The Point-Slope Form

**OBJECTIVES**

1. Given a point and a slope, find the graph of a line
2. Given a point and the slope, find the equation of a line
3. Given two points, find the equation of a line

Often in mathematics it is useful to be able to write the equation of a line, given its slope and any point on the line. In this section, we will derive a third special form for a line for this purpose.

Suppose that a line has slope \( m \) and that it passes through the known point \( P(x_1, y_1) \). Let \( Q(x, y) \) be any other point on the line. Once again we can use the definition of slope and write

\[
m = \frac{y - y_1}{x - x_1}
\]

Multiplying both sides of equation (1) by \( x - x_1 \), we have

\[
m(x - x_1) = y - y_1
\]

or

\[
y - y_1 = m(x - x_1)
\]

Equation (2) is called the point-slope form for the equation of a line, and all points lying on the line [including \((x_1, y_1)\)] will satisfy this equation. We can state the following general result.

**Rules and Properties:** Point-Slope Form for the Equation of a Line

The equation of a line with slope \( m \) that passes through point \((x_1, y_1)\) is given by

\[
y - y_1 = m(x - x_1)
\]

**Example 1**

Finding the Equation of a Line

Write the equation for the line that passes through point \((3, -1)\) with a slope of 3.

Letting \((x_1, y_1) = (3, -1)\) and \( m = 3 \) in point-slope form, we have

\[
y - (-1) = 3(x - 3)
\]

or

\[
y + 1 = 3x - 9
\]

We can write the final result in slope-intercept form as

\[
y = 3x - 10
\]

**CHECK YOURSELF 1**

Write the equation of the line that passes through point \((-2, -4)\) with a slope of \(\frac{3}{2}\). Write your result in slope-intercept form.
Because we know that two points determine a line, it is natural that we should be able to write the equation of a line passing through two given points. Using the point-slope form together with the slope formula will allow us to write such an equation.

**Example 2**

Finding the Equation of a Line

Write the equation of the line passing through (2, 4) and (4, 7).

First, we find \( m \), the slope of the line. Here

\[
\frac{y_2 - y_1}{x_2 - x_1} = m
\]

\[
\frac{7 - 4}{4 - 2} = m
\]

\[
m = \frac{3}{2}
\]

Now we apply the point-slope form with \( m = \frac{3}{2} \) and \((x_1, y_1) = (2, 4)\):

\[
y - 4 = \frac{3}{2}(x - 2)
\]

\[
y - 4 = \frac{3}{2}x - 3
\]

\[
y = \frac{3}{2}x + 1
\]

**NOTE** We could just as well have chosen to let \((x_1, y_1) = (4, 7)\). The resulting equation will be the same in either case. Take time to verify this for yourself.

**CHECK YOURSELF 2**

Write the equation of the line passing through \((-2, 5)\) and \((1, 3)\). Write your result in slope-intercept form.

A line with slope zero is a horizontal line. A line with an undefined slope is vertical. The next example illustrates the equations of such lines.

**Example 3**

Finding the Equation of a Line

(a) Find the equation of a line passing through \((7, -2)\) with a slope of zero.

We could find the equation by letting \( m = 0 \). Substituting the ordered pair \((7, -2)\) into the slope-intercept form, we can solve for \( b \).

\[
y = mx + b
\]

\[
-2 = 0(7) + b
\]

\[
-2 = b
\]

So,

\[
y = 0 \cdot x - 2 \quad \text{or} \quad y = -2
\]

It is far easier to remember that any line with a zero slope is a horizontal line and has the form

\[
y = b
\]
The value for \( b \) will always be the \( y \) coordinate for the given point.

\( \text{(b) Find the equation of a line with undefined slope passing through } (4, -5). \)

A line with undefined slope is vertical. It will always be of the form \( x = a \), in which \( a \) is the \( x \) coordinate for the given point. The equation is
\[ x = 4 \]

**CHECK YOURSELF 3**

(a) Find the equation of a line with zero slope that passes through point \((-3, 5)\).

(b) Find the equation of a line passing through \((-3, -6)\) with undefined slope.

Alternate methods for finding the equation of a line through two points exist and have particular significance in other fields of mathematics, such as statistics. The following example shows such an alternate approach.

**Example 4**

**Finding the Equation of a Line**

Write the equation of the line through points \((-2, 3)\) and \((4, 5)\).

First, we find \( m \), as before:
\[
m = \frac{5 - 3}{4 - (-2)} = \frac{2}{6} = \frac{1}{3}
\]

We now make use of the slope-intercept equation, but in a slightly different form.

Because \( y = mx + b \), we can write
\[ b = y - mx \]

Now letting \( x = -2, y = 3, \) and \( m = \frac{1}{3} \), we can calculate \( b \):
\[
b = 3 - \left(\frac{1}{3}\right)(-2) = 3 + \frac{2}{3} = \frac{11}{3}
\]

With \( m = \frac{1}{3} \) and \( b = \frac{11}{3} \), we can apply the slope-intercept form to write the equation of the desired line. We have
\[ y = \frac{1}{3}x + \frac{11}{3} \]

**CHECK YOURSELF 4**

Repeat the Check Yourself 2 exercise, using the technique illustrated in Example 4.

We now know that we can write the equation of a line once we have been given appropriate geometric conditions, such as a point on the line and the slope of that line. In some applications the slope may be given not directly but through specified parallel or perpendicular lines.
Finding the Equation of a Line

Find the equation of the line passing through \((-4, -3)\) and parallel to the line determined by \(3x + 4y = 12\).

First, we find the slope of the given parallel line, as before:

\[
3x + 4y = 12 \\
4y = -3x + 12 \\
y = -\frac{3}{4}x + 3
\]

\[\text{NOTE} \quad \text{The slope of the given line is } -\frac{3}{4}.\]

Now because the slope of the desired line must also be \(-\frac{3}{4}\), we can use the point-slope form to write the required equation:

\[
y - (-3) = -\frac{3}{4}[x - (-4)]
\]

This simplifies to

\[
y = -\frac{3}{4}x - 6
\]

and we have our equation in slope-intercept form.

**CHECK YOURSELF 5**

Find the equation of the line passing through \((5, 4)\) and perpendicular to the line with equation \(2x - 5y = 10\).

*Hint: Recall that the slopes of perpendicular lines are negative reciprocals of each other.*

The following chart summarizes the various forms of the equation of a line.

<table>
<thead>
<tr>
<th>Form</th>
<th>Equation for Line L</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>(ax + by = c)</td>
<td>Constants (a) and (b) cannot both be zero.</td>
</tr>
<tr>
<td>Slope-intercept</td>
<td>(y = mx + b)</td>
<td>Line (L) has (y) intercept ((0, b)) with slope (m).</td>
</tr>
<tr>
<td>Point-slope</td>
<td>(y - y_1 = m(x - x_1))</td>
<td>Line (L) passes through point ((x_1, y_1)) with slope (m).</td>
</tr>
<tr>
<td>Horizontal</td>
<td>(y = k)</td>
<td>Slope is zero.</td>
</tr>
<tr>
<td>Vertical</td>
<td>(x = h)</td>
<td>Slope is undefined.</td>
</tr>
</tbody>
</table>

**CHECK YOURSELF ANSWERS**

1. \(y = \frac{3}{2}x - 1\)  
2. \(y = -\frac{2}{3}x + \frac{11}{3}\)  
3. (a) \(y = 5\); (b) \(x = -3\)  
4. \(y = -\frac{2}{3}x + \frac{11}{3}\)  
5. \(y = -\frac{5}{2}x + \frac{33}{2}\)
7.3 Exercises

Write the equation of the line passing through each of the given points with the indicated slope. Give your results in slope-intercept form, where possible.

1. \((0, 2), m = 3\)  
2. \((0, -4), m = -2\)

3. \((0, 2), m = \frac{3}{2}\)  
4. \((0, -3), m = -2\)

5. \((0, 4), m = 0\)  
6. \((0, 5), m = -\frac{3}{5}\)

7. \((0, -5), m = \frac{5}{4}\)  
8. \((0, -4), m = -\frac{3}{4}\)

9. \((1, 2), m = 3\)  
10. \((-1, 2), m = 3\)

11. \((-2, -3), m = -3\)  
12. \((1, -4), m = -4\)

13. \((5, -3), m = \frac{2}{5}\)  
14. \((4, 3), m = 0\)

15. \((2, -3), m\) is undefined  
16. \((2, -5), m = \frac{1}{4}\)

Write the equation of the line passing through each of the given pairs of points. Write your result in slope-intercept form, where possible.

17. \((2, 3)\) and \((5, 6)\)  
18. \((3, -2)\) and \((6, 4)\)

19. \((-2, -3)\) and \((2, 0)\)  
20. \((-1, 3)\) and \((4, -2)\)

21. \((-3, 2)\) and \((4, 2)\)  
22. \((-5, 3)\) and \((4, 1)\)
ANSWERS

23. (2, 0) and (0, −3)  
24. (2, −3) and (2, 4)

25. (0, 4) and (−2, −1)  
26. (−4, 1) and (3, 1)

Write the equation of the line $L$ satisfying the given geometric conditions.

27. $L$ has slope 4 and $y$ intercept $(0, −2)$.

28. $L$ has slope $\frac{2}{3}$ and $y$ intercept $(0, 4)$.

29. $L$ has $x$ intercept $(4, 0)$ and $y$ intercept $(0, 2)$.

30. $L$ has $x$ intercept $(-2, 0)$ and slope $\frac{3}{4}$.

31. $L$ has $y$ intercept $(0, 4)$ and a 0 slope.

32. $L$ has $x$ intercept $(-2, 0)$ and an undefined slope.

33. $L$ passes through point $(3, 2)$ with a slope of 5.

34. $L$ passes through point $(-2, -4)$ with a slope of $-\frac{3}{2}$.

35. $L$ has $y$ intercept $(0, 3)$ and is parallel to the line with equation $y = 3x - 5$.

36. $L$ has $y$ intercept $(0, -3)$ and is parallel to the line with equation $y = 2 \cdot \frac{3}{4}x + 1$.

37. $L$ has $y$ intercept $(0, 4)$ and is perpendicular to the line with equation $y = -2x + 1$.

38. $L$ has $y$ intercept $(0, 2)$ and is parallel to the line with equation $y = -1$.

39. $L$ has $y$ intercept $(0, 3)$ and is parallel to the line with equation $y = 2$.

40. $L$ has $y$ intercept $(0, 2)$ and is perpendicular to the line with equation $2x - 3y = 6$.

41. $L$ passes through point $(-3, 2)$ and is parallel to the line with equation $y = 2x - 3$.

42. $L$ passes through point $(-4, 3)$ and is parallel to the line with equation $y = -2x + 1$.

43. $L$ passes through point $(3, 2)$ and is parallel to the line with equation $y = \frac{4}{3}x + 4$. 
44. \( L \) passes through point \((-2, -1)\) and is perpendicular to the line with equation \( y = 3x + 1 \).

45. \( L \) passes through point \((5, -2)\) and is perpendicular to the line with equation \( y = -3x - 2 \).

46. \( L \) passes through point \((3, 4)\) and is perpendicular to the line with equation \( y = \frac{3}{5}x + 2 \).

47. \( L \) passes through \((-2, 1)\) and is parallel to the line with equation \( x + 2y = 4 \).

48. \( L \) passes through \((-3, 5)\) and is parallel to the \( x \) axis.

49. Describe the process for finding the equation of a line if you are given two points on the line.

50. How would you find the equation of a line if you were given the slope and the \( x \) intercept?

51. A temperature of \(10^\circ C\) corresponds to a temperature of \(50^\circ F\). Also \(40^\circ C\) corresponds to \(104^\circ F\). Find the linear equation relating \(F\) and \(C\).

52. In planning for a new item, a manufacturer assumes that the number of items produced \(x\) and the cost in dollars \(C\) of producing these items are related by a linear equation. Projections are that 100 items will cost $10,000 to produce and that 300 items will cost $22,000 to produce. Find the equation that relates \(C\) and \(x\).

53. A word processing station was purchased by a company for $10,000. After 4 years it is estimated that the value of the station will be $4000. If the value in dollars \(V\) and the time the station has been in use \(t\) are related by a linear equation, find the equation that relates \(V\) and \(t\).

54. Two years after an expansion, a company had sales of $42,000. Four years later the sales were $102,000. Assuming that the sales in dollars \(S\) and the time in years \(t\) are related by a linear equation, find the equation relating \(S\) and \(t\).

Getting Ready for Section 7.4 [Section 2.7]

Graph each of the following inequalities.

- (a) \( x < 3 \)
- (b) \( x \geq -2 \)
- (c) \( 2x \leq 8 \)
- (d) \( 3x \geq -9 \)
- (e) \( -3x < 12 \)
- (f) \( -2x \leq 10 \)
- (g) \( \frac{2}{3}x \leq 4 \)
- (h) \( -\frac{3}{4}x \geq 6 \)
Answers

1. \( y = 3x + 2 \)  
3. \( y = \frac{3}{2}x + 2 \)  
5. \( y = 4 \)  
7. \( y = \frac{5}{4}x - 5 \)  
9. \( y = 3x - 1 \)  
11. \( y = -3x - 9 \)  
13. \( y = \frac{2}{5}x - 5 \)  
15. \( x = 2 \)  
17. \( y = x + 1 \)  
19. \( y = \frac{3}{4}x - \frac{3}{2} \)  
21. \( y = 2 \)  
23. \( y = \frac{3}{2}x - 3 \)  
25. \( y = \frac{5}{2}x + 4 \)  
27. \( y = 4x - 2 \)  
29. \( y = -\frac{1}{2}x + 2 \)  
31. \( y = 4 \)  
33. \( y = 5x - 13 \)  
35. \( y = 3x + 3 \)  
37. \( y = \frac{1}{2}x + 4 \)  
39. \( y = 3 \)  
41. \( y = 2x + 8 \)  
43. \( y = \frac{4}{3}x - 2 \)  
45. \( y = \frac{1}{3}x - \frac{11}{3} \)  
47. \( y = -\frac{1}{2}x \)  
49. \( \text{graph} \)  
51. \( F = \frac{9}{5}C + 32 \)  

53. \( V = -1500t + 10,000 \)

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

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