. $4x + 3y = 12$

10. $2x + 5y = 10$

11. $y = 9$

12. $2x - 3y = 6$

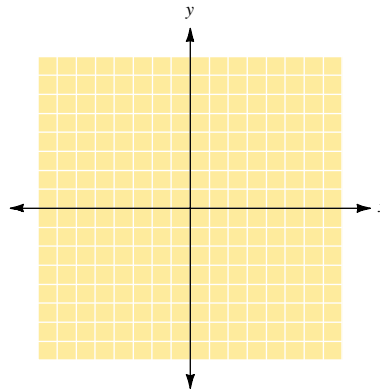
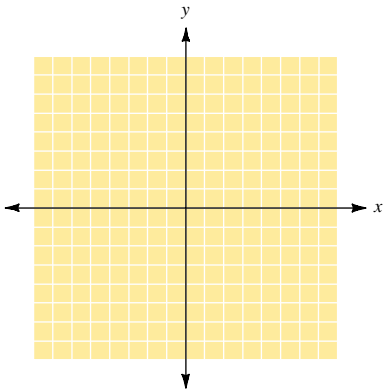
13. $3x - 2y = 8$

14. $x = 5$

Write the equation of the line with given slope and y intercept. Then graph each line, using the slope and y intercept.

15. Slope: 3; y intercept: (0, 5)

16. Slope: -2; y intercept: (0, 4)



Name _____

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ANSWERS

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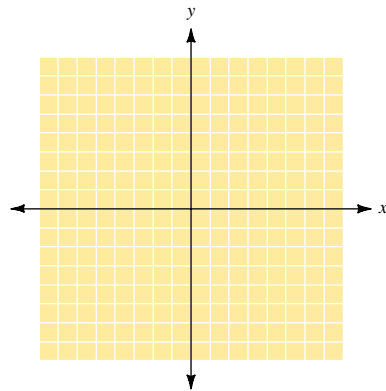
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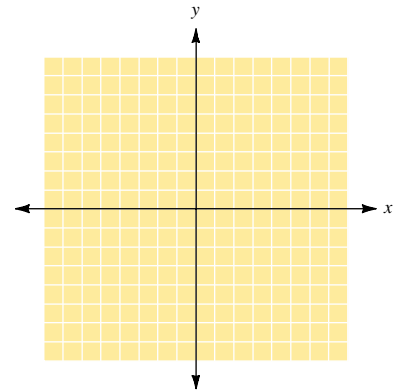
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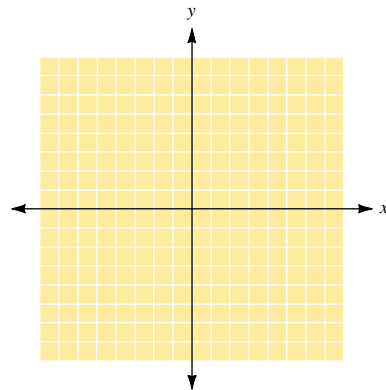
17. Slope: -3 ; y intercept: $(0, 4)$



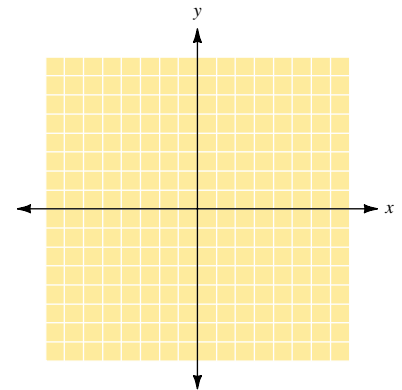
18. Slope: 5 ; y intercept: $(0, -2)$



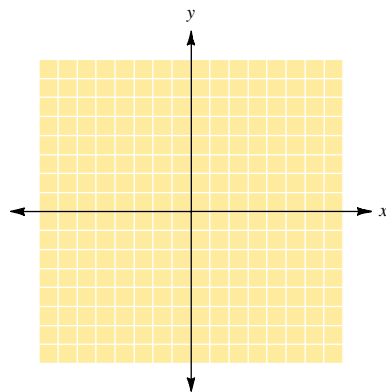
19. Slope: $\frac{1}{2}$; y intercept: $(0, -2)$



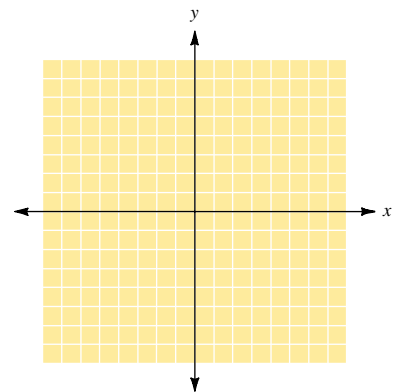
20. Slope: $-\frac{3}{4}$; y intercept: $(0, 8)$



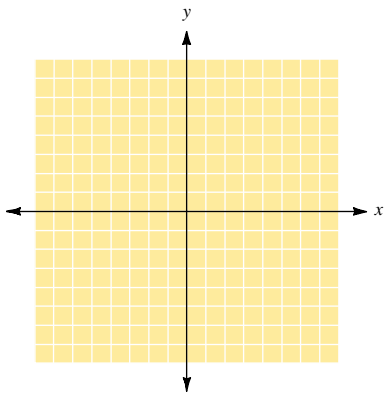
21. Slope: $-\frac{2}{3}$; y intercept: $(0, 0)$



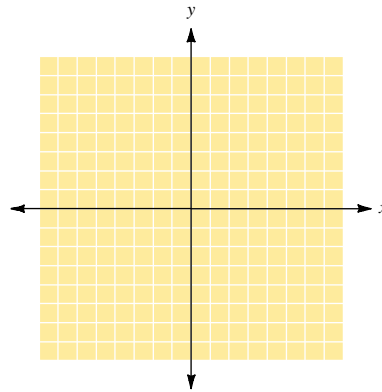
22. Slope: $\frac{2}{3}$; y intercept: $(0, -2)$



23. Slope: $\frac{3}{4}$; y intercept: (0, 3)



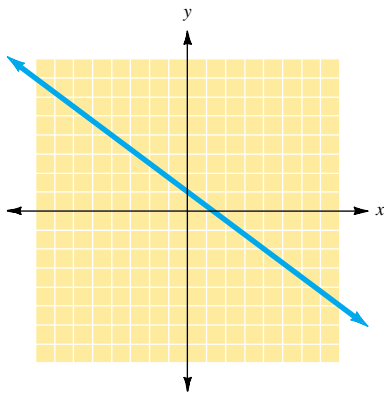
24. Slope: -3 ; y intercept: (0, 0)



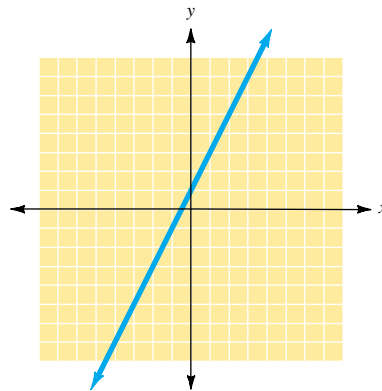
In exercises 25 to 32, match the graph with one of the equations below.

- (a) $y = 2x$, (b) $y = x + 1$, (c) $y = -x + 3$, (d) $y = 2x + 1$,
 (e) $y = -3x - 2$, (f) $y = \frac{2}{3}x + 1$, (g) $y = -\frac{3}{4}x + 1$, (h) $y = -4x$

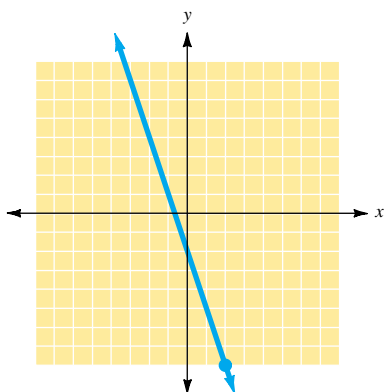
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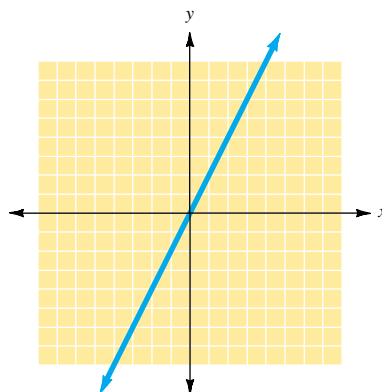
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ANSWERS

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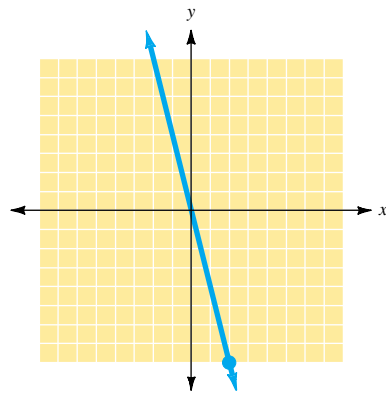
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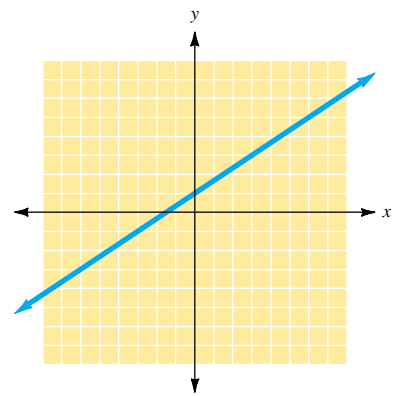
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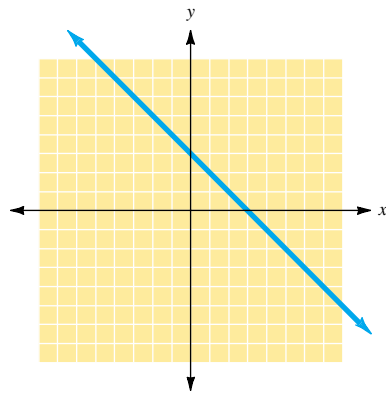
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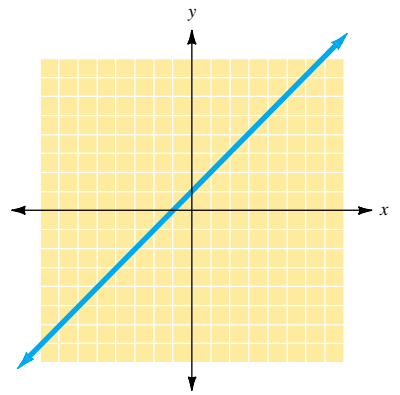
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In which quadrant(s) are there no solutions for each line?



33. $y = 2x + 1$

34. $y = 3x + 2$

35. $y = -x + 1$

36. $y = -2x + 5$

37. $y = -2x - 5$

38. $y = -5x - 7$

39. $y = 3$




40. $x = -2$

41. **Recycling.** The equation $y = 0.10x + 200$ describes the award money in a recycling contest. What are the slope and the y intercept for this equation?
42. **Fundraising.** The equation $y = 15x - 100$ describes the amount of money a high school class might earn from a paper drive. What are the slope and y intercept for this equation?
43. **Science.** On a certain February day in Philadelphia, the temperature at 6:00 A.M. was 10°F . By 2:00 P.M. the temperature was up to 26°F . What was the hourly rate of temperature change?
44. **Slope of a roof.** A roof rises 8.75 feet (ft) in a horizontal distance of 15.09 ft. Find the slope of the roof to the nearest hundredth.
45. **Slope of airplane descent.** An airplane covered 15 miles (mi) of its route while decreasing its altitude by 24,000 ft. Find the slope of the line of descent that was followed. (1 mi = 5280 ft.) Round to the nearest hundredth.

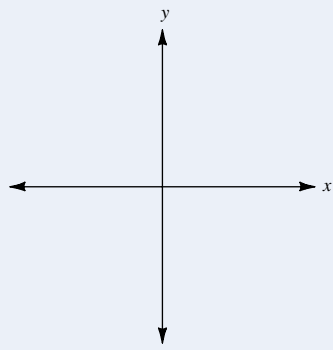


46. **Slope of road descent.** Driving down a mountain, Tom finds that he has descended 1800 ft in elevation by the time he is 3.25 mi horizontally away from the top of the mountain. Find the slope of his descent to the nearest hundredth.
47. Complete the following statement: “The difference between undefined slope and zero slope is”
48. Complete the following: “The slope of a line tells you”
49. In a study on nutrition conducted in 1984, 18 normal adults aged 23 to 61 years old were measured for body fat, which is given as percentage of weight. The mean (average) body fat percentage for women 40 years old was 28.6 percent, and for women 53 years old was 38.4 percent. Work with a partner to decide how to show this information on a scatterplot. Try to find a linear equation that will tell you percentage of body fat based on a woman’s age. What does your equation give for 20 years of age? For 60? Do you think a linear model works well for predicting body fat percentage in women as they age?



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49.  _____

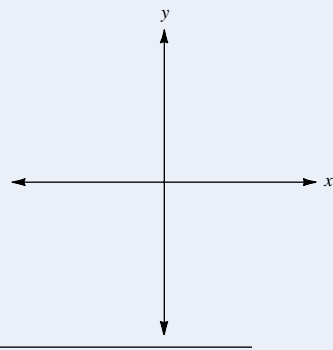
50. 



50. On two occasions last month, Sam Johnson rented a car on a business trip. Both times it was the same model from the same company, and both times it was in San Francisco. Sam now has to fill out an expense account form and needs to know how much he was charged per mile and the base rate. On both occasions he dropped the car at the airport booth and just got the total charge, not the details. All Sam knows is that he was charged \$210 for 625 miles on the first occasion and \$133.50 for 370 miles on the second trip. Sam has called accounting to ask for help. Plot these two points on a graph, and draw the line that goes through them. What question does the slope of the line answer for Sam? How does the y intercept help? Write a memo to Sam explaining the answers to his question and how a knowledge of algebra and graphing has helped you find the answers.



51. _____



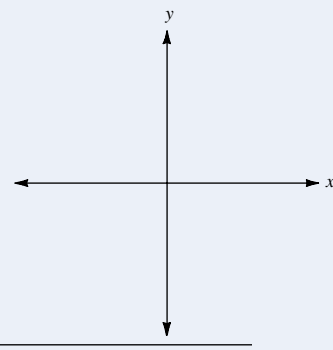
51. On the same graph, sketch the following lines:



$$y = 2x - 1 \quad \text{and} \quad y = 2x + 3$$

What do you observe about these graphs? Will the lines intersect?

52. _____

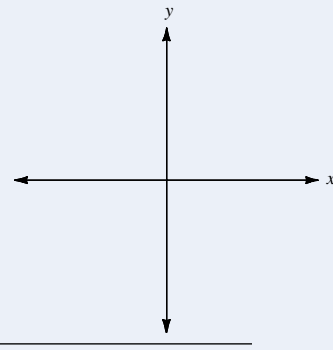


52. Repeat exercise 51 using



$$y = -2x + 4 \quad \text{and} \quad y = -2x + 1$$

53. _____



53. On the same graph, sketch the following lines:



$$y = \frac{2}{3}x \quad \text{and} \quad y = -\frac{3}{2}x$$

What do you observe concerning these graphs? Find the product of the slopes of these two lines.

54. _____

54. Repeat exercise 53 using



$$y = \frac{4}{3}x \quad \text{and} \quad y = -\frac{3}{4}x$$

55. Based on exercises 53 and 54, write the equation of a line that is perpendicular to

$$y = \frac{3}{5}x$$



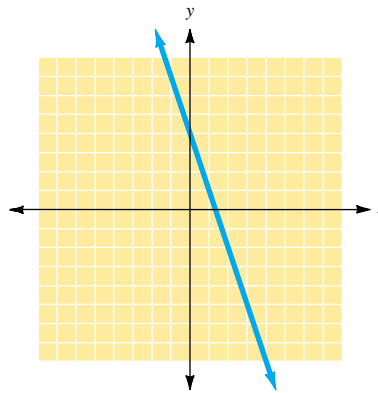
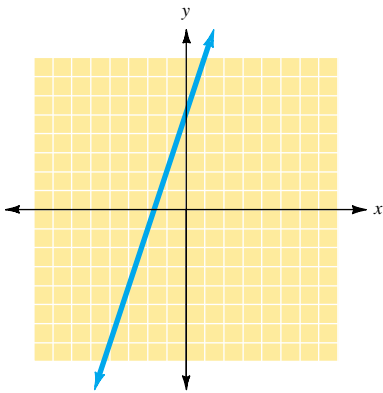
Getting Ready for Section 7.2 [Section 6.4]

Find the slope of the line connecting the given points.

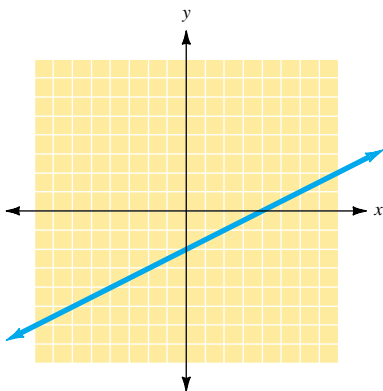
- (a) $(-4, 6)$ and $(3, 20)$ (b) $(2, 8)$ and $(-6, -8)$ (c) $(5, -7)$ and $(-5, 3)$
 (d) $(2, 8)$ and $(2, 5)$ (e) $(6, 9)$ and $(3, 9)$ (f) $(4, 6)$ and $(-4, -2)$

Answers

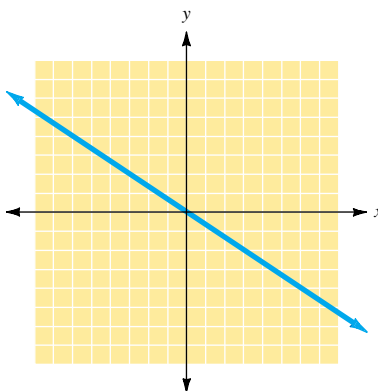
1. Slope 3, y intercept $(0, 5)$ 3. Slope -2 , y intercept $(0, -5)$
 5. Slope $\frac{3}{4}$, y intercept $(0, 1)$ 7. Slope $\frac{2}{3}$, y intercept $(0, 0)$
 9. Slope $-\frac{4}{3}$, y intercept $(0, 4)$ 11. Slope 0, y intercept $(0, 9)$
 13. Slope $\frac{3}{2}$, y intercept $(0, -4)$
 15. $y = 3x + 5$ 17. $y = -3x + 4$



19. $y = \frac{1}{2}x - 2$

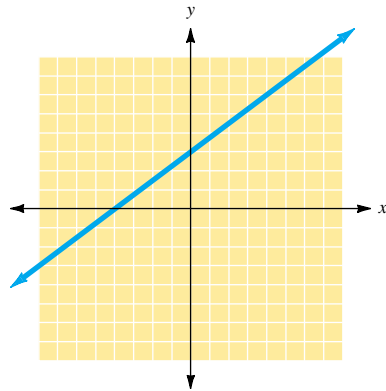


21. $y = -\frac{2}{3}x$



55. _____
 a. _____
 b. _____
 c. _____
 d. _____
 e. _____
 f. _____

23. $y = \frac{3}{4}x + 3$



25. *g* 27. *e* 29. *h* 31. *c* 33. IV 35. III 37. I
 39. III and IV 41. Slope = 0.10; *y* intercept = (0, 200) 43. 2°/hr

45. -0.30

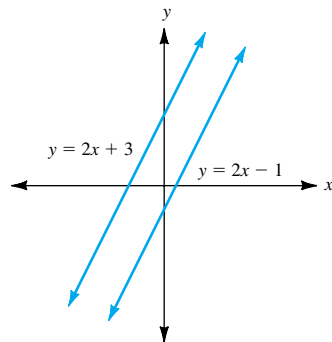
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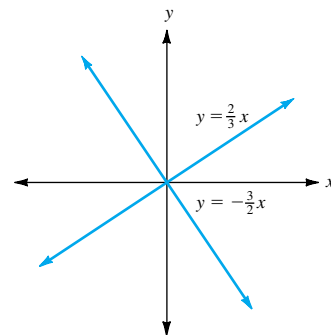
49.



51. Parallel lines; no



53. Perpendicular lines; -1



55. $y = -\frac{5}{3}x$ a. 2 b. 2 c. -1 d. Undefined e. 0 f. 1