

7.6

Applications of Rational Equations

7.6 OBJECTIVES

1. Solve a literal equation that involves a rational expression
2. Solve a distance problem that involves a rational equation
3. Solve a work problem
4. Convert units


In the previous section, we solved rational equations. In this section we will see several applications of those techniques. Given a literal equation such as

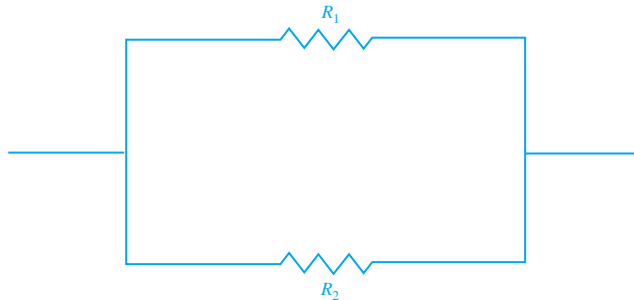
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

we can use the techniques of Section 7.5 to solve for one of the variables.

Example 1

Solving a Literal Equation

NOTE This is a parallel electric circuit. The symbol for a resistor is .



NOTE Recall that the numbers 1 and 2 are *subscripts*. We read R_1 as “ R sub 1” and R_2 as “ R sub 2.”

If two resistors with resistances R_1 and R_2 are connected in parallel, the combined resistance R can be found from

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Solve the formula for R .

First, the LCD is RR_1R_2 , and we multiply:

$$RR_1R_2 \cdot \frac{1}{R} = RR_1R_2 \cdot \frac{1}{R_1} + RR_1R_2 \cdot \frac{1}{R_2}$$

Simplifying yields

$$R_1R_2 = RR_2 + RR_1 \quad \text{Factor out } R \text{ on the right.}$$

$$R_1R_2 = R(R_2 + R_1) \quad \text{Divide by } R_2 + R_1 \text{ to isolate } R.$$

$$\frac{R_1R_2}{R_2 + R_1} = R \quad \text{or} \quad R = \frac{R_1R_2}{R_1 + R_2}$$

NOTE Reversing the left and right sides of an equation uses the symmetric property of equality.



CHECK YOURSELF 1

Solve for D_1 .

$$\frac{1}{F} = \frac{1}{D_1} + \frac{1}{D_2}$$

NOTE This formula involves the focal length of a convex lens.

Many distance problems also lead to rational equations. You will recall that distance (d), rate (r), and time (t) are related by the following formula

Rules and Properties: The Distance Relationship I

$$d = r \cdot t$$

Treating this as a literal equation, we can produce two variations that frequently lead to rational equations.

Rules and Properties: The Distance Relationship II

$$r = \frac{d}{t}$$

and

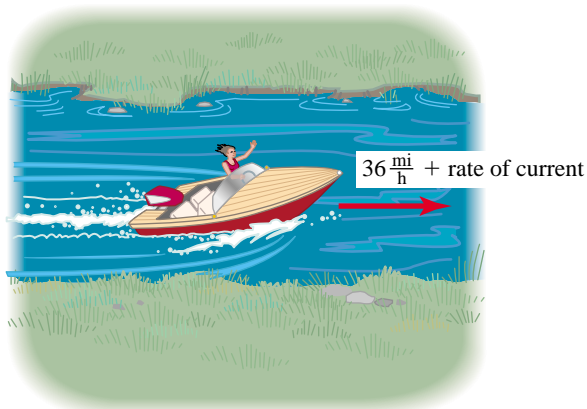
Rules and Properties: The Distance Relationship III

$$t = \frac{d}{r}$$

Example 2

Finding a Rate

A boat, which moves at 36 mi/h in still water, travels 28 mi downstream in the same amount of time that it takes to travel 20 mi upstream. Find the speed of the current.



When solving an application that involves the distance relationship, it is usually best to begin by completing the following table.

	Distance	Rate	Time
Upstream			
Downstream			

Letting r represent the rate of the current, we have the following information:

	Distance	Rate	Time
Upstream	20	$36 - r$	
Downstream	28	$36 + r$	

Using relationship III above, we can complete the last column of the table.

	Distance	Rate	Time
Upstream	20	$36 - r$	$\frac{20}{(36 - r)}$
Downstream	28	$36 + r$	$\frac{28}{(36 + r)}$

Having completed the table, we go back to the original problem. The key phrase is “in the same amount of time.” That means that the time going upstream is the same as the time going downstream. This leads to the equation

$$\frac{28}{(36 + r)} = \frac{20}{(36 - r)}$$

The LCD is $(36 + r)(36 - r)$. Clearing the fractions, we get

$$28(36 - r) = 20(36 + r)$$

$$1008 - 28r = 720 + 20r$$

$$288 = 48r$$

$$r = 6$$

The rate of the current is 6 mi/h.



CHECK YOURSELF 2

A boat, which moves at 30 mi/h in still water, travels 3 mi downstream in the same amount of time that it takes to travel 2 mi upstream. Find the speed of the current.

Another type of application that frequently leads to a rational equation is something called a **work problem**. Solving a work problem is simplified by using the following work principles.

Rules and Properties: Work Principle I

If a job takes t hours to complete, then, for each hour,

$$\frac{1}{t}$$

represents the portion of the job that has been completed.

This principle confirms that, if a job takes 2 h to do, then $\frac{1}{2}$ of the job is done each hour. If a job takes 10 h to do, then $\frac{1}{10}$ of the job is done each hour.

Extending the idea behind the first principle, we find the second work principle.

Rules and Properties: Work Principle II

If two entities are working on the same job, and the first would take a hours to complete the job alone and the second b hours to complete the job alone, then the expression

$$\frac{1}{a} + \frac{1}{b}$$

represents the portion of the job completed each hour if both entities are at work.

Combining these two principles, we get an equation that will allow us to find the total time it takes to complete a job.

Rules and Properties: Work Principle III

If two entities are working on the same job, and the first would take a hours to complete the job alone and the second b hours to complete the job alone, then the equation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$$

can be used to find t , the time it will take to complete the job.

We will use this equation in the next example.

Solving a Work Problem

Jason and Hilger are required to paint over the graffiti on a wall. If Jason worked alone, it would take him 20 h to repaint. Working alone, Hilger could do the job in 15 h. How long will it take them to do the painting if they work together?



Using the work principle, we get the equation

$$\frac{1}{20} + \frac{1}{15} = \frac{1}{t}$$

Clearing the fractions (LCD = 60t) yields

$$3t + 4t = 60$$

$$7t = 60$$

$$t = \frac{60}{7}$$

Working together, they will finish the job in $8\frac{4}{7}$ h. Note that, when working together, it will always take less time to complete a job than it will take either individual!



CHECK YOURSELF 3

Filling a hot tub with a hose will take 4 h. Draining the same tub takes 7 h. If the drain is open, how long will it take to fill the tub? (Hint: Treat the draining time as a negative number.)



The techniques used with rational equations are also used when performing **unit conversions**. The following table contains some of the common conversion factors.

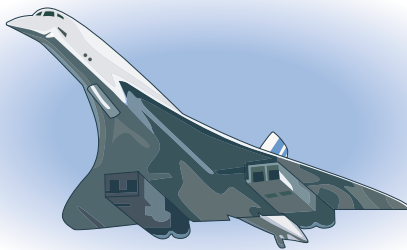
Common Conversion Factors	
1 mile	= 5280 feet
1 hour	= 60 minutes
1 minute	= 60 seconds
1 kilogram	= 1000 grams
1 second	= 1,000,000 microseconds
1 gigabyte	= 1,000,000,000 bytes

Each of the conversion factors can be rewritten as one of two fractions that are equal to one. For example, we can rewrite the statement 1 kilogram = 1000 grams as either

$$\frac{1 \text{ kilogram}}{1000 \text{ grams}} = 1 \quad \text{or} \quad \frac{1000 \text{ grams}}{1 \text{ kilogram}} = 1$$

Example 4**Converting Units**

An SST can fly over 2000 mi/h. To the nearest tenth, how many miles does it cover each second?



First, note that 2000 mi/h can be written in several different forms. When doing unit conversion, it is best to write a rate (like 2000 mi/h) as a ratio, so we write it as

$$\frac{2000 \text{ mi}}{1 \text{ h}}$$

This is now multiplied by the appropriate conversion ratios. To figure out what those ratios should be, it is best to first write the units of the desired result.

$$\frac{2000 \text{ mi}}{1 \text{ h}} = \frac{\text{mi}}{1 \text{ s}}$$

To convert hours to seconds, use the conversion ratio $\frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \frac{1 \text{ h}}{3600 \text{ s}}$

$$\frac{2000 \text{ mi}}{1 \text{ h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = \frac{2000 \text{ mi}}{3600 \text{ s}} = \frac{5 \text{ mi}}{9 \text{ s}} \approx \frac{0.55 \text{ mi}}{1 \text{ s}}$$

When traveling at 2000 mi/h, the SST covers 0.55 mi every second.

**CHECK YOURSELF 4**

A roller coaster reaches a maximum speed of 75 ft/s. To the nearest tenth, convert the speed to miles per hour.

**CHECK YOURSELF ANSWERS**

1. $\frac{FD_2}{D_2 - F}$ 2. 6 mi/h 3. $\frac{28}{3}$ h 4. 51.1 mi/h

7.6 Exercises

Name _____

Section _____ Date _____

In exercises 1 to 8, solve each equation for the indicated variable.

1. $\frac{1}{x} = \frac{1}{a} - \frac{1}{b}$ for x

2. $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ for a

3. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_1

4. $\frac{1}{F} = \frac{1}{D_1} + \frac{1}{D_2}$ for D_2

5. $y = \frac{x+1}{x-1}$ for x

6. $y = \frac{x-3}{x-2}$ for x

7. $t = \frac{A-P}{Pr}$ for P

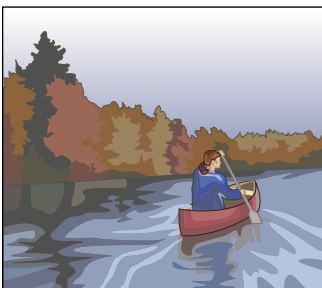
8. $I = \frac{nE}{R+nr}$ for n

Solve the following problems.

9. **Motion.** A motorboat can travel 20 mi/h in still water. If the boat can travel 3 mi downstream on a river in the same time it takes to travel 2 mi upstream, what is the rate of the river's current?



10. **Motion.** A small jet has an airspeed (the rate in still air) of 300 mi/h. During one day's flights, the pilot noted that the plane could fly 85 mi with a tailwind in the same time it took to fly 65 mi against that same wind. What was the rate of the wind?
11. **Motion.** A plane flew 720 mi with a steady 30-mi/h tailwind. The pilot then returned to the starting point, flying against that same wind. If the round-trip flight took 10 h, what was the plane's airspeed?
12. **Motion.** Janet and Michael took a canoeing trip, traveling 6 mi upstream along a river, against a 2 mi/h current. They then returned downstream to the starting point of their trip. If their entire trip took 4 h, what was their rate in still water?



13. **Motion.** Po Ling can bicycle 75 mi in the same time it takes her to drive 165 mi. If her driving rate is 30 mi/h faster than her rate on the bicycle, find each rate.

ANSWERS

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20. _____

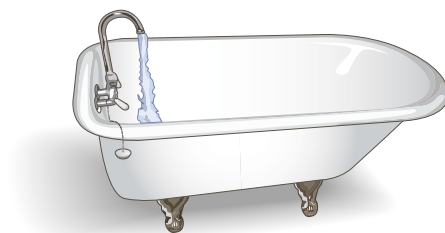
21. _____

22. _____

- 14. Motion.** A passenger train can travel 275 mi in the same time a freight train takes to travel 225 mi. If the speed of the passenger train is 10 mi/h more than that of the freight train, find the speed of each train.
- 15. Motion.** A light plane took 1 h longer to fly 540 mi on the first portion of a trip than to fly 360 mi on the second. If the rate was the same for each portion, what was the flying time for each leg of the trip?
- 16. Motion.** Gilbert took 2 h longer to drive 240 mi on the first day of a business trip than to drive 144 mi on the second day. If his rate was the same both days, what was his driving time for each day?
- 17. Motion.** An express train and a passenger bus leave the same city, at the same time, for a destination 350 mi away. The rate of the train is 20 mi/h faster than the rate of the bus. If the train arrives at its destination 2 h ahead of the bus, find each rate.
- 18. Motion.** A private plane and a commercial plane take off from an airport at the same time for a city 720 mi away. The rate of the private plane is 180 mi/h less than that of the commercial plane. If the commercial plane arrives 2 h ahead of the private plane, find each plane's rate.
- 19. Work.** One road crew can pave a section of highway in 15 h. A second crew, working with newer equipment, can do the same job in 10 h. How long would it take to pave that same section of highway if both crews worked together?



- 20. Work.** One computer printer can print a company's weekly payroll checks in 60 min. A second printer would take 90 min to complete the job. How long would it take the two printers, operating together, to print the checks?
- 21. Work.** An inlet pipe can fill a tank in 10 h. An outlet pipe can drain that same tank in 30 h. The inlet valve is opened, but the outlet valve is accidentally left open. How long will it take to fill the tank with both valves open?
- 22. Work.** A bathtub can be filled in 8 min. It takes 12 min for the bathtub to drain. If the faucet is turned on but the drain is also left open, how long will it take to fill the tub?



23. **Work.** An electrician can wire a house in 20 h. If she works with an apprentice, the same job can be completed in 12 h. How long would it take the apprentice, working alone, to wire the house?
24. **Work.** A landscaper can prepare and seed a new lawn in 12 h. If he works with an assistant, the job takes 8 h. How long would it take the assistant, working alone, to complete the job?
25. **Work.** An experienced roofer can work twice as fast as her helper. Working together, they can shingle a new section of roof in 4 h. How long would it take the experienced roofer, working alone, to complete the same job?



26. **Work.** One model copier operates at 3 times the speed of another. Working together, the copiers can copy a report in 8 min. The faster model breaks down, and the other model must be used. How long will the job take with the machine that is available?
27. **Work.** A college uses two optical scanners to grade multiple-choice tests. One model takes 12 min longer to complete the scoring of a test than the other model. If by both models working together the test can be scored in 8 min, how long would each model take to score the same test, used by itself?
28. **Work.** Virginia can complete her company's monthly report in 5 h less time than Carl. If they work together, the report will take them 6 h to finish. How long would it take Virginia, working alone?
29. **Number analysis.** The sum of the reciprocals of two consecutive integers is equal to 11 times the reciprocal of the product of those integers. What are the two integers?
30. **Number analysis.** The sum of the reciprocals of two consecutive even integers is equal to 10 times the reciprocal of the product of those integers. Find the two integers.
31. **Number analysis.** If the same number is added to the numerator and denominator of $\frac{2}{5}$, the result is $\frac{4}{5}$. What is that number?
32. **Number analysis.** If the same number is subtracted from the numerator and denominator of $\frac{11}{15}$, the result is $\frac{1}{3}$. Find that number.

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ANSWERS

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40. _____

33. **Number analysis.** One positive number is 2 more than another. If the sum of the reciprocals of the two numbers is $\frac{7}{24}$, what are those numbers?
34. **Number analysis.** One integer is 3 less than another. If the sum of the reciprocals of the two numbers is $\frac{7}{10}$, find the two integers.
35. A car is traveling at the rate of 60 ft/s. To the nearest tenth, convert the speed to miles per hour.
36. How many seconds are there in the month of June?
37. Arlene's hard drive has 2 gigabytes of memory. How many bytes is this?
38. Mr. Fernandez can walk 5 mi in 2 h. What is his speed in feet per second, to the nearest tenth?
39. Trucks in Sam's home state are taxed at the rate of 0.15¢ per gram. His truck weighs 1620 kg. What is his annual tax?



40. Suppose you travel to a certain destination at an average speed of 30 mi/h. What should your speed be on the return trip to bring your overall average speed to 60 mi/h?



Answers

1. $\frac{ab}{b-a}$ 3. $\frac{RR_2}{R_2 - R}$ 5. $\frac{y+1}{y-1}$ 7. $\frac{A}{1+rt}$ 9. 4 mi/h
11. 150 mi/h 13. Bicycling 25 mi/h, driving 55 mi/h 15. 3 h, 2 h
17. Bus 50 mi/h, train 70 mi/h 19. 6 h 21. 15 h 23. 30 h 25. 6 h
27. 12 min, 24 min 29. 5, 6 31. 10 33. 6, 8 35. 40.9 mi/h
37. 2,000,000,000 39. \$2430