OBJECTIVES

1. Find the least common multiple (LCM) of two numbers
2. Find the least common multiple (LCM) of a group of numbers
3. Compare the size of two fractions

In this chapter, we are discussing the process used for adding or subtracting two fractions. One of the most important concepts we use in the addition and subtraction of fractions is that of **multiples**.

**Definitions: Multiples**

The *multiples* of a number are the product of that number with the natural numbers 1, 2, 3, 4, 5, . . . .

**Example 1**

**Listing Multiples**

List the multiples of 3.

The multiples of 3 are

\[ 3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, \ldots \]

or

\[ 3, 6, 9, 12, \ldots \]

**NOTE** Notice that the multiples, except for 3 itself, are larger than 3.

An easy way of listing the multiples of 3 is to think of counting by threes.

**CHECK YOURSELF 1**

List the first seven multiples of 4.

Sometimes we need to find common multiples of two or more numbers.

**Definitions: Common Multiples**

If a number is a multiple of each of a group of numbers, it is called a *common multiple* of the numbers; that is, it is a number that is evenly divisible by all the numbers in the group.
Finding Common Multiples

Find four common multiples of 3 and 5.

Some common multiples of 3 and 5 are

15, 30, 45, 60

Example 2

CHECK YOURSELF 2

List the first six multiples of 6. Then look at your list from Check Yourself 1 and list some common multiples of 4 and 6.

For our later work, we will use the least common multiple of a group of numbers.

Definitions: Least Common Multiple

The least common multiple (LCM) of a group of numbers is the smallest number that is a multiple of each number in the group.

It is possible to simply list the multiples of each number and then find the LCM by inspection.

Example 3

Finding the Least Common Multiple (LCM)

Find the least common multiple of 6 and 8.

Multiples

6: 6, 12, 18, 24, 30, 36, 42, 48, . . .

8: 8, 16, 24, 32, 40, 48, . . .

We see that 24 is the smallest number common to both lists. So 24 is the LCM of 6 and 8.

CHECK YOURSELF 3

Find the least common multiple of 20 and 30 by listing the multiples of each number.

The technique of the last example will work for any group of numbers. However, it becomes tedious for larger numbers. Let’s outline a different approach.
Some students prefer a slightly different method of lining up the factors to help in remembering the process of finding the LCM of a group of numbers.

**Example 4**

**Finding the Least Common Multiple (LCM)**

To find the LCM of 10 and 18, factor:

\[
10 = 2 \times 5 \\
18 = 2 \times 3 \times 3 \\
\]

Bring down the factors.

2 and 5 appear, at most, one time in any one factorization. And 3 appears two times in one factorization.

\[2 \times 3 \times 3 \times 5 = 90\]

So 90 is the LCM of 10 and 18.

**CHECK YOURSELF 4**

*Use the method of Example 4 to find the LCM of 24 and 36.*

The procedure is the same for a group of more than two numbers.

**Example 5**

**Finding the Least Common Multiple (LCM)**

To find the LCM of 12, 18, and 20, we factor:

\[
12 = 2 \times 2 \times 3 \\
18 = 2 \times 3 \times 3 \\
20 = 2 \times 2 \times 3 \times 5 \\
\]

The different factors that appear are 2, 3, and 5.

\[2 \times 2 \times 3 \times 3 \times 5 = 180\]

So 180 is the LCM of 12, 18, and 20.
CHAPTER 3  ADDING AND SUBTRACTING FRACTIONS

CHECK YOURSELF 5

Find the LCM of 3, 4, and 6.

The process of finding the least common multiple is very useful when adding, subtracting, or comparing unlike fractions (fractions with different denominators).

Suppose you are asked to compare the sizes of the fractions $\frac{3}{7}$ and $\frac{4}{7}$. Because each unit in the diagram is divided into seven parts, it is easy to see that $\frac{4}{7}$ is larger than $\frac{3}{7}$.

![Diagram showing comparison of fractions]

Four parts of seven are a greater portion than three parts. Now compare the size of the fractions $\frac{2}{5}$ and $\frac{3}{7}$.

![Diagram showing comparison of fractions]

We cannot compare fifths with sevenths! $\frac{2}{5}$ and $\frac{3}{7}$ are not like fractions. Because they name different ways of dividing the whole, deciding which fraction is larger is not nearly so easy.

To compare the sizes of fractions, we change them to equivalent fractions having a common denominator. This common denominator must be a common multiple of the original denominators.

Rules and Properties:  The Fundamental Principle of Fractions

\[
\frac{a}{b} = \frac{a \times c}{b \times c}
\]
Example 6
Finding Common Denominators to Order Fractions

Compare the sizes of $\frac{2}{5}$ and $\frac{3}{7}$.

The original denominators are 5 and 7. Because 35 is a common multiple of 5 and 7, let’s use 35 as our common denominator.

Think, “What must we multiply 5 by to get 35?” The answer is 7. Multiply the numerator and denominator by that number.

\[
\begin{align*}
\times 7 & \quad \frac{2}{5} = \frac{14}{35}, \\
\times 7 & \quad \frac{3}{7} = \frac{15}{35}, \\
\times 5 & \quad \text{Multiply the numerator and denominator by 5.}
\end{align*}
\]

Because $\frac{2}{5} = \frac{14}{35}$ and $\frac{3}{7} = \frac{15}{35}$, we see that $\frac{3}{7}$ is larger than $\frac{2}{5}$.

NOTE $\frac{2}{5}$ and $\frac{14}{35}$ are equivalent fractions. They name the same part of a whole.

NOTE 15 of 35 parts represents a greater portion of the whole than 14 parts.

CHECK YOURSELF 6

Which is larger, $\frac{5}{9}$ or $\frac{4}{7}$?

Let’s consider an example that uses the inequality notation.

Example 7
Using an Inequality Symbol with Two Fractions

Use the inequality symbol < or > to complete the statement below.

$\frac{5}{8} \quad \frac{3}{5}$

Once again we must compare the sizes of the two fractions, and this is done by converting the fractions to equivalent fractions with a common denominator. Here we will use 40 as that denominator.

\[
\begin{align*}
\times 5 & \quad \frac{5}{8} = \frac{25}{40}, \\
\times 8 & \quad \frac{3}{5} = \frac{24}{40}, \\
\times 5 & \quad \text{Multiply the numerator and denominator by 5.}
\end{align*}
\]

Because $\frac{5}{8} \left( \text{or} \frac{25}{40} \right)$ is larger than $\frac{3}{5} \left( \text{or} \frac{24}{40} \right)$, we write

$\frac{5}{8} > \frac{3}{5}$

NOTE The inequality symbol “points” to the smaller quantity.
CHECK YOURSELF 2

Use the symbol < or > to complete the statement.

\[
\frac{5}{9} \quad \frac{5}{9} \quad \frac{6}{11} \quad \frac{6}{11}
\]

CHECK YOURSELF ANSWERS

1. The first seven multiples of 4 are 4, 8, 12, 16, 20, 24, and 28.
2. 6, 12, 18, 24, 30, 36; some common multiples of 4 and 6 are 12, 24, and 36.
3. The multiples of 20 are 20, 40, 60, 80, 100, 120, \ldots; the multiples of 30 are 30, 60, 90, 120, 150, \ldots; the least common multiple of 20 and 30 is 60, the smallest number common to both lists.
4. \(2 \times 2 \times 2 \times 3 \times 3 = 72\)
5. 12
6. \(\frac{4}{7}\) is larger
7. \(\frac{5}{9} > \frac{6}{11}\)
3.2 Exercises

Find the least common multiple (LCM) for each of the following groups of numbers. Use whichever method you wish.

1. 2 and 3  
2. 3 and 5  
3. 4 and 6  
4. 6 and 9  
5. 10 and 20  
6. 12 and 36  
7. 9 and 12  
8. 20 and 30  
9. 12 and 16  
10. 10 and 15  
11. 12 and 15  
12. 12 and 21  
13. 18 and 36  
14. 25 and 50  
15. 25 and 40  
16. 10 and 14  
17. 3, 5, and 6  
18. 2, 8, and 10  
19. 18, 21, and 28  
20. 8, 15, and 20  
21. 20, 30, and 45  
22. 12, 20, and 35

ANSWERS

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 
19. 
20. 
21. 
22. 
Arrange the given fractions from smallest to largest.

23. \( \frac{12}{17} \) \( \frac{9}{10} \)

24. \( \frac{4}{9} \) \( \frac{5}{11} \)

25. \( \frac{5}{8} \) \( \frac{3}{5} \)

26. \( \frac{9}{10} \) \( \frac{8}{9} \)

27. \( \frac{3}{8} \) \( \frac{1}{3} \) \( \frac{1}{4} \)

28. \( \frac{7}{12} \) \( \frac{5}{18} \) \( \frac{1}{3} \)

29. \( \frac{11}{12} \) \( \frac{4}{5} \) \( \frac{5}{6} \)

30. \( \frac{5}{8} \) \( \frac{9}{16} \) \( \frac{13}{32} \)

Complete the statements, using the symbol < or >.

31. \( \frac{5}{6} \) < \( \frac{2}{5} \)

32. \( \frac{3}{4} \) < \( \frac{10}{11} \)

33. \( \frac{4}{9} \) < \( \frac{3}{7} \)

34. \( \frac{7}{10} \) < \( \frac{11}{15} \)

35. \( \frac{7}{20} \) < \( \frac{9}{25} \)

36. \( \frac{5}{12} \) < \( \frac{7}{18} \)

37. \( \frac{5}{16} \) < \( \frac{7}{20} \)

38. \( \frac{7}{12} \) < \( \frac{9}{15} \)

39. A company uses two types of boxes, 8 cm and 10 cm long. They are packed in larger cartons to be shipped. What is the shortest length container that will accommodate boxes of either size without any room left over? (Each container can contain only boxes of one size—no mixing allowed.)
40. There is an alternate approach to finding the least common multiple of two numbers. The LCM of two numbers can be found by dividing the product of the two numbers by the greatest common factor (GCF) of those two numbers. For example, the GCF of 24 and 36 is 12. If we use the above formula, we obtain

\[
\text{LCM of 24 and 36} = \frac{24 \cdot 36}{12} = 72
\]

(a) Use the above formula to find the LCM of 150 and 480.
(b) Verify the result by finding the LCM using the method of prime factorization.
(c) The above approach can be extended so that it can be used to find the LCM of three numbers. Describe this extension.
(d) Use the results of part (c) to find the LCM of 48, 315, and 450.

Solve the following applications.

41. Drill bits. Three drill bits are marked \(\frac{3}{8}, \frac{5}{16}\), and \(\frac{11}{32}\). Which drill bit is largest?

42. Bolt size. Bolts can be purchased with diameters of \(\frac{3}{8}, \frac{1}{4}\), or \(\frac{3}{16}\) inches (in.). Which is smallest?

43. Plywood size. Plywood comes in thicknesses of \(\frac{5}{8}, \frac{3}{4}, \frac{1}{2}\), and \(\frac{3}{8}\) in. Which size is thickest?

44. Doweling. Doweling is sold with diameters of \(\frac{1}{2}, \frac{9}{16}, \frac{5}{8}\), and \(\frac{3}{8}\) in. Which size is smallest?

45. Elian is asked to create a fraction equivalent to \(\frac{1}{4}\). His answer is \(\frac{4}{7}\). What did he do wrong? What would be a correct answer?
46. A sign on a busy highway says Exit 5A is \( \frac{3}{4} \) mile away and Exit 5B is \( \frac{5}{8} \) mile away. Which exit is first?

47. Complete the following Crossword puzzle.

**ACROSS**

2. The LCM of 11 and 13
4. The GCF of 120 and 300
7. The GCF of 13 and 52
8. The GCF of 360 and 540

**DOWN**

1. The LCM of 8, 14, and 21
3. The LCM of 16 and 12
5. The LCM of 2, 5, and 13
6. The GCF of 54 and 90

**Answers**

1. 6 3. 12  5. 20  7. 36  9. 48 11. 12 = 2 × 2 × 3; 15 = 3 × 5; the LCM is 2 × 2 × 3 × 5 = 60 13. 36 15. 200 17. 30  19. 252  21. 180  23. \( \frac{12}{17} \), \( \frac{9}{10} \) 25. \( \frac{3}{5} \), \( \frac{5}{8} \) 27. \( \frac{1}{4} \), \( \frac{1}{3} \), \( \frac{3}{8} \) 29. \( \frac{4}{5} \), \( \frac{5}{6} \), \( \frac{11}{12} \) 31. > 33. > 35. < 37. < 39. 41. \( \frac{3}{8} \) 43. \( \frac{3}{4} \) in. 45.