# 4.6 Converting from Fractions to Decimals

## 4.6 OBJECTIVES

1. Convert a common fraction to a decimal
2. Convert a common fraction to a repeating decimal
3. Convert a mixed number to a decimal

Because a common fraction can be interpreted as division, you can divide the numerator of the common fraction by its denominator to convert a common fraction to a decimal. The result is called a **decimal equivalent.**

### Example 1

#### Converting a Fraction to a Decimal Equivalent

Write $\frac{5}{8}$ as a decimal.

**NOTE** Remember that 5 can be written as 5.0, 5.00, 5.000, and so on. In this case, we continue the division by adding zeros to the dividend until a 0 remainder is reached.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8}\phantom{00} \\ 20\phantom{0} \\ \underline{16}\phantom{0} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Because $\frac{5}{8}$ means 5 ÷ 8, divide 8 into 5.

We see that $\frac{5}{8} = 0.625$; 0.625 is the decimal equivalent of $\frac{5}{8}$.

**CHECK YOURSELF 1**

*Find the decimal equivalent of* $\frac{7}{8}$.

Some fractions are used so often that we have listed their decimal equivalents for your reference.

**NOTE** The division used to find these decimal equivalents stops when a 0 remainder is reached. The equivalents are called **terminating decimals.**

**Some Common Decimal Equivalents**

| | | | |
|---|---|---|---|
| $\frac{1}{2} = 0.5$ | $\frac{1}{4} = 0.25$ | $\frac{1}{5} = 0.2$ | $\frac{1}{8} = 0.125$ |
| | $\frac{3}{4} = 0.75$ | $\frac{2}{5} = 0.4$ | $\frac{3}{8} = 0.375$ |
| | | $\frac{3}{5} = 0.6$ | $\frac{5}{8} = 0.625$ |
| | | $\frac{4}{5} = 0.8$ | $\frac{7}{8} = 0.875$ |

If a decimal equivalent does not terminate, you can round the result to approximate the fraction to some specified number of decimal places. Consider Example 2.

## Example 2

### Converting a Fraction to a Decimal Equivalent

Write $\frac{3}{7}$ as a decimal. Round the answer to the nearest thousandth.

$$
\begin{array}{r}
0.4285 \\
7\overline{)3.0000} \\
\underline{2\,8}\phantom{000} \\
20\phantom{00} \\
\underline{14}\phantom{00} \\
60\phantom{0} \\
\underline{56}\phantom{0} \\
40 \\
\underline{35} \\
5
\end{array}
$$

In this example, we are choosing to round to three decimal places, so we must add enough zeros to carry the division to four decimal places.

So $\frac{3}{7} = 0.429$ (to the nearest thousandth).

**CHECK YOURSELF 2**

*Find the decimal equivalent of* $\frac{5}{11}$ *to the nearest thousandth.*

If a decimal equivalent does *not* terminate, it will *repeat* a sequence of digits. These decimals are called **repeating decimals.**

### Converting a Fraction to a Repeating Decimal

**(a)** Write $\frac{1}{3}$ as a decimal.

$$
\begin{array}{r}
0.333 \\
3\overline{)1.000} \\
\underline{9}\phantom{00} \\
10\phantom{0} \\
\underline{9}\phantom{0} \\
10 \\
\underline{9}
\end{array}
$$

The digit 3 will just repeat itself indefinitely because each new remainder will be 1.

Adding more zeros and going on will simply lead to more threes in the quotient.

We can say $\frac{1}{3} = 0.333...$

The three dots mean "and so on" and tell us that 3 will repeat itself indefinitely.

**(b)** Write $\frac{5}{12}$ as a decimal.

```
     0.4166. . .
12 ) 5.0000
     4 8
       20
       12
        80
        72
         80
         72
          8
```

In this example, the digit 6 will just repeat itself because the remainder, 8, will keep occurring if we add more zeros and continue the division.

## CHECK YOURSELF 3

*Find the decimal equivalent of each fraction.*

**(a)** $\frac{2}{3}$ **(b)** $\frac{7}{12}$

Some important decimal equivalents (rounded to the nearest thousandth) are shown below for reference.

$\frac{1}{3} = 0.333$ $\frac{1}{6} = 0.167$ $\frac{2}{3} = 0.667$ $\frac{5}{6} = 0.833$

Another way to write a repeating decimal is with a bar placed over the digit or digits that repeat. For example, we can write

0.37373737 . . .

as

$0.\overline{37}$

The bar placed over the digits indicates that "37" repeats indefinitely.

### Converting a Fraction to a Repeating Decimal

Write $\frac{5}{11}$ as a decimal.

```
       0.4545
  11 ) 5.0000
       4 4
         60
         55
5 -->    50
         44
          60
          55
           5
```

As soon as a remainder repeats itself, as 5 does here, the pattern of digits will repeat in the quotient.

$\frac{5}{11} = 0.\overline{45}$

$= 0.4545. . .$

## CHECK YOURSELF 4

*Use the bar notation to write the decimal equivalent of $\frac{5}{7}$. (Be patient. You'll have to divide for a while to find the repeating pattern.)*

You can find the decimal equivalents for mixed numbers in a similar way. Find the decimal equivalent of the fractional part of the mixed number, and then combine that with the whole-number part. Example 5 illustrates this approach.

### Example 5

### Converting a Mixed Number to a Decimal Equivalent

Find the decimal equivalent of $3\frac{5}{16}$.

$\frac{5}{16} = 0.3125$ First find the equivalent of $\frac{5}{16}$ by division.

$3\frac{5}{16} = 3.3125$ Add 3 to the result.

## CHECK YOURSELF 5

*Find the decimal equivalent of $2\frac{5}{8}$.*

We learned something important in this section. To find the decimal equivalent of a fraction, we use long division. Because the remainder must be less than the divisor, the remainder must *either repeat or become 0.* Thus *every common fraction* will have a *repeating* or a *terminating* decimal as its decimal equivalent.

## CHECK YOURSELF ANSWERS

**1.** 0.875 **2.** $\frac{5}{11} = 0.455$ (to the nearest thousandth)

**3.** **(a)** 0.666. . . ; **(b)** $\frac{7}{12} = 0.583\ldots$ The digit 3 will continue indefinitely.

**4.** $\frac{5}{7} = 0.\overline{714285}$ **5.** 2.625

Name ______________

Section ________ Date ________

## 4.6 Exercises

Find the decimal equivalents for each of the following fractions.

**1.** $\frac{3}{4}$ **2.** $\frac{4}{5}$ **3.** $\frac{9}{20}$

**4.** $\frac{3}{10}$ **5.** $\frac{1}{5}$ **6.** $\frac{1}{8}$

**7.** $\frac{5}{16}$ **8.** $\frac{11}{20}$ **9.** $\frac{7}{10}$

**10.** $\frac{7}{16}$ **11.** $\frac{27}{40}$ **12.** $\frac{17}{32}$

Find the decimal equivalents rounded to the indicated place.

**13.** $\frac{5}{6}$ thousandth **14.** $\frac{7}{12}$ hundredth **15.** $\frac{4}{15}$ thousandth

Write the decimal equivalents, using the bar notation.

**16.** $\frac{1}{18}$ **17.** $\frac{4}{9}$ **18.** $\frac{3}{11}$

Find the decimal equivalents for each of the following mixed numbers.

**19.** $5\frac{3}{5}$ **20.** $7\frac{3}{4}$ **21.** $4\frac{7}{16}$

Find the decimal equivalent for each fraction.

**22.** $\frac{1}{11}$ **23.** $\frac{1}{111}$ **24.** $\frac{1}{1111}$

**25.** From the pattern of exercises 22 to 24, can you guess the decimal representation for $\frac{1}{11{,}111}$?

### ANSWERS

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______
25. ______

**ANSWERS**

26. ______

27. ______

28. ______

29. ______

30. ______

31. ______

32. ______

33. ______

Insert > or < to form a true statement.

**26.** $\frac{18}{21}$ ___ 0.863 **27.** $\frac{31}{34}$ ___ 0.9118 **28.** $\frac{21}{37}$ ___ 0.5664 **29.** $\frac{13}{17}$ ___ 0.7657

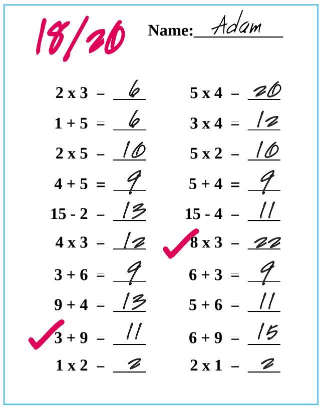
**30.** Every fraction has a decimal equivalent that either terminates (for example, $\frac{1}{4} = 0.25$) or repeats (for example, $\frac{2}{9} = 0.\overline{2}$). Work with a group to try to discover which fractions have terminating decimals and which have repeating decimals. You may assume that the numerator of each fraction you consider is 1, and focus your attention on the denominator. Be able to predict (successfully!) whether a given fraction is a "terminator" or a "repeater." (*Hint:* Study the prime factorization of each denominator you work with.)

**31.** Write the decimal equivalent of each fraction, using bar notation:

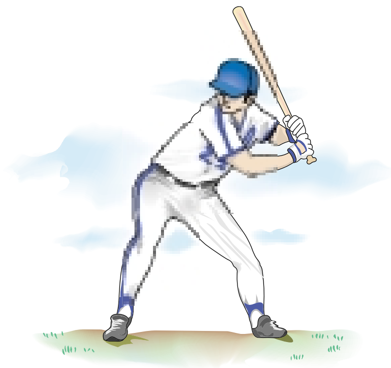
$\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}$

Based on these results, predict the decimal equivalent of $\frac{9}{9}$.

**32.** On a math quiz, Adam correctly answered 18 of 20 questions, or $\frac{18}{20}$ of the quiz. Write the decimal equivalent of this fraction.




**33.** In a weekend baseball tournament, Joel had 4 hits in 13 times at bat. That is, he hit safely $\frac{4}{13}$ of the time. Write the decimal equivalent for Joel's hitting, rounding to three decimal places. (That number is Joel's batting average.)



ANSWERS

34. ____

35. ____

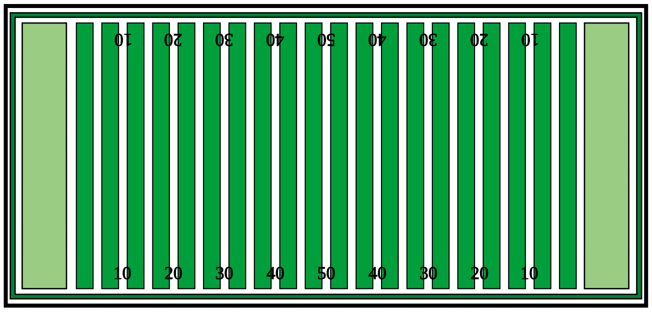
36. ____

**34.** The following table gives the wins and losses of the teams in the National League East as of September 18. The winning percentage of each team is calculated by writing the number of wins over the total games played, and converting this fraction to a decimal. Calculate the ratio of wins to total games played for every team, rounding to three decimal places.

| Team | Wins | Losses |
|---|---|---|
| Atlanta | 92 | 56 |
| New York | 90 | 58 |
| Philadelphia | 70 | 77 |
| Montreal | 62 | 85 |
| Florida | 57 | 89 |

**35.** The following table gives the wins and losses of all the teams in the Western Division of the National Football Conference for a recent season. Determine the ratio of wins to total games played for every team, rounding to three decimal points each of the teams.

| Team | Wins | Losses |
|---|---|---|
| San Francisco | 13 | 3 |
| Atlanta | 7 | 9 |
| Carolina | 7 | 9 |
| New Orleans | 6 | 10 |
| St. Louis | 5 | 11 |




**36.** The following table gives the free throws attempted (FTA) and the free throws made (FTM) for the top five players in the NBA for a recent season. Calculate the free throw percentage for each player by writing the FTM over the FTA and converting this fraction to a decimal.

| Player | FTM | FTA |
|---|---|---|
| Jeff Mulin | 154 | 164 |
| Jeff Hornacek | 285 | 322 |
| Ray Allen | 342 | 391 |
| Jamal Anderson | 275 | 315 |
| Kevin Johnson | 162 | 186 |

## Answers

**1.** 0.75 **3.** 0.45 **5.** 0.2 **7.** 0.3125 **9.** 0.7 **11.** 0.675
**13.** 0.833 **15.** 0.267 **17.** $0.\overline{4}$ **19.** 5.6 **21.** 4.4375 **23.** 0.009
**25.** 0.00009 **27.** $<$ **29.** $<$ **31.**  **33.** 0.308

**35.** 0.813, 0.438, 0.438, 0.375, 0.313