3.1 Ordered Pairs and Relations

OBJECTIVES

1. Identify ordered pairs
2. Identify a relation
3. Identify the domain and range of a function

In Chapter 1, we introduced the idea of a set of numbers. In this chapter, we will look at pairs of numbers.

Given the two related values \( x \) and \( y \), we write the pair as \((x, y)\). The set of all possible ordered pairs of real numbers is written as \( \{(x, y) | x \in R, y \in R\} \).

The ordered pair \((2, -3)\) is different from the ordered pair \((-3, 2)\). By contrast, the set \( \{2, -3\} \) is identical to the set \( \{-3, 2\} \).

Example 1

Identifying Ordered Pairs

Which of the following are ordered pairs?

(a) \((2, -\pi)\)  
(b) \(\{2, -4\}\)  
(c) \((1, 3, -1)\)  
(d) \(\{(1, -5), (9, 0)\}\)  
(e) \(2, 5\)

Only (a) is an ordered pair. (b) is a set (it uses braces instead of parentheses), (c) has three numbers instead of two, (d) is a set of ordered pairs, and (e) is simply a list of two numbers.

CHECK YOURSELF 1

Which of the following are ordered pairs?

(a) \(\left\{ \frac{1}{2}, -3 \right\}\)  
(b) \(-3, \frac{1}{3}\)  
(c) \(\{(5, 0)\}\)  
(d) \((1, -5)\)  
(e) \(-3, 6\)

Ordered pairs are not necessarily made up of two numbers. Given something like (John Doe, 123-45-6789), we have an ordered pair. In this case, it is a name paired with that person’s Social Security number (SSN).

Definitions: Relation

A set of ordered pairs is called a relation.

We usually denote a relation with a capital letter.

Given

\( R = \{(John \ Doe, 123-45-6789), (Jacob \ Smith, 987-65-4321), (Julia \ Jones, 111-22-3333)\} \)

we have a relation, which we call \( R \). In this case, there are three ordered pairs in the relation \( R \).
Within this relation, there are two interesting sets. The first is the set of names, which happens to be the set of first elements. The second is the set of SSNs, which is the set of second elements. Each of these sets has a name.

**Definitions:** Domain

The set of first elements in a relation is called the **domain** of the relation.

**Example 2**

**Finding the Domain of a Relation**

Find the domain of each relation.

(a) \( A = \{ (\text{Ben Bender, 58}), (\text{Carol Clairol, 32}), (\text{David Duval, 29}) \} \)

The domain of \( A \) is \{Ben Bender, Carol Clairol, David Duval\}.

(b) \( B = \left\{ \left( 5, \frac{1}{2} \right), (-4, -5), (-12, 10), (-16, \pi) \right\} \)

The domain of \( B \) is \{5, -4, -12, -16\}.

**CHECK YOURSELF 2**

Find the domain of each relation.

(a) \( R = \{ (\text{Secretariat, 10}), (\text{Seattle Slew, 8}), (\text{Charismatic, 5}), (\text{Gallant Man, 7}) \} \)

(b) \( S = \left\{ \left( -\frac{1}{2}, \frac{3}{4} \right), (0, 0), (1, 5), (\pi, \pi) \right\} \)

**NOTE**

\( X \rightarrow \text{Domain} \)

\( Y \rightarrow \text{Range} \)

Many students find it helpful to remember that domain and range occur in alphabetical order.

**Definitions:** Range

The set of second elements in a relation is called the **range** of the relation.
Example 3

Finding the Range of a Relation

Find the range for each relation.

(a) \( A = \{(\text{Ben Bender, 58}), (\text{Carol Clairol, 32}), (\text{David Duval 29})\}\)

The range of \( A \) is \( 58, 32, 29 \)

(b) \( B = \left\{ \left(5, \frac{1}{2}\right), (-4, -5), (-12, 10), (-16, \pi) \right\} \)

The range of \( B \) is \( \left\{ \frac{1}{2}, -5, 10, \pi \right\} \)

CHECK YOURSELF 3

Find the range of each relation.

(a) \( R = \{(\text{Secretariat, 10}), (\text{Seattle Slew, 8}), (\text{Charismatic, 5}), (\text{Gallant Man, 7})\}\)

(b) \( S = \left\{ \left(-\frac{1}{2}, \frac{3}{4}\right), (0, 0), (1, 5), (\pi, \pi) \right\} \)

CHECK YOURSELF ANSWERS

1. (b) and (d) are ordered pairs
2. (a) The domain of \( R \) is \{Secretariat, Seattle Slew, Charismatic, Gallant Man\};

   (b) The domain of \( S \) is \( \left\{ -\frac{1}{2}, 0, 1, \pi \right\} \)

3. (a) The range of \( R \) is \( \{10, 8, 5, 7\} \); (b) The range of \( S \) is \( \left\{ \frac{3}{4}, 0, 5, \pi \right\} \)
3.1 Exercises

In exercises 1 to 4, identify the ordered pairs.

1. (a) (3, −5)   (b) {7, 9}   (c) (2, 5)   (d) 5, 2   (e) ((3, 1), 4)
2. (a) {7, 23}   (b) (1, 0, (5, 6))   (c) \( \left( \frac{1}{2}, -1 \right) \)   (d) [5, 6]   (e) (23, 7)
3. (a) 18, 67   (b) (−3, −9)   (c) {3, 9}   (d) (3, 7, −3)   (e) [12, 56]
4. (a) {45, 67}   (b) (9, 3)   (c) 5, 8   (d) (11, −3, 9)   (e) [5, 2]

In exercises 5 to 20, find the domain and range of each relation.

5. \( A = \{(\text{Colorado}, 21), (\text{Edmonton}, 5), (\text{Calgary}, 18), (\text{Vancouver}, 17)\} \)
6. \( B = \{(\text{Eric Lindros}, 88), (\text{Mark Recchi}, 8), (\text{John LeClair}, 10), (\text{Keith Primeau}, 25)\} \)
7. \( C = \{(\text{John Adams}, -16), (\text{John Kennedy}, -23), (\text{Richard Nixon}, -5), (\text{Harry Truman}, -11)\} \)
8. \( E = \{(\text{Utah}, 27), (\text{San Antonio}, 28), (\text{Minnesota}, 24), (\text{Denver}, 19)\} \)
9. \( F = \left\{ \left( \text{St. Louis}, \frac{1}{2} \right), \left( \text{Denver}, -\frac{3}{4} \right), \left( \text{Green Bay}, \frac{7}{8} \right), \left( \text{Dallas}, -\frac{4}{5} \right) \right\} \)
10. \( G = \left\{ \left( \text{Chamber}, \pi \right), \left( \text{Testament}, 2\pi \right), \left( \text{Rainmaker}, \frac{1}{2} \right), \left( \text{Street Lawyer}, 6 \right) \right\} \)
11. \{(1, 2), (3, 4), (5, 6), (7, 8), (9, 10)\}
12. \{(2, 3), (3, 5), (4, 7), (5, 9), (6, 11)\}
13. \{(1, 2), (4, 6), (3, 3), (5, 4), (6, 1)\}
14. \{(3, 4), (5, 7), (6, 1), (2, 2), (4, 3)\}
15. \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)\}
16. \{(3, 4), (3, 6), (3, 8), (3, 9), (3, 10)\}
17. \{(1, 5), (2, 5), (3, 6), (2, 4), (4, 5)\}
18. \{(2, 8), (3, 9), (2, 9), (3, 8), (4, 7)\}
19. \{(-1, 3), (-2, 4), (-3, 5), (4, 4), (5, 6)\}
20. \{(-2, 4), (1, 4), (-3, 4), (5, 4), (7, 4)\}
22. Food Purchases. In the snack department of the local supermarket, candy costs $1.58 per pound. For 1 to 5 pounds, write the cost of candy as a set of ordered pairs.

In exercises 23 to 26, write a set of ordered pairs that describes each situation. Give the domain and range of each relation.

23. The first element is an integer between −3 and 3. The second coordinate is the cube of the first coordinate.

24. The first element is a positive integer less than 6. The second coordinate is the sum of the first coordinate and −2.

25. The first element is the number of hours worked 10, 20, 30, 40; the second coordinate is the salary at $6 per hour.

26. The first coordinate is the number of toppings on a pizza (up to 4); the second coordinate is the price of the pizza, which is $9 plus $1 per topping.

### Answers

1. (a) and (c)  
3. (b)  
5. D: {Colorado, Edmonton, Calgary, Vancouver}; R: {21, 5, 18, 17}  
9. D: {St. Louis, Denver, Green Bay, Dallas}; R: \( \left\{ \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{4}{5} \right\} \)  
11. D: {1, 3, 5, 7, 9}; R: {2, 4, 6, 8, 10}  
13. D: {1, 3, 4, 5, 6}; R: {1, 2, 3, 4, 6}  
15. D: {1}; R: {2, 3, 4, 5, 6}  
17. D: {1, 2, 3, 4}; R: {4, 5, 6}  
19. D: {−1, −2, −3, 4, 5}; R: {3, 4, 5, 6}  
21. \( \left\{ \left( 1, \frac{9}{8} \right), \left( 2, \frac{7}{8} \right), \left( 3, \frac{4}{8} \right), \left( 4, \frac{1}{4} \right), \left( 5, 9 \right) \right\} \)  
23. {−2, −8}, {−1, −1}, {0, 0}, {1, 1}, {2, 8}; D: {−2, −1, 0, 1, 2}; R: {−8, −1, 0, 1, 8}  
25. {(10, 60), (20, 120), (30, 180), (40, 240)}; D: {10, 20, 30, 40}; R: {60, 120, 180, 240}