## Objectives

1. Identify the numerator and denominator of a fraction
2. Use fractions to name parts of a whole
3. Identify proper fractions
4. Write improper fractions as mixed numbers
5. Write mixed numbers as improper fractions

Previous sections dealt with whole numbers and the operations that are performed on them. We are now ready to consider a new kind of number, a **fraction**.

### Definitions: Fraction

Whenever a unit or a whole quantity is divided into parts, we call those parts **fractions** of the unit.

**NOTE** Our word *fraction* comes from the Latin stem *fractio*, which means "breaking into pieces."

**NOTE** *Common fraction* is technically the correct term. We will normally just use *fraction* in these materials if there is no room for confusion.

In Figure 1, the whole has been divided into five equal parts. We use the symbol \( \frac{2}{5} \) to represent the shaded portion of the whole.

![Figure 1](image)

The symbol \( \frac{2}{5} \) is called a **common fraction**, or more simply a fraction. A fraction is written in the form \( \frac{a}{b} \), in which \( a \) and \( b \) represent whole numbers and \( b \) cannot be equal to 0.

We give the numbers \( a \) and \( b \) special names. The **denominator**, \( b \), is the number on the bottom. This tells us how many equal parts the unit or whole has been divided. The **numerator**, \( a \), is the number on the top. This tells us how many parts of the unit are used.

In Figure 1, the **denominator** is 5; the unit or whole (the circle) has been divided into five equal parts. The **numerator** is 2. We have taken two parts of the unit.

\[
\begin{align*}
\frac{2}{5} & \quad \text{Numerator} \\
5 & \quad \text{Denominator}
\end{align*}
\]

### Example 1

**Labeling Fraction Components**

The fraction \( \frac{4}{7} \) names the shaded part of the rectangle in Figure 2.
CHAPTER 2  MULTIPLYING AND DIVIDING FRACTIONS

CHECK YOURSELF 1

What fraction names the shaded part of this diagram? Identify the numerator and denominator.

Fractions can also be used to name a part of a collection or a set of identical objects.

Example 2

Naming a Fractional Part

The fraction $\frac{5}{6}$ names the shaded part of Figure 3. We have shaded five of the six identical objects.

CHECK YOURSELF 2

What fraction names the shaded part of this diagram?
**Example 3**

**Naming a Fractional Part**

In a class of 23 students, 15 are women. We can name the part of the class that is women as $\frac{15}{23}$.

**CHECK YOURSELF 3**

Seven replacement parts out of a shipment of 50 were faulty. What fraction names the portion of the shipment that was faulty?

A fraction can also be thought of as indicating division. The symbol $\frac{a}{b}$ also means $a \div b$.

**Example 4**

**Interpreting Division as a Fraction**

$\frac{2}{3}$ names the quotient when 2 is divided by 3. So $\frac{2}{3} = 2 \div 3$.

**CHECK YOURSELF 4**

*Using the numbers 5 and 9, write $\frac{5}{9}$ in another way.*

We can use the relative size of the numerator and denominator of a fraction to separate fractions into different categories.

**Definitions: Proper Fraction**

If the numerator is *less than* the denominator, the fraction names a number less than 1 and is called a **proper fraction**.

**Definitions: Improper Fraction**

If the numerator is *greater than or equal to* the denominator, the fraction names a number greater than or equal to 1 and is called an **improper fraction**.

**Example 5**

**Categorizing Fractions**

(a) $\frac{2}{3}$ is a proper fraction because the numerator is less than the denominator (Figure 4).

(b) $\frac{4}{3}$ is an improper fraction because the numerator is larger than the denominator (Figure 5).
(c) Also, $\frac{8}{8}$ is an improper fraction because it names exactly 1 unit; the numerator is equal to the denominator (Figure 6).

Another way to write a fraction that is larger than 1 is called a **mixed number**.

**Definitions:** Mixed Number

A **mixed number** is the sum of a whole number and a proper fraction.

**Example 6**

**Identifying a Mixed Number**

$2\frac{3}{4}$ is a mixed number. It represents the sum of the whole number 2 and the fraction $\frac{3}{4}$. Look at the diagram below, which represents $2\frac{3}{4}$.

The addition sign is usually not written.

**CHECK YOURSELF 6**

Give the mixed number that names the shaded portion of the given diagram.
For our later work it will be important to be able to change back and forth between improper fractions and mixed numbers. Because an improper fraction represents a number that is greater than or equal to 1, we have the following rule:

Rules and Properties: Improper Fractions to Mixed Numbers

An improper fraction can always be written as either a mixed number or as a whole number.

To do this, remember that you can think of a fraction as indicating division. The numerator is divided by the denominator. This leads us to the following rule:

Step by Step: To Change an Improper Fraction to a Mixed Number

Step 1 Divide the numerator by the denominator.
Step 2 If there is a remainder, write the remainder over the original denominator.

Example 7
Converting a Fraction to a Mixed Number

Convert $\frac{17}{5}$ to a mixed number.

Divide 17 by 5.

$3 \frac{2}{5}$

In diagram form:

\[
\begin{array}{c}
\text{Original denominator} \\
\text{Quotient} \\
\text{Remainder}
\end{array}
\]

CHECK YOURSELF 7

Convert $\frac{32}{5}$ to a mixed number.
**Example 8**

**Converting a Fraction to a Mixed Number**

Convert \(\frac{21}{7}\) to a mixed or a whole number.

Divide 21 by 7.

\[
7 \longdiv{21} \quad \frac{21}{7} = 3
\]

**NOTE** If there is no remainder, the improper fraction is equal to some whole number; in this case 3.

**CHECK YOURSELF 8.**

Convert \(\frac{48}{6}\) to a mixed or a whole number.

It is also easy to convert mixed numbers to improper fractions. Just use the following rule:

**Step by Step:** To Change a Mixed Number to an Improper Fraction

**Step 1** Multiply the denominator of the fraction by the whole-number portion of the mixed number.

**Step 2** Add the numerator of the fraction to that product.

**Step 3** Write that sum over the original denominator to form the improper fraction.

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**Example 9**

**Converting Mixed Numbers to Improper Fractions**

(a) Convert \(3\frac{2}{5}\) to an improper fraction.

\[
3 \times 5 + 2 = \frac{(5 \times 3) + 2}{5} = \frac{17}{5}
\]

In diagram form:
Each of the three units has 5 fifths, so the whole-number portion is $5 \times 3$, or 15, fifths. Then add the 2 fifths from the fractional portion for 17 fifths.

(b) Convert $4\frac{5}{7}$ to an improper fraction.

\[ 4\frac{5}{7} = \frac{(7 \times 4) + 5}{7} = \frac{33}{7} \]

NOTE Multiply the denominator, 7, by the whole number, 4, and add the numerator, 5.

CHECK YOURSELF 9

Convert $5\frac{3}{8}$ to an improper fraction.

One special kind of improper fraction should be mentioned at this point: a fraction with a denominator of 1.

NOTE This is the same as our earlier rule for dividing by 1.

Definitions: Fractions with a Denominator of 1

Any fraction with a denominator of 1 is equal to the numerator alone.

For example, $\frac{5}{1} = 5$ and $\frac{12}{1} = 12$.

You probably do many conversions between mixed and whole numbers without even thinking about the process that you follow, as Example 10 illustrates.

Example 10

Converting Quarter Dollars to Dollars

Maritza has 53 quarters in her bank. How many dollars does she have?

Because there are 4 quarters in each dollar, 53 quarters can be written as

\[ \frac{53}{4} \]

Converting the amount to dollars is the same as rewriting it as a mixed number.

\[ \frac{53}{4} = 13\frac{1}{4} \]

She has 13\frac{1}{4} dollars, which you would probably write as $13.25$. (Note: We will discuss decimal point usage later in this text.)

CHECK YOURSELF 10

Kevin is doing the inventory in the convenience store in which he works. He finds there are 11 half gallons (gal) of milk. Write the amount of milk as a mixed number of gallons.
CHECK YOURSELF ANSWERS

1. $\frac{3}{8}$ ← Numerator
   $\frac{3}{8}$ ← Denominator

2. $\frac{2}{7}$
3. $\frac{7}{50}$
4. $5 \div 9$

5. Proper fractions: Improper fractions: 6. $\frac{5}{6}$ 7. $\frac{2}{5}$ 8. 9. $\frac{43}{8}$
   $\frac{10}{11}$ 3 7 5 8 6 13 15
   $\frac{4}{11}$ 4 8 5 6 10 8

10. $\frac{11}{2} = \frac{51}{2}$ gal
2.3 **Exercises**

Identify the numerator and denominator of each fraction.

1. \( \frac{6}{11} \)  
2. \( \frac{5}{12} \)  
3. \( \frac{3}{11} \)  
4. \( \frac{9}{14} \)

What fraction names the shaded part of each of the following figures?

5.  
6.  
7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.

Solve the following applications.

15. **Test scores.** You missed 7 questions on a 20-question test. What fraction names the part you got correct? The part you got wrong?

16. **Basketball.** Of the five starters on a basketball team, two fouled out of a game. What fraction names the part of the starting team that fouled out?

**ANSWERS**

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.  
15.  
16.
17. **Car sales.** A used-car dealer sold 11 of the 17 cars in stock. What fraction names the portion sold? What fraction names the portion not sold?

18. **Purchases.** At lunch, five people out of a group of nine had hamburgers. What fraction names the part of the group who had hamburgers? What fraction names the part who did not have hamburgers?

19. Using the numbers 2 and 5, show another way of writing \( \frac{2}{5} \).

20. Using the numbers 4 and 5, show another way of writing \( \frac{4}{5} \).

Identify each number as a proper fraction, an improper fraction, or a mixed number.

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<td>( \frac{5}{7} )</td>
<td>( \frac{3}{7} )</td>
<td>( \frac{13}{17} )</td>
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Give the mixed number that names the shaded portion of each diagram.

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Change to a mixed or whole number.

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<td>( \frac{59}{5} )</td>
<td>( \frac{58}{7} )</td>
<td>( \frac{73}{8} )</td>
<td>( \frac{151}{12} )</td>
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<td>45.</td>
<td>46.</td>
<td>47.</td>
<td>48.</td>
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<td>( \frac{24}{6} )</td>
<td>( \frac{160}{8} )</td>
<td>( \frac{9}{1} )</td>
<td>( \frac{8}{1} )</td>
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Change to an improper fraction.

49. $\frac{2}{3}$  
50. $\frac{5}{6}$  
51. 8  
52. $\frac{5}{8}$  
53. $\frac{2}{9}$  
54. 7  
55. $\frac{3}{7}$  
56. $\frac{2}{9}$  
57. $\frac{6}{13}$  
58. $\frac{3}{10}$  
59. $\frac{10}{5}$  
60. $\frac{2}{5}$  
61. $100\frac{2}{3}$  
62. $150\frac{1}{4}$  
63. $118\frac{3}{4}$  
64. $250\frac{3}{4}$

Solve the following applications.

65. **Savings.** Clayton has 64 quarters in his bank. How many dollars does he have?

66. **Savings.** Amy has 19 quarters in her purse. How many dollars does she have?

67. **Inventory.** Manuel counted 35 half gallons of orange juice in his store. Write the amount of orange juice as a mixed number of gallons.

68. **Inventory.** Sarah has 19 half gallons of turpentine in her paint store. Write the amount of turpentine as a mixed number of gallons.

69. The U.S. census information can be found in your library, or on the Web, at [www.census.gov](http://www.census.gov). Use the 1990 census to determine the following:

(a) Fraction of the population of the United States contained in your state  
(b) Fraction of the population of the United States 65 years of age or older  
(c) Fraction of the United States that is female

70. Using the information on the given label, determine the following:

(a) Fraction of calories that comes from fat  
(b) Fraction of fat grams that comes from saturated fat
71. Based on a 2000-calorie diet, consult with your local health officials and determine what fraction of your daily diet each of the following should contribute.

(a) Fat (b) Carbohydrate (c) Unsaturated fat (d) Protein

Using this information, work with your classmates to plan a daily menu that will satisfy the requirements.

72. Suppose the national debt had to be paid by individuals.

(a) How would the amount each individual owed be determined?
(b) Would this be a proper or improper fraction?

73. In statistics, we describe the probability of an event by dividing the number of ways the event can happen by the total number of possibilities. Using this definition, determine the probability of each of the following:

(a) Obtaining a head on a single toss of a coin
(b) Obtaining a prime number when a die is tossed

Answers

1. 6 is the numerator; 11 is the denominator
3. 3 is the numerator; 11 is the denominator
5. \( \frac{3}{4} \)
7. \( \frac{5}{6} \)
9. \( \frac{5}{7} \)
11. \( \frac{11}{12} \)

13. \( \frac{5}{8} \)
15. \( \frac{13}{20} \)
17. \( \frac{11}{17} \)
19. \( \frac{2}{5} \)
21. Proper

23. Mixed number
25. Improper
27. Improper
29. Mixed number

31. Proper
33. \( \frac{3}{4} \)
35. \( \frac{5}{8} \)
37. \( \frac{4}{5} \)
39. \( \frac{4}{5} \)
41. \( \frac{11}{5} \)

43. \( \frac{9}{8} \)
45. 4. There is no remainder in the division.

47. 9

49. \( \frac{14}{3} \)
51. \( \frac{8}{1} \)
53. \( \frac{56}{9} \)
55. \( \frac{3}{7} = (7 \times 3) + 3 = \frac{24}{7} \)
57. \( \frac{97}{13} \)

59. \( \frac{10}{5} = \frac{(5 \times 10) + 2}{5} = \frac{52}{5} \)
61. \( \frac{302}{3} \)
63. \( \frac{475}{4} \)
65. $16

67. \( \frac{17}{2} \) gal
69. \( \frac{71}{1} \)

73. (a) \( \frac{1}{2} \) (b) \( \frac{3}{6} = \frac{1}{2} \)