Exponential Notation and the Order of Operations

**OBJECTIVES**

1. Use exponent notation
2. Evaluate expressions containing powers of whole numbers
3. Know the order of operations
4. Evaluate expressions that contain several operations

**Overcoming Math Anxiety**

**Preparing for a Test**

Preparation for a test really begins on the first day of class. Everything you have done in class and at home has been part of that preparation. However, there are a few things that you should focus on in the last few days before a scheduled test.

1. Plan your test preparation to end at least 24 hours before the test. The last 24 hours is too late, and besides, you will need some rest before the test.
2. Go over your homework and class notes with pencil and paper in hand. Write down all of the problem types, formulas, and definitions that you think might give you trouble on the test.
3. The day before the test, take the page(s) of notes from step 2, and transfer the most important ideas to a 3 × 5 card.
4. Just before the test, review the information on the card. You will be surprised at how much you remember about each concept.
5. Understand that, if you have been successful at completing your homework assignments, you can be successful on the test. This is an obstacle for many students, but it is an obstacle that can be overcome. Truly anxious students are often surprised that they scored as well as they did on a test. They tend to attribute this to blind luck. It is not. It is the first sign that you really do “get it.” Enjoy the success.

Earlier we described multiplication as a shorthand for repeated addition. There is also a shorthand for repeated multiplication. It uses powers of a whole number.

**Example 1**

**Writing Repeated Multiplication as a Power**

\[ 3 \times 3 \times 3 \times 3 \text{ can be written as } 3^4. \]

This is read as “3 to the fourth power.”

In this case, repeated multiplication is written as the power of a number.

In this example, 3 is the base of the expression, and the raised number, 4, is the exponent, or power.

**NOTE**

Recall that

\[ 3 + 3 + 3 + 3 = 4 \times 3 \]

Repeated addition was written as multiplication.

**NOTE**

René Descartes, a French philosopher and mathematician, is generally credited with first introducing our modern exponent notation in about 1637.
CHAPTER 1  OPERATIONS ON WHOLE NUMBERS

CHECK YOURSELF 1

Write $2 \times 2 \times 2 \times 2 \times 2$ as a power of 2.

Definitions: Exponents

The exponent tells us the number of times the base is to be used as a factor.

Example 2

Evaluating a Number Raised to a Power

$2^5$ is read “2 to the fifth power.”

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Here 2 is the base, and 5 is the exponent.

CHECK YOURSELF 2

Read and evaluate $3^4$.

Example 3

Evaluating a Number Raised to a Power

Evaluate $5^3$ and $8^2$.

$5^3 = 5 \times 5 \times 5 = 125$  Use 3 factors of 5.

$8^2 = 8 \times 8 = 64$  Use 2 factors of 8.

And $8^2$ is read “8 to the second power” or “8 squared.”

CHECK YOURSELF 3

Evaluate.

(a) $6^2$  
(b) $2^4$

We need two special definitions for powers of whole numbers.

Definitions: Raising a Number to the First Power

A whole number raised to the first power is just that number.

For example, $9^1 = 9$
Evaluating Numbers Raised to the Power of Zero or One

- \( 8^0 = 1 \)
- \( 4^0 = 1 \)
- \( 5^1 = 5 \)
- \( 3^1 = 3 \)

We talked about powers of 10 earlier when we multiplied by numbers that end in 0. Because the powers of 10 have a special importance, let's list some of them.

- \( 10^0 = 1 \)
- \( 10^1 = 10 \)
- \( 10^2 = 10 \times 10 = 100 \)
- \( 10^3 = 10 \times 10 \times 10 = 1000 \)
- \( 10^4 = 10 \times 10 \times 10 \times 10 = 10,000 \)
- \( 10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000 \)

Do you see why the powers of 10 are so important?

The powers of 10 correspond to the place values of our number system, ones, tens, hundreds, thousands, and so on.

This is what we meant earlier when we said that our number system was based on the number 10.

If multiplication is combined with addition or subtraction, you must know which operation to do first in finding the expression's value. We can easily illustrate this problem. How should we simplify the following statement?

\[ 3 + 4 \times 5 = ? \]

Both multiplication and addition are involved in this expression, and we must decide which to do first to find the answer.

1. Multiplying first gives us
   \[ 3 + 20 = 23 \]
2. Adding first gives us

$$7 \times 5 = 35$$

The answers differ depending on which operation is done first!

Only one of these results can be correct, which is why mathematicians developed a rule to tell us the order in which the operations should be performed. The rules are as follows.

**Step by Step: The Order of Operations**

If multiplication, division, addition, and subtraction are involved in the same expression, do the operations in the following order:

**Step 1** Do all multiplication or division in order from left to right.

**Step 2** Do all addition or subtraction in order from left to right.

**Example 5**

Using the Order of Operations

(a) $$3 \times 4 + 5 = 12 + 5 = 17$$  Multiply first, then add or subtract.

(b) $$5 + 3 \times 6 = 5 + 18 = 23$$

(c) $$16 - 2 \times 3 = 16 - 6 = 10$$

(d) $$7 \times 8 - 20 = 56 - 20 = 36$$

(e) $$5 \times 6 + 4 \times 3 = 30 + 12 = 42$$

**CHECK YOURSELF 5**

Evaluate.

(a) $$8 + 3 \times 5$$  (b) $$15 \times 5 - 3$$  (c) $$4 \times 3 + 2 \times 6$$

We now want to extend our rule for the order of operations. Let’s see what happens when parentheses or exponents are involved in an expression.

**Step by Step: The Order of Operations**

Mixed operations in an expression should be done in the following order:

**Step 1** Do any operations inside parentheses.

**Step 2** Evaluate any powers.

**Step 3** Do all multiplication or division in order from left to right.

**Step 4** Do all addition or subtraction in order from left to right.

**Example 6**

Evaluating an Expression

Evaluate $$4 \times 2^3$$.

**Step 1** There are no parentheses
Step 2 Evaluate powers
4 × 2³ = 4 × 8

Step 3 Multiply or divide
4 × 8 = 32

CHECK YOURSELF 6
Evaluate.

3 × 3²

Example 7
Evaluating an Expression
Evaluate (2 + 3)² + 4 × 3.

Step 1 Do operations inside parentheses
(2 + 3)² + 4 × 3 = 5² + 4 × 3

Step 2 Evaluate powers
5² + 4 × 3 = 25 + 4 × 3

Step 3 Multiply or divide
25 + 4 × 3 = 25 + 12

Step 4 Add or subtract
25 + 12 = 37

CHECK YOURSELF 7
Evaluate.

4 + (8 − 5)²

Example 8
Using the Order of Operations
(a) Evaluate 20 ÷ 2 × 5.

Because the multiplication and division appear next to each other, work in order from left to right. Try it the other way and see what happens!

20 ÷ 2 × 5
= 10 × 5
= 50
So 20 ÷ 2 × 5 = 50.
(b) Evaluate \((5 + 13) \div 6\).

\[
\begin{align*}
(5 + 13) & \div 6 \\
& = 18 \div 6 \\
& = 3
\end{align*}
\]

So \((5 + 13) \div 6 = 3\).

**CHECK YOURSELF 8**

Evaluate.

(a) \(36 \div 4 \times 2\)  
(b) \((8 + 22) \div 5\)

**CHECK YOURSELF ANSWERS**

1. \(2^6\)  
2. “three to the fourth power” is 81  
3. (a) 36; (b) 16  
4. (a) 1; (b) 7  
5. (a) 23; (b) 72; (c) 24  
6. 27  
7. 13  
8. (a) 18; (b) 6
1.7 Exercises

Evaluate.

1. \(3^2\)  
2. \(2^3\)  
3. \(2^4\)

4. \(5^2\)  
5. \(5^1\)  
6. \(6^0\)

7. \(9^0\)  
8. \(7^1\)  
9. \(10^3\)

10. \(10^2\)  
11. \(10^6\)  
12. \(10^7\)

13. \(2 \times 4^3\)  
14. \((2 \times 4)^3\)  
15. \(5 + 2^2\)

16. \((5 + 2)^2\)  
17. \((3 \times 2)^4\)  
18. \(3 \times 2^4\)

19. \(2 \times 6^2\)  
20. \((2 \times 6)^2\)  
21. \(14 - 3^2\)

22. \(12 + 4^2\)  
23. \((3 + 2)^3 - 20\)  
24. \(5 + (9 - 5)^2\)

25. \((7 - 4)^4 - 30\)  
26. \((5 + 2)^2 + 20\)  
27. \(8 \div 4 + 2\)

28. \(3 \times 5 + 2\)  
29. \(24 - 6 \div 3\)  
30. \(3 + 9 \div 3\)

31. \((24 - 6) \div 3\)  
32. \((3 + 9) \div 3\)  
33. \(12 + 3 \div 3\)

34. \(6 \times 12 \div 3\)  
35. \(18 \div 6 \times 3\)  
36. \(30 \div 5 \times 2\)

37. \(30 \div 6 - 12 \div 3\)  
38. \(5 + 8 \div 4 - 3\)  
39. \(4^2 \div 2\)

40. \(2 \times 4^3\)  
41. \(5^2 \times 3\)  
42. \(6^2 \div 3\)

43. \(3 \times 3^3\)  
44. \(2^3 \times 3\)  
45. \((3^3 + 3) \div 10\)

46. \((2^4 + 4) \div 5\)  
47. \(15 \div (5 - 3 + 1)\)  
48. \(20 \div (3 + 4 - 2)\)

49. \(27 \div (2^2 + 5)\)  
50. \(48 \div (2^3 + 4)\)

ANSWERS

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50.
Numbers such as 3, 4, and 5 are called Pythagorean triples, after the Greek mathematician Pythagoras (sixth century B.C.), because

\[3^2 + 4^2 = 5^2\]

Which of the following sets of numbers are Pythagorean triples?

51. 6, 8, 10  
52. 6, 11, 12  
53. 5, 12, 13  
54. 7, 24, 25  
55. 8, 16, 18  
56. 8, 15, 17

57. Is \((a + b)^p\) equal to \(a^p + b^p\)?

Try a few numbers and decide if you think this is true for all whole numbers, for some whole numbers, or never true. Write an explanation of your findings, and give examples.

58. Does \((a \cdot b)^p = a^p \cdot b^p\)?

Try a few numbers and decide if you think this is true for all whole numbers, for some whole numbers, or never true. Write an explanation of your findings, and give examples.

Answers

1. 9  
3. 16  
5. 5  
7. 1  
9. 1000  
11. 1,000,000  
13. 128  
15. 9  
17. 1296  
19. 72  
21. 5  
23. 105  
25. 51  
27. 4  
29. 22  
31. 6  
33. 13  
35. 9  
37. 1  
39. 8  
41. 75  
43. 81  
45. 3  
47. 5  
49. 3  
51. Yes  
53. Yes  
55. No  
57. Yes
Using a Scientific Calculator to Evaluate an Expression

Multiplication, like addition and subtraction, is easy to do on your calculator. To multiply $23 \times 37$ on the calculator:

1. Press the clear key.
2. Enter the first factor. 23
3. Press the times key. $\times$
4. Enter the second factor. 37
5. Press the equals key. $=$ \[851\]

The display shows \[851\]

**Example 1**

Using a Scientific Calculator to Multiply Three or More Numbers

To multiply $3 \times 5 \times 7$, use this sequence:

$3 \times 5 \times 7 \; \Rightarrow$ When you press the times key the second time, note that the product of 3 and 5 is in the display.

Display \[105\]

**CHECK YOURSELF 1**

Multiply $9 \times 11 \times 15$ with your calculator.

**Example 2**

Using a Scientific Calculator to Evaluate an Expression

To evaluate $3 \times 4 + 5$, use this sequence:

$3 \times 4 + 5 \; \Rightarrow$

Display \[17\]

**CHECK YOURSELF 2**

Evaluate $5 \times 6 + 11$ with your calculator.

So far, our calculator has done the multiplication and then the addition. Let’s change the order of the addition and multiplication in an expression and see what the calculator does.

**Example 3**

Using a Scientific Calculator to Evaluate an Expression

To evaluate $6 + 3 \times 5$, use this sequence:

$6 + 3 \times 5 \; \Rightarrow$
Display \[21\]
Again the calculator follows the “order of operations.” Try this with your calculator.

**CHECK YOURSELF 3**

_**Evaluate 9 + 3 \times 7 with your calculator.**_

Most calculators have parentheses keys that will allow you to evaluate more complicated expressions easily.

**Example 4**

**Using a Scientific Calculator to Evaluate an Expression**

To evaluate \(3 \times (4 + 5)\), use this sequence:

\[
3 \times [4 + 5] =
\]

Display \[27\]
Now the calculator does the addition in the parentheses as the first step. Then it does the multiplication.

**CHECK YOURSELF 4**

_**Evaluate 4 \times (2 + 9) with your calculator.**_

**CHECK YOURSELF ANSWERS**

1. 1485 2. 41 3. 30 4. 44
Calculator Exercises

Multiply, using your calculator.

1. \[
\begin{array}{c}
57 \\
\times 89 \\
\end{array}
\]

2. \[
\begin{array}{c}
98 \\
\times 25 \\
\end{array}
\]

3. \[
\begin{array}{c}
256 \\
\times 508 \\
\end{array}
\]

4. \[
\begin{array}{c}
285 \\
\times 820 \\
\end{array}
\]

5. \[
\begin{array}{c}
23,456 \\
\times 2358 \\
\end{array}
\]

6. \[
\begin{array}{c}
18,569 \\
\times 3286 \\
\end{array}
\]

7. \[12 \times 15 \times 8\]

8. \[32 \times 5 \times 18\]

9. \[78 \times 145 \times 36\]

10. \[358 \times 39 \times 928\]

11. \[24 \times 35 \times 48 \times 36\]

12. \[37 \times 15 \times 42 \times 29\]

Use your calculator to evaluate each of the following expressions.

13. \[4 \times 5 - 7\]

14. \[3 \times 7 + 8\]

15. \[9 + 3 \times 7\]

16. \[6 \times 0 + 3\]

17. \[4 + 5 \times 0\]

18. \[23 - 4 \times 5\]

19. \[5 \times (4 + 7)\]

20. \[8 \times (6 + 5)\]

21. \[5 \times 4 + 5 \times 7\]

22. \[8 \times 6 + 8 \times 5\]
Solve the following applications using a calculator.

23. Sales. A car dealer kept the following record of a month’s sales. Complete the table.

<table>
<thead>
<tr>
<th>Model</th>
<th>Number Sold</th>
<th>Profit per Sale</th>
<th>Monthly Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subcompact</td>
<td>38</td>
<td>$528</td>
<td></td>
</tr>
<tr>
<td>Compact</td>
<td>33</td>
<td>647</td>
<td></td>
</tr>
<tr>
<td>Standard</td>
<td>19</td>
<td>912</td>
<td></td>
</tr>
</tbody>
</table>

Monthly Total Profit

24. Salary. You take a job paying $1 the first day. On each following day your pay doubles. That is, on day 2 your pay is $2, on day 3 the pay is $4, and so on. Complete the table.

<table>
<thead>
<tr>
<th>Day</th>
<th>Daily Pay</th>
<th>Total Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1</td>
<td>$1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
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<td>7</td>
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<td>8</td>
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<tr>
<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers

1. 5073  3. 130,048  5. 55,309,248  7. 1440  9. 407,160  
11. 1,451,520  13. 13  15. 30  17. 4  19. 55  21. 55

23. Monthly profit:

$20,064  
21,351  
17,328  
$58,743