5.5 Graphing Systems of Linear Inequalities in Two Variables

5.5 OBJECTIVES

1. Graph a system of linear inequalities in two variables
2. Solve an application of a system of linear inequalities

In Section 5.1, we dealt with finding the solution set of a system of linear equations. That solution set represented the points of intersection of the graphs of the equations in the system. In this section, we extend that idea to include systems of linear inequalities.

In this case, the solution set is all ordered pairs that satisfy each inequality. The graph of the solution set of a system of linear inequalities is then the intersection of the graphs of the individual inequalities. Let’s look at an example.

Example 1

Solving a System by Graphing

Solve the following system of linear inequalities by graphing

\[ x + y > 4 \]
\[ x - y < 2 \]

We start by graphing each inequality separately. The boundary line is drawn, and using \((0, 0)\) as a test point, we see that we should shade the half plane above the line in both graphs.

\[ x - y < 2 \]
\[ x + y > 4 \]

**NOTE** Notice that the boundary line is dashed to indicate that points on the line do not represent solutions.

In practice, the graphs of the two inequalities are combined on the same set of axes, as is shown below. The graph of the solution set of the original system is the intersection of the graphs drawn above.

**NOTE** Points on the lines are not included in the solution set.
Most applications of systems of linear inequalities lead to **bounded regions**. This requires a system of three or more inequalities, as shown in Example 2.

**Example 2**

Solving a System by Graphing

Solve the following system of linear inequalities by graphing.

\[
\begin{align*}
2x - y &< 4 \\
x + y &< 3
\end{align*}
\]

On the same set of axes, we graph the boundary line of each of the inequalities. We then choose the appropriate half planes (indicated by the arrow that is perpendicular to the line) in each case, and we locate the intersection of those regions for our graph.

**Note** The vertices of the shaded region are given because they have particular significance in later applications of this concept. Can you see how the coordinates of the vertices were determined?

**Check Yourself 1**

Solve the following system of linear inequalities by graphing.

\[
\begin{align*}
2x - y &< 4 \\
x + y &< 3
\end{align*}
\]

**Check Yourself 2**

Solve the following system of linear inequalities by graphing.

\[
\begin{align*}
2x - y &< 8 & x &\geq 0 \\
x + y &< 7 & y &\geq 0
\end{align*}
\]

Let’s expand on Example 4, Section 5.2, to see an application of our work with systems of linear inequalities. Consider Example 3.
Example 3

Solving a Business-Based Application

A manufacturer produces a standard model and a deluxe model of a 25-in. television set. The standard model requires 12 h of labor to produce, and the deluxe model requires 18 h. The labor available is limited to 360 h per week. Also, the plant capacity is limited to producing a total of 25 sets per week. Draw a graph of the region representing the number of sets that can be produced, given these conditions.

As suggested earlier, we let \( x \) represent the number of standard-model sets produced and \( y \) the number of deluxe-model sets. Because the labor is limited to 360 h, we have

\[
12x + 18y \leq 360 \quad (1)
\]

The total production, here \( x + y \) sets, is limited to 25, so we can write

\[
x + y \leq 25 \quad (2)
\]

For convenience in graphing, we divide both members of inequality (1) by 6, to write the equivalent system

\[
2x + 3y \leq 60
\]

\[
x + y \leq 25
\]

\[
x \geq 0
\]

\[
y \geq 0
\]

We now graph the system of inequalities as before. The shaded area represents all possibilities in terms of the number of sets that can be produced.

**NOTE** We have \( x \geq 0 \) and \( y \geq 0 \) because the number of sets produced cannot be negative.

**NOTE** The shaded area is called the feasible region. All points in the region meet the given conditions of the problem and represent possible production options.
CHECK YOURSELF 3

A manufacturer produces TVs and CD players. The TVs require 10 h of labor to produce and the CD players require 20 h. The labor hours available are limited to 300 h per week. Existing orders require that at least 10 TVs and at least 5 CD players be produced per week. Draw a graph of the region representing the possible production options.

CHECK YOURSELF ANSWERS

1. \(2x - y < 4\)
   \(x + y < 3\)

2. \(2x - y \leq 8\)
   \(x + y \leq 7\)
   \(x \geq 0\)
   \(y \geq 0\)

3. Let \(x\) be the number of TVs and \(y\) be the number of CD players. The system is
   \(10x + 20y \leq 300\)
   \(x \geq 10\)
   \(y \geq 5\)
Exercises

In exercises 1 to 18, solve each system of linear inequalities by graphing.

1. $x + 2y \leq 4$
   $x - y \geq 1$

2. $3x - y > 6$
   $x + y < 6$

3. $3x + y < 6$
   $x + y > 4$

4. $2x + y \geq 8$
   $x + y \geq 4$

5. $x + 3y \leq 12$
   $2x - 3y \leq 6$

6. $x - 2y > 8$
   $3x - 2y > 12$

7. $3x + 2y \leq 12$
   $x \geq 2$

8. $2x + y \leq 6$
   $y \geq 1$

9. $2x + y < 8$
   $x > 1$
   $y > 2$

10. $3x - y \leq 6$
    $x \geq 1$
    $y \leq 3$

11. $x + 2y \leq 8$
    $2 \leq x \leq 6$
    $y \geq 0$

12. $x + y < 6$
    $0 \leq y \leq 3$
    $x \geq 1$
In exercises 19 and 20, draw the appropriate graph.

19. **Manufacturing.** A manufacturer produces both two-slice and four-slice toasters. The two slice toaster takes 6 h of labor to produce and the four-slice toaster 10 h. The labor available is limited to 300 h per week, and the total production capacity is 40 toasters per week. Draw a graph of the feasible region, given these conditions, in which \( x \) is the number of two-slice toasters and \( y \) is the number of four-slice toasters.
20. **Production.** A small firm produces both AM and AM/FM car radios. The AM radios take 15 h to produce, and the AM/FM radios take 20 h. The number of production hours is limited to 300 h per week. The plant’s capacity is limited to a total of 18 radios per week, and existing orders require that at least 4 AM radios and at least 3 AM/FM radios be produced per week. Draw a graph of the feasible region, given these conditions, in which $x$ is the number of AM radios and $y$ the number of AM/FM radios.

21. When you solve a system of linear inequalities, it is often easier to shade the region that is not part of the solution, rather than the region that is. Try this method, then describe its benefits.

22. Describe a system of linear inequalities for which there is no solution.

23. Write the system of inequalities whose graph is the shaded region.

24. Write the system of inequalities whose graph is the shaded region.
Answers

1. 

3. 

5. 

7. 

9. 

11. 

13. 

15. 

17. 

19. 

21. 

23. \[ y \leq 2x + 3 \]
   \[ y \leq -3x + 5 \]
   \[ y \geq -x - 1 \]