5.4 Solving Proportions

**OBJECTIVE**

1. Solve a proportion for an unknown value

A proportion consists of four values. If three of the four values of a proportion are known, you can always find the missing or unknown value.

In the proportion $\frac{a}{3} = \frac{10}{15}$, the first value is unknown. We have chosen to represent the unknown value with the letter $a$. Using the proportion rule, we can proceed as follows.

$$\frac{a}{3} = \frac{10}{15}$$

$$15 \cdot a = 3 \cdot 10 \quad \text{or} \quad 15 \cdot a = 30$$

The equals sign tells us that $15 \cdot a$ and 30 are just different names for the same number. This type of statement is called an equation.

**Definitions:** Equation

An equation is a statement that two expressions are equal.

One important property of an equation is that we can divide both sides by the same nonzero number. Here let's divide by 15.

$$15 \cdot a = 30$$

$$\frac{15 \cdot a}{15} = \frac{30}{15}$$

$$\frac{1}{15} \cdot a = 2$$

Divide by the coefficient of the variable. Do you see why we divided by 15? It leaves our unknown $a$ by itself in the left term.

$$a = 2$$

You should always check your result. It is easy in this case. We found a value of 2 for $a$. Replace the unknown $a$ with that value. Then verify that the fractions are proportional. We started with $\frac{a}{3} = \frac{10}{15}$ and found a value of 2 for $a$. So we write

$$3 \cdot 10 = 2 \cdot 15$$

$$30 = 30$$

The value of 2 for $a$ is correct.

The procedure for solving a proportion is summarized as follows.
Step by Step: To Solve a Proportion

Step 1 Use the proportion rule to write the equivalent equation \(a \cdot d = b \cdot c\).

Step 2 Divide both terms of the equation by the coefficient of the variable.

Step 3 Use the value found to replace the unknown in the original proportion. Check that the ratios or the rates are proportional.

Example 1

Solving Proportions for Unknown Values

Find the unknown value.

(a) \(\frac{8}{x} = \frac{6}{9}\)

Step 1 Using the proportion rule, we have the following.

\[6 \cdot x = 8 \cdot 9\]

or \(6x = 72\)

Step 2 Locate the coefficient of the variable, 6, and divide both sides of the equation by that coefficient.

\[\frac{6x}{6} = \frac{72}{6}\]

\[x = 12\]

Step 3 To check, replace \(x\) with 12 in the original proportion.

\[\frac{8}{12} = \frac{6}{9}\]

Multiply:

\[12 \cdot 6 = 8 \cdot 9\]

\[72 = 72\] The value of 12 for \(x\) checks.

(b) \(\frac{3}{5} = \frac{b}{25}\)

Step 1 Use the proportion rule.

\[5 \cdot b = 3 \cdot 25\]

or \(5 \cdot b = 75\)

Step 2 Locate the coefficient of the variable, 5, and divide both sides of the equation by that coefficient.

\[\frac{5b}{5} = \frac{75}{5}\]

\[b = 15\]
Step 3  To check, replace $b$ with 15 in the original proportion.

\[
\frac{3}{5} = \frac{15}{25}
\]

Multiply:

\[
5 \cdot 15 = 3 \cdot 25
\]

\[
75 = 75 \quad \text{The value of 15 checks for } b.
\]

CHECK YOURSELF 1

Solve the proportions for $n$. Check your result.

(a) \( \frac{4}{5} = \frac{n}{25} \)  
(b) \( \frac{7}{9} = \frac{42}{n} \)

In solving for a missing term in a proportion, we may find an equation involving fractions or decimals. Example 2 involves finding the unknown value in such cases.

Example 2

Solving Proportions for Unknown Values

(a) Solve the proportion for $x$.

\[
\frac{1}{4} = \frac{4}{x}
\]

\[
\frac{1}{4} \cdot x = 12
\]

We divide by the coefficient of $x$.

In this case it is $\frac{1}{4}$.

\[
\frac{1}{4} = \frac{12}{1}
\]

Remember: $\frac{12}{1}$ is $12 \div \frac{1}{4}$.

\[
x = \frac{12}{4}
\]

Invert the divisor and multiply.

\[
x = 48
\]

To check, replace $x$ with 48 in the original proportion.

\[
\frac{1}{4} \cdot \frac{4}{3} = \frac{4}{48}
\]

\[
3 \cdot \frac{4}{48} = \frac{1}{4} \cdot 48
\]

\[
12 = 12
\]
(b) Solve the proportion for \( a \).

\[
\frac{0.5}{2} = \frac{3}{a}
\]

\[
0.5a = 6
\]

**NOTE** Here we must divide 6 by 0.5 to find the unknown value. The steps of that division are shown below for review.

\[
\begin{array}{c}
0.5a = 6 \\
\frac{0.5a}{0.5} = \frac{6}{0.5} \\
a = 12
\end{array}
\]

We will leave it to you to confirm that \( 0.5 \cdot 12 = 2 \cdot 3 \).

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**CHECK YOURSELF 2**

(a) Solve for \( a \).

\[
\frac{1}{2} = \frac{3}{a}
\]

\[
\frac{2}{5} = \frac{3}{a}
\]

(b) Solve for \( x \).

\[
\frac{0.4}{x} = \frac{2}{30}
\]

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**CHECK YOURSELF ANSWERS**

1. (a) \( 5n = 100 \)

To check: \( \frac{5n}{5} = \frac{100}{5} \)

\( n = 20 \)

\( 5 \cdot 20 = 4 \cdot 25 \)

(b) \( 7n = 42 \cdot 9 \)

To check: \( \frac{7n}{7} = \frac{42 \cdot 9}{7} \)

\( n = 6 \)

\( 7 \cdot 6 = 9 \cdot 42 \)

2. (a) 30; (b) 6
5.4 Exercises

Solve for the unknown in each of the following proportions.

1. \( \frac{x}{3} = \frac{6}{9} \)
2. \( \frac{x}{6} = \frac{3}{9} \)
3. \( \frac{10}{n} = \frac{15}{6} \)

4. \( \frac{4}{3} = \frac{8}{n} \)
5. \( \frac{4}{7} = \frac{y}{14} \)
6. \( \frac{7}{m} = \frac{14}{8} \)

7. \( \frac{5}{8} = \frac{a}{16} \)
8. \( \frac{5}{7} = \frac{x}{35} \)
9. \( \frac{8}{p} = \frac{6}{3} \)

10. \( \frac{4}{15} = \frac{8}{n} \)
11. \( \frac{11}{a} = \frac{2}{44} \)
12. \( \frac{5}{x} = \frac{15}{9} \)

13. \( \frac{35}{40} = \frac{7}{n} \)
14. \( \frac{x}{8} = \frac{15}{24} \)
15. \( \frac{a}{42} = \frac{5}{7} \)

16. \( \frac{7}{12} = \frac{m}{24} \)
17. \( \frac{18}{12} = \frac{12}{p} \)
18. \( \frac{x}{32} = \frac{7}{8} \)

19. \( \frac{x}{18} = \frac{64}{72} \)
20. \( \frac{20}{15} = \frac{100}{a} \)
21. \( \frac{6}{n} = \frac{75}{100} \)

22. \( \frac{36}{x} = \frac{8}{6} \)
23. \( \frac{5}{35} = \frac{a}{28} \)
24. \( \frac{20}{24} = \frac{p}{18} \)

25. \( \frac{12}{100} = \frac{3}{x} \)
26. \( \frac{b}{7} = \frac{21}{49} \)
27. \( \frac{p}{24} = \frac{25}{120} \)

28. \( \frac{5}{x} = \frac{20}{88} \)
29. \( \frac{1}{2} = \frac{3}{a} \)
30. \( \frac{x}{5} = \frac{2}{3} \)

ANSWERS

1. 
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30.
**Answers**

1. \( 9x = 18; x = 2 \)  
3. 4  
5. 8  
7. 10  
9. 4  
11. 242

13. \( 35n = 280; n = 8 \)  
15. 30

17. \( 18p = 144; p = 8 \)  
19. 16  
21. 8

23. \( 35a = 140; a = 4 \)  
25. 25  
27. 5  
29. 12  
31. 2

33. 24  
35. 12  
37. 80  
39. 0.55