\[ I(r) = -0.5 + 2.7r \]

where \( r \) represents R & D expenditures.

(A) Complete the following table. Round values of \( I(r) \) to one decimal place.

<table>
<thead>
<tr>
<th>( r ) (R &amp; D)</th>
<th>0.66</th>
<th>0.75</th>
<th>0.85</th>
<th>0.99</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net income</td>
<td>1.2</td>
<td>1.5</td>
<td>1.8</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>( I(r) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(B) Sketch by hand the graph of \( I \) and the data on the same axes.

(C) Use the modeling function \( I \) to estimate the income if the company spends $1.5 billion on research and development. $2 billion.

Section 1-4 Functions: Graphs and Properties

Basic Concepts

Increasing and Decreasing Functions

Local Maxima and Minima

Piecewise-Defined Functions

The Greatest Integer Function

One of the primary goals of this course is to provide you with a set of mathematical tools that can be used, in conjunction with a graphing utility, to analyze graphs that arise quite naturally in important applications. In this section, we discuss some basic concepts that are commonly used to describe graphs of functions.

Basic Concepts

Each function that has a real number domain and range has a graph—the graph of the ordered pairs of real numbers that constitute the function. When functions are graphed, domain values usually are associated with the horizontal axis and range values with the vertical axis. Thus, the graph of a function \( f \) is the same as the graph of the equation

\[ y = f(x) \]

where \( x \) is the independent variable and the abscissa of a point on the graph of \( f \). The variables \( y \) and \( f(x) \) are dependent variables, and either is the ordinate of a point on the graph of \( f \) (see Fig. 1).

The abscissa of a point where the graph of a function intersects the \( x \) axis is called an \textbf{x intercept} or \textbf{zero} of the function. The \( x \) intercept is also a real solution or \textbf{root} of the equation \( f(x) = 0 \). The ordinate of a point where the graph of a function crosses the \( y \) axis is called the \textbf{y intercept} of the function. The \( y \) intercept is given by \( f(0) \), provided \( 0 \) is in the domain of \( f \). Note that a function can have more than one \( x \) intercept but can never have more than one \( y \) intercept—a consequence of the vertical line test discussed in the preceding section.

\textbf{Example 1}

Finding \( x \) and \( y \) Intercepts

Find the \( x \) and \( y \) intercepts (correct to one decimal place) of \( f(x) = x^3 + x - 3 \).
Solution

From the graph of \( f \) in Figure 2(a), we see that the \( y \) intercept is \( f(0) = -3 \) and that there is an \( x \) intercept between 1 and 2. Zooming in on the graph near \( x = 1 \) [Figs. 2(b) and 2(c)], we see that the \( x \) intercept (to one decimal place) is 1.2.

\[ f(x) = x^3 + x - 3 \]

**FIGURE 2**

(a) 
(b) 
(c)

**MATCHED PROBLEM 1**

Find the \( x \) and \( y \) intercepts (correct to one decimal place) of \( f(x) = x^3 + x + 5 \).

Suppose we want a more accurate approximation of the \( x \) intercept of the function \( f(x) = x^3 + x - 3 \) discussed in Example 1.

(A) Use zoom and trace to approximate the \( x \) intercept to five decimal places. How many times did you have to zoom in on the graph to obtain this accuracy?

(B) Consult the manual for your graphing utility to learn how to use its built-in routine for approximating \( x \) intercepts. **Root** and **zero** are two common names for this routine. Use your graphing utility to approximate the \( x \) intercept to five decimal places (Fig. 3).

**FIGURE 3**

As you undoubtedly discovered in Explore/Discuss 1, using a built-in routine for approximating \( x \) intercepts is much faster than using zoom and trace. One of the great advantages of a graphing utility is the ability to approximate \( x \) intercepts or, equivalently, solutions to equations of the form \( f(x) = 0 \).

The domain of a function is the set of all the \( x \) coordinates of points on the graph of the function and the range is the set of all the \( y \) coordinates. It is instructive to view the domain and range as subsets of the coordinate axes as in Figure 4. Note the effective use of interval notation in describing the domain and range of the functions in this figure. In Figure 4(a) a solid dot is used to indicate that a point is on the graph of the function and in Figure 4(b) an open dot to
indicate that a point is not on the graph of the function. An open or solid dot at the end of a graph indicates that the graph terminates there, while an arrowhead indicates that the graph continues beyond the portion shown with no significant changes in shape [see Fig. 4(b)].

**FIGURE 4**
Domain and range.

![Domain and range diagram](image)

**Example 2**

**Finding the Domain and Range from a Graph**

Find the domain and range of the function $f$ whose graph is shown in Figure 5.

**FIGURE 5**

![Graph of function](image)

**Solution**

The dots at each end of the graph of $f$ indicate that the graph terminates at these points. Thus, the $x$ coordinates of the points on the graph are between $-3$ and 6.

The open dot at $(-3, 4)$ indicates that $-3$ is not in the domain of $f$, while the closed dot at $(6, -3)$ indicates that 6 is in the domain of $f$. That is,

$$\text{Domain: } -3 < x \leq 6 \quad \text{or} \quad (-3, 6]$$

The $y$ coordinates are between $-5$ and 4 and, as before, the open dot at $(-3, 4)$ indicates that 4 is not in the range of $f$ and the closed dot at $(3, -5)$ indicates that $-5$ is in the range of $f$. Thus,

$$\text{Range: } -5 \leq y < 4 \quad \text{or} \quad [-5, 4)$$
Find the domain and range of the function \( f \) given by the graph in Figure 6.

In Example 2, the domain and range for the function \( f \) were easily determined from the graph of \( f \), since the entire graph of \( f \) was visible. More often we can only show part of a graph of a function defined by an equation, since the graph may continue to move indefinitely far away from the origin, and it may not be clear what happens to the graph as it moves further away. When a function is defined by an equation, such as

\[
f(x) = 3\sqrt{x} \quad \text{or} \quad g(x) = 4\sqrt{x} - 0.4x \quad (1)
\]

the domain is often easily determined from the equation, but the range is not. Since \( \sqrt{x} \) represents a real number only for \( x \geq 0 \), functions \( f \) and \( g \) in equation (1) have the same domain, \([0, \infty)\). What about the range for \( f \) and for \( g \)? Looking at their graphs in standard viewing windows (Fig. 7), we might conclude that the range of each function is \([0, \infty)\). And we would be wrong!

Using new window variables produces the graphs in Figure 8. The graph of \( f \) does appear to continue upward, but the graph of \( g \) changes direction and starts to go down. Using more advanced mathematics, it can be shown that \([0, \infty)\) is the correct range for \( f \) and the correct range for \( g \) is \((-\infty, 10]\).
Before we continue, we must discuss another point about the graphs in Figure 7. Notice that the graphing utility we used to produce Figure 7 skipped over all the negative values of \(x\) because the functions were not defined as real numbers for \(x < 0\). Most graphing utilities will disregard any values of \(x\) between \(X_{\text{min}}\) and \(X_{\text{max}}\) for which the function is not defined as a real number. But some do not. Instead, some graphing utilities display an error message if the interval \([X_{\text{min}}, X_{\text{max}}]\) contains any values of \(x\) for which the function is not defined. If yours does this, you will have to be certain to exclude such \(x\) values from the viewing window.

**Increasing and Decreasing Functions**

Graph each function in the standard viewing window, then write a verbal description of the behavior exhibited by the graph as \(x\) moves from left to right.

(A) \(f(x) = 2 - x\)  
(B) \(f(x) = x^3\)  
(C) \(f(x) = 5\)  
(D) \(f(x) = 9 - x^2\)

We now take a look at increasing and decreasing properties of functions. Intuitively, a function is increasing over an interval \(I\) in its domain if its graph rises as the independent variable increases (moves from left to right) over \(I\). A function is decreasing over \(I\) if its graph falls as the independent variable increases over \(I\) (Fig. 9).

**FIGURE 9**  
Increasing, decreasing, and constant functions.

(a) Decreasing on \((-\infty, \infty)\)  
(b) Increasing on \((-\infty, \infty)\)  
(c) Constant on \((-\infty, \infty)\)  
(d) Decreasing on \((-\infty, 0]\)  
Increasing on \([0, \infty)\)
More formally, we define increasing, decreasing, and constant functions as follows:

**INCREASING, DECREASING, AND CONSTANT FUNCTIONS**

Let \( I \) be an interval in the domain of a function \( f \). Then,

1. \( f \) is increasing on \( I \) if \( f(b) > f(a) \) whenever \( b > a \) in \( I \).
2. \( f \) is decreasing on \( I \) if \( f(b) < f(a) \) whenever \( b > a \) in \( I \).
3. \( f \) is constant on \( I \) if \( f(a) = f(b) \) for all \( a \) and \( b \) in \( I \).

**EXAMPLE 3**

**Describing a Graph**

Use the terms increasing, decreasing, and constant to describe the graph of

\[
 f(x) = 60x - x^3 \quad \text{for} \quad -10 \leq x \leq 10
\]

**Solution**

We begin by graphing \( f \) in the standard viewing window [Fig. 10(a)]. Since this view of the graph is not very informative, we need to adjust the viewing window. The trace procedure can be used to determine appropriate values for \( \text{Ymin} \) and \( \text{Ymax} \). (The trace procedure displays the coordinates of a point on the graph even if the point is not visible in the viewing window.) While tracing along the graph in Figure 10(a) from \( \text{Xmin} = -10 \) to \( \text{Xmax} = 10 \), notice that all the \( y \) values are between \(-400\) and \(400\), inclusive. Using \( \text{Ymin} = -400 \) and \( \text{Ymax} = 400 \) produces the graph in Figure 10(b).

The graph changes direction twice, once near \( x = -4.468 \) and again near \( x = 4.468 \). Using zoom and trace procedures near 4.468, we see that the graph changes direction at \( x = 4.47 \), correct to two decimal places [Fig. 10(c)]. Similar procedures show that the graph also changes direction at \( x = -4.47 \) (details omitted). Thus, this function decreases as \( x \) goes from \(-10\) to \(-4.47\), increases as \( x \) goes from \(-4.47\) to \(4.47\), and decreases as \( x \) goes from \(4.47\) to \(10\). Or using interval notation, \( f \) is decreasing on \([-10, -4.47]\) and \([4.47, 10]\), and increasing on \([-4.47, 4.47]\).

**MATCHED PROBLEM 3**

Repeat Example 3 with \( f(x) = x^3 - 40x \).
In Example 3, we used zoom and trace procedures to approximate the points where the graph changes direction and we limited our discussion to the graphing interval \([X_{\text{min}}, X_{\text{max}}]\). Techniques for locating the exact points where graphs change direction and for analyzing increasing and decreasing behavior are discussed extensively in calculus.

**Local Maxima and Minima**

The points where the graph of a function changes direction play an essential role in the analysis of functions and graphs. These points are also critical to the solution of many applied problems involving the maximum or minimum values of a function. We define local maximum and minimum values of a function as follows:

**LOCAL MAXIMA AND LOCAL MINIMA**

The functional value \(f(c)\) is called a **local maximum** if there is an interval \((a, b)\) containing \(c\) such that

\[ f(x) \leq f(c) \text{ for all } x \in (a, b) \]

The functional value \(f(c)\) is called a **local minimum** if there is an interval \((a, b)\) containing \(c\) such that

\[ f(x) \geq f(c) \text{ for all } x \in (a, b) \]

The functional value \(f(c)\) is called a **local extremum** if it is either a local maximum or a local minimum.

Most graphing utilities have built-in routines for approximating local maxima and minima that don’t require you to zoom in on the graph (consult your manual for details). Graph \(f(x) = 60x - x^3\) for \(-10 \leq x \leq 10, \ -400 \leq y \leq 400\) and use a built-in routine to find the local maximum and local minimum [see Fig. 10(b)].

**Example 4**

**Maximizing Revenue**

The revenue (in dollars) from the sale of \(x\) bicycle locks is given by

\[ R(x) = 21x - 0.016x^2 \quad 0 \leq x \leq 1,300 \]
(A) Find the revenue for $x = 100, 200, \ldots, 1,300$. Use these values to estimate the number of locks that must be sold to maximize the revenue. What is the estimated maximum revenue?

(B) Graph $y = R(x)$ and find the exact number of locks that must be sold to maximize the revenue. What is the maximum revenue now?

Solutions

(A) Use a graphing utility to compute the required values of $R$ (Fig. 11).

Examining the values in Figure 11, we see that selling 700 locks will produce a revenue of $6,860. This is the best estimate we can obtain from the values in these tables. It is likely that there exist values of $x$ not in these tables that will produce larger values of $R(x)$.

(B) Using a built-in maximum routine (Fig. 12) and rounding to the nearest unit, we see that the maximum revenue of $6,891 occurs when 656 locks are sold.

MATCHED PROBLEM

The profit (in dollars) from the sale of $x$ bicycle locks is given by

$$P(x) = 17.5x - 0.016x^2 - 2,000 \quad 0 \leq x \leq 1,300$$

(A) Find the profit for $x = 100, 200, \ldots, 1,300$. Use these values to estimate the number of locks that must be sold to maximize the profit. What is the estimated maximum profit?

(B) Graph $y = P(x)$ and find the number of locks that must be sold to maximize the profit. What is the maximum profit now?

Technically speaking, the revenue function $R$ in Example 4 is defined only for integer values of $x$, $x = 0, 1, \ldots, 1,300$. However, for the purposes of mathematical analysis and as an aid in visualizing the behavior of the function $R$, we connect this discrete set of points with a continuous curve. There is nothing wrong with this, as long as you realize that it may not always be possible to realistically interpret values of $R$ when $x$ is not an integer. For example, the maximum value of $R$ actually occurs at $x = 656.25$, but since it is not possible to manufacture one-fourth of a lock, we simply conclude that $x = 656$ is the number of locks that produces the maximum revenue.
### Piecewise-Defined Functions

The **absolute value function** is defined by

\[ f(x) = |x| = \begin{cases} 
-x & \text{if } x < 0 \\
 x & \text{if } x \geq 0 
\end{cases} \]

The graph of \(|x|\) is shown in Figure 13. Most graphing utilities use abs or ABS to denote this function, and the graph is produced directly using \(y_1 = \text{abs}(x)\).

Notice that the absolute value function is defined by different formulas for different parts of its domain. Functions whose definitions involve more than one formula are called **piecewise-defined functions**. As the next example illustrates, piecewise-defined functions occur naturally in many applications.

### Example 5

#### Rental Charges

A car rental agency charges $0.25 per mile if the mileage does not exceed 100. If the total mileage exceeds 100, the agency charges $0.25 per mile for the first 100 miles and $0.15 per mile for any additional mileage.

(A) If \(x\) represents the number of miles a rented vehicle is driven, express the mileage charge \(C(x)\) as a function of \(x\).

(B) Complete the following table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C(x))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(C) Sketch the graph of \(y = C(x)\) by hand, using a graphing utility as an aid, and indicate the points in the table on the graph with solid dots.

#### Solutions

(A) If \(0 \leq x \leq 100\), then

\[ C(x) = 0.25x \]

If \(x > 100\), then

\[
\begin{align*}
\text{Charge for the first 100 miles} & \quad \text{Charge for the additional mileage} \\
C(x) &= 0.25(100) + 0.15(x - 100) \\
&= 25 + 0.15x - 15 \\
&= 10 + 0.15x
\end{align*}
\]

Thus, we see that \(C\) is a piecewise-defined function:

\[ C(x) = \begin{cases} 
0.25x & \text{if } 0 \leq x \leq 100 \\
10 + 0.15x & \text{if } x > 100 
\end{cases} \]

(B) Piecewise-defined functions are evaluated by first determining which rule applies and then using the appropriate rule to find the value of the function. To begin, we enter both rules in a graphing utility and use the table routine...
(Fig. 14). To complete the table, we use the values of $C(x)$ from the $y_1$ column if $0 \leq x \leq 100$, and from the $y_2$ column if $x > 100$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(x)$</td>
<td>$0.00$</td>
<td>$12.50$</td>
<td>$25$</td>
<td>$32.50$</td>
<td>$40$</td>
</tr>
</tbody>
</table>

(C) Using a graph of both rules in the same viewing window as an aid (Fig. 15), we sketch the graph of $y = C(x)$ and add the points from the table to produce Figure 16.

Another car rental agency charges $0.30 per mile when the total mileage does not exceed 75, and $0.30 per mile for the first 75 miles plus $0.20 per mile for the additional mileage when the total mileage exceeds 75.

(A) If $x$ represents the number of miles a rented vehicle is driven, express the mileage charge $C(x)$ as a function of $x$.

(B) Complete the following table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(x)$</td>
<td>$0.00$</td>
<td>$12.50$</td>
<td>$15$</td>
<td>$25$</td>
<td>$32.50$</td>
</tr>
</tbody>
</table>

(C) Sketch the graph of $y = C(x)$ by hand, using a graphing utility as an aid, and indicate the points in the table on the graph with solid dots.

Refer to Figures 14 and 16 in the solution to Example 5. Notice that the two formulas in the definition of $C$ produce the same value at $x = 100$ and that the graph of $C$ contains no breaks. Informally, a graph (or portion of a graph) is said to be **continuous** if it contains no breaks or gaps and can be drawn without lifting a pen from the paper. A graph is **discontinuous** at any points where there is
a break or a gap. For example, the graph of the function in Figure 17 is discontinuous at \( x = 1 \). The entire graph cannot be drawn without lifting a pen from the paper. (A formal presentation of continuity can be found in calculus texts.)

**The Greatest Integer Function**

We conclude this section with a discussion of an interesting and useful function called the greatest integer function.

The greatest integer of a real number \( x \), denoted by \([x]\), is the integer \( n \) such that \( n \leq x < n + 1 \); that is, \([x]\) is the largest integer less than or equal to \( x \). For example,

\[
[3.45] = 3 \quad [-2.13] = -3 \quad \text{Not} \quad -2
\]

\([7] = 7 \quad [-8] = -8 \quad [0] = 0

The greatest integer function \( f \) is defined by the equation \( f(x) = [x] \). A piecewise definition of \( f \) for \(-2 \leq x < 3\) is shown below and a sketch of the graph of \( f \) for \(-5 \leq x \leq 5\) is shown in Figure 18. Since the domain of \( f \) is all real numbers, the piecewise definition continues indefinitely in both directions, as does the stairstep pattern in the figure. Thus, the range of \( f \) is the set of all integers. The greatest integer function is an example of a more general class of functions called step functions.

\[
f(x) = [x] = \begin{cases} 
\vdots & \\
-2 & \text{if} \ -2 \leq x < -1 \\
-1 & \text{if} \ -1 \leq x < \ 0 \\
0 & \text{if} \ \ 0 \leq x < \ 1 \\
1 & \text{if} \ \ 1 \leq x < \ 2 \\
2 & \text{if} \ \ 2 \leq x < \ 3 \\
\vdots & 
\end{cases}
\]

Notice in Figure 18 that at each integer value of \( x \) there is a break in the graph, and between integer values of \( x \) there is no break. Thus, the greatest integer function is discontinuous at each integer \( n \) and continuous on each interval of the form \([n, n + 1]\).
Most graphing utilities will graph the greatest integer function, usually denoted by \( \text{int} \), but these graphs require careful interpretation. Comparing the sketch of \( y = \lfloor x \rfloor \) in Figure 18 with the graph of \( y = \text{int}(x) \) in Figure 19(a), we see that the graphing utility has connected the end points of the horizontal line segments. This gives the appearance that the graph is continuous when it is not. To obtain a correct graph, consult the manual to determine how to change the graphing mode on your graphing utility from connected mode to dot mode [Fig. 19(b)].

To avoid misleading graphs, use the dot mode on your graphing utility when graphing a function with discontinuities.

**EXAMPLE 6**

**Computer Science**

Let

\[
    f(x) = \frac{\lfloor 10x + 0.5 \rfloor}{10}
\]

Find

(A) \( f(6) \)  \quad (B) \( f(1.8) \)  \quad (C) \( f(3.24) \)  \quad (D) \( f(4.582) \)  \quad (E) \( f(-2.68) \)

What operation does this function perform?

**Solutions**

(A) \( f(6) \)  \quad = \frac{\lfloor 60.5 \rfloor}{10} = \frac{60}{10} = 6

(B) \( f(1.8) \)  \quad = \frac{\lfloor 18.5 \rfloor}{10} = \frac{18}{10} = 1.8

(C) \( f(3.24) \)  \quad = \frac{\lfloor 32.9 \rfloor}{10} = \frac{32}{10} = 3.2

(D) \( f(4.582) \)  \quad = \frac{\lfloor 46.32 \rfloor}{10} = \frac{46}{10} = 4.6

(E) \( f(-2.68) \)  \quad = \frac{\lfloor -26.3 \rfloor}{10} = \frac{-27}{10} = -2.7

Comparing the values of \( x \) and \( f(x) \) in the table, we conclude that this function rounds decimal fractions to the nearest tenth.
Let \( f(x) = [x + 0.5] \). Find
\begin{align*}
(A) \ f(6) & \quad (B) \ f(1.8) & \quad (C) \ f(3.24) & \quad (D) \ f(-4.3) & \quad (E) \ f(-2.69)
\end{align*}
What operation does this function perform?

Answers to Matched Problems

1. \( x \) intercept: \(-1.5\); \( y \) intercept: 5
2. Domain: \(-4 < x < 5 \) or \((-4, 5)\)
   Range: \(-4 < y \leq 3 \) or \((-4, 3]\)
3. \( f \) is increasing on \([-10, -3.65]\) and \([3.65, 10]\), and decreasing on \([-3.65, 3.65]\).
4. (A) The sale of 500 locks will produce an estimated maximum profit of $2,750.
   (B) The maximum profit of $2,785 occurs when 547 locks are sold.
5. (A) \( C(x) = \begin{cases} 0.3x & \text{if } 0 \leq x \leq 75 \\ 7.5 + 0.2x & \text{if } x > 75 \end{cases} \)
   (B) \begin{array}{c|c|c|c|c}
   x & 0 & 50 & 75 & 100 & 150 \\
   \hline
   C(x) & \$0 & \$15 & \$22.50 & \$27.50 & \$37.50
   \end{array}
   (C) \begin{array}{c}
   \begin{align*}
   y &= 0.3x \\
   y &= 7.5 + 0.2x
   \end{align*}
   \end{array}
6. (A) 6 \quad (B) 2 \quad (C) 3 \quad (D) -4 \quad (E) -3; \ f \) rounds decimal fractions to the nearest integer.

EXERCISE 1-4

1. For the function \( f \), find
   (A) Domain
   (B) Range
   (C) \( x \) intercepts
   (D) \( y \) intercept
   (E) Intervals over which \( f \) is increasing
   (F) Intervals over which \( f \) is decreasing
   (G) Intervals over which \( f \) is constant
   (H) Any points of discontinuity
   2. Repeat Problem 1 for the function \( g \).
   3. Repeat Problem 1 for the function \( h \).
   4. Repeat Problem 1 for the function \( k \).
5. Repeat Problem 1 for the function \( p \).
6. Repeat Problem 1 for the function \( q \).

In Problems 7–16, examine the graph of the function on the interval \([-10, 10]\) to determine the intervals over which the function is increasing, the intervals over which the function is decreasing, and the intervals over which the function is constant. Approximate the endpoints of the intervals to the nearest integer.

7. \( f(x) = |x + 2| - 5 \)
8. \( g(x) = 6 - |x - 3| \)
9. \( h(x) = x + |x + 3| \)
10. \( k(x) = |x - 2| - x \)
11. \( m(x) = |x - 3| - |x + 4| \)
12. \( n(x) = |x + 5| - |x - 2| \)
13. \( p(x) = |x - 1| + |x + 3| \)
14. \( q(x) = |x + 2| + |x - 4| \)
15. \( r(x) = |x + 4| - |x| + |x - 4| \)
16. \( s(x) = |x| - |x + 5| - |x - 3| \)

Problems 17–22 describe the graph of a continuous function \( f \) over the interval \([-5, 5]\). Sketch by hand the graph of a function that is consistent with the given information.

17. The function \( f \) is increasing on \([-5, -2]\), constant on \([-2, 2]\), and decreasing on \([2, 5]\).
18. The function \( f \) is decreasing on \([-5, -2]\), constant on \([-2, 2]\), and increasing on \([2, 5]\).
19. The function \( f \) is decreasing on \([-5, -2]\), constant on \([-2, 2]\), and decreasing on \([2, 5]\).
20. The function \( f \) is increasing on \([-5, -2]\), constant on \([-2, 2]\), and increasing on \([2, 5]\).
21. The function \( f \) is decreasing on \([-5, -2]\), increasing on \([-2, 2]\), and decreasing on \([2, 5]\).
22. The function \( f \) is increasing on \([-5, -2]\), decreasing on \([-2, 2]\), and increasing on \([2, 5]\).

Each function \( f \) in Problems 23–28 has exactly one local extreme value \( f(c) \) and two \( x \) intercepts. Explore the graph of \( f \) with a graphing utility to locate the intercepts and to determine if the local extremum is a local maximum or a local minimum. Approximate \( c, f(c) \), and the \( x \) intercepts to one decimal place.

23. \( f(x) = 0.7x^2 - 5x + 3 \)
24. \( f(x) = 5 - 0.3x^2 - x \)
25. \( f(x) = 7\sqrt{x} - 2x \)
26. \( f(x) = 3x - 8\sqrt{x} \)
27. \( f(x) = x^2 - 4\sqrt{x} - 4 \)
28. \( f(x) = 2\sqrt{x} - |x| + 2 \)

In Problems 29–36, sketch the graph by hand, using a graphing utility as an aid, and find the domain, range, and any points of discontinuity.

29. \( f(x) = \begin{cases} x + 1 & \text{if } -1 \leq x < 0 \\ -x + 1 & \text{if } 0 \leq x \leq 1 \end{cases} \)
30. \( f(x) = \begin{cases} x & \text{if } -2 \leq x < 1 \\ -x + 2 & \text{if } 1 \leq x \leq 2 \end{cases} \)
31. \( f(x) = \begin{cases} -2 & \text{if } -3 \leq x < -1 \\ 4 & \text{if } -1 \leq x \leq 2 \end{cases} \)
32. \( f(x) = \begin{cases} 1 & \text{if } -2 \leq x < 2 \\ -3 & \text{if } 2 \leq x \leq 5 \end{cases} \)
33. \( f(x) = \begin{cases} x + 2 & \text{if } x < -1 \\ x - 2 & \text{if } x \geq -1 \end{cases} \)
34. \( f(x) = \begin{cases} -1 - x & \text{if } x \leq 2 \\ 5 - x & \text{if } x > 2 \end{cases} \)
35. \( g(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ -x^2 - 1 & \text{if } x > 0 \end{cases} \)
36. \( h(x) = \begin{cases} 
-x^2 - 2 & \text{if } x < 0 \\
x^2 + 2 & \text{if } x > 0 
\end{cases} \)

In Problems 37–42, write a verbal description of the graph of the given function over the interval \([-10, 10]\) using increasing and decreasing terminology, and indicating any local maximum and minimum values. Approximate to two decimal places the coordinates of any points used in your description.

37. \( f(x) = x^3 + 4.3x - 32 \)
38. \( g(x) = -x^2 + 6.9x + 25 \)
39. \( h(x) = x^3 - x^2 - 74x + 60 \)
40. \( k(x) = -x^2 + x^2 + 82x - 25 \)
41. \( p(x) = |x^2 - x - 18| \)
42. \( q(x) = |x^2 - 2x - 30| \)

Problems 43–48 describe the graph of a function \( f \) that is continuous on the interval \([-5, 5]\), except as noted. Sketch by hand the graph of a function that is consistent with the given information.

43. The function \( f \) is increasing on \([-5, 0)\), discontinuous at \( x = 0 \), increasing on \((0, 5)\), \( f(-2) = 0 \), and \( f(2) = 0 \).
44. The function \( f \) is decreasing on \([-5, 0)\), discontinuous at \( x = 0 \), decreasing on \((0, 5)\), \( f(-3) = 0 \), and \( f(3) = 0 \).
45. The function \( f \) is discontinuous at \( x = 0 \), \( f(-3) = -2 \) is a local maximum, and \( f(2) = 3 \) is a local minimum.
46. The function \( f \) is discontinuous at \( x = 0 \), \( f(-3) = 2 \) is a local minimum, and \( f(2) = -3 \) is a local maximum.
47. The function \( f \) is discontinuous at \( x = -2 \) and \( x = 2 \), \( f(-3) = -2 \) and \( f(3) = -2 \) are local maxima, and \( f(0) = 0 \) is a local minimum.
48. The function \( f \) is discontinuous at \( x = -2 \) and \( x = 2 \), \( f(-3) = 2 \) and \( f(3) = 2 \) are local minima, and \( f(0) = 0 \) is a local maximum.

C

In Problems 49–54, graph \( y = f(x) \) in a standard viewing window. Assuming that the graph continues as indicated beyond the part shown in this viewing window, find the domain, range, and any points of discontinuity. (Use the dot mode on your graphing utility.)

49. \( f(x) = \frac{|5x - 10|}{x - 2} \)
50. \( f(x) = \frac{4x + 12}{|x + 3|} \)
51. \( f(x) = x + \frac{|4x - 4|}{x - 1} \)
52. \( f(x) = x + \frac{|2x + 2|}{x + 1} \)

53. \( f(x) = |x| - \frac{|9 - 3x|}{x - 3} \)
54. \( f(x) = |x| + \frac{|2x + 4|}{x + 2} \)

In Problems 55–60, write a piecewise definition of \( f \) and sketch by hand the graph of \( f \), using a graphing utility as an aid. Include sufficient intervals to clearly illustrate both the definition and the graph. Find the domain, range, and any points of discontinuity.

55. \( f(x) = \frac{|x|}{2} \)
56. \( f(x) = \frac{|x|}{3} \)
57. \( f(x) = \frac{|3x|}{2} \)
58. \( f(x) = \frac{|2x|}{3} \)
59. \( f(x) = x - \frac{|x|}{2} \)
60. \( f(x) = \frac{|x|}{3} - x \)

61. The function \( f \) is continuous and increasing on the interval \([1, 9]\) with \( f(1) = -5 \) and \( f(9) = 4 \).
   (A) Sketch a graph of \( f \) that is consistent with the given information.
   (B) How many times does your graph cross the \( x \) axis? Could the graph cross more times? Fewer times? Support your conclusions with additional sketches and/or verbal arguments.

62. Repeat Problem 61 if the function does not have to be continuous.

63. The function \( f \) is continuous on the interval \([-5, 5]\) with \( f(-5) = -4 \), \( f(1) = 3 \), and \( f(5) = -2 \).
   (A) Sketch a graph of \( f \) that is consistent with the given information.
   (B) How many times does your graph cross the \( x \) axis? Could the graph cross more times? Fewer times? Support your conclusions with additional sketches and/or verbal arguments.

64. Repeat Problem 63 if \( f \) is continuous on \([-8, 8]\) with \( f(-8) = -6 \), \( f(-4) = 3 \), \( f(3) = -2 \), and \( f(8) = 5 \).

65. The function \( f \) is continuous on \([0, 10]\), \( f(5) = -5 \) is a local minimum, and \( f \) has no other local extrema on this interval.
   (A) Sketch a graph of \( f \) that is consistent with the given information.
   (B) How many times does your graph cross the \( x \) axis? Could the graph cross more times? Fewer times? Support your conclusions with additional sketches and/or verbal arguments.

66. Repeat Problem 65 if \( f(5) = 1 \) and all other information is unchanged.

Applications

67. Delivery Charges. A nationwide package delivery service charges \$15 for overnight delivery of packages weighing
1 pound or less. Each additional pound (or fraction thereof) costs an additional $3. Let \( C(x) \) be the charge for overnight delivery of a package weighing \( x \) pounds.

(A) Write a piecewise definition of \( C \) for \( 0 < x \leq 6 \) and sketch the graph of \( C \) by hand.

(B) Can the function \( f \) defined by \( f(x) = 15 + 3\lceil x \rceil \) be used to compute the delivery charges for all \( x \), \( 0 < x \leq 6 \)? Justify your answer.

68. Telephone Charges. Calls to 900 numbers are charged to the caller. A 900 number hot line for tips and hints for video games charges $4 for the first minute of the call and $2 for each additional minute (or fraction thereof). Let \( C(x) \) be the charge for a call lasting \( x \) minutes.

(A) Write a piecewise definition of \( C \) for \( 0 < x \leq 6 \) and sketch the graph of \( C \) by hand.

(B) Can the function \( f \) defined by \( f(x) = 4 + 2\lceil x \rceil \) be used to compute the charges for all \( x \), \( 0 < x \leq 6 \)? Justify your answer.

* 69. Sales Commissions. An appliance salesperson receives a base salary of $200 a week and a commission of 4% on all sales over $3,000 during the week. In addition, if the weekly sales are $8,000 or more, the salesperson receives a $100 bonus. If \( x \) represents weekly sales (in dollars), express the weekly earnings \( E(x) \) as a function of \( x \), and sketch its graph. Identify any points of discontinuity. Find \( E(5,750) \) and \( E(9,200) \).

* 70. Service Charges. On weekends and holidays, an emergency plumbing repair service charges $2.00 per minute for the first 30 minutes of a service call and $1.00 per minute for each additional minute. If \( x \) represents the duration of a service call in minutes, express the total service charge \( S(x) \) as a function of \( x \), and sketch its graph. Identify any points of discontinuity. Find \( S(25) \) and \( S(45) \).

71. Computer Science. Let \( f(x) = 10[0.5 + x/10] \). Evaluate \( f \) at \( 4, -4, 6, -6, 24, 25, 247, -243, -245 \), and \( -246 \). What operation does this function perform?

72. Computer Science. Let \( f(x) = 100[0.5 + x/100] \). Evaluate \( f \) at \( 40, -40, 60, -60, 740, 750, 7,551, -601, -649 \), and \( -651 \). What operation does this function perform?

* 73. Computer Science. Use the greatest integer function to define a function \( f \) that rounds real numbers to the nearest hundredth.

* 74. Computer Science. Use the greatest integer function to define a function \( f \) that rounds real numbers to the nearest thousandth.

75. Revenue. The revenue (in dollars) from the sale of \( x \) car seats for infants is given by

\[
R(x) = 60x - 0.035x^2 \quad 0 \leq x \leq 1,700
\]

(A) Examine the values of the revenue function \( R(x) \) for \( x = 100, 200, \ldots, 1,700 \) to estimate the number of car seats that must be sold to maximize the revenue. What is the estimated maximum revenue?

(B) Graph \( y = R(x) \) and find the number of car seats that must be sold to maximize the revenue. What is the maximum revenue now?

76. Profit. The profit (in dollars) from the sale of \( x \) car seats for infants is given by

\[
P(x) = 38x - 0.035x^2 - 4,000 \quad 0 \leq x \leq 1,700
\]

(A) Examine the values of the profit function \( P(x) \) for \( x = 100, 200, \ldots, 1,700 \) to estimate the number of car seats that must be sold to maximize the profit. What is the estimated maximum profit?

(B) Graph \( y = P(x) \) and find the number of car seats that must be sold to maximize the profit. What is the maximum profit now?

77. Manufacturing. A box is to be made out of a piece of cardboard that measures 18 by 24 inches. Squares, \( x \) inches on a side, will be cut from each corner and then the ends and sides will be folded up (see the figure). The volume of the box is given by

\[
V(x) = x(24 - 2x)(18 - 2x) \quad 0 \leq x \leq 9
\]

(A) Examine the values of the volume function \( V(x) \) for \( x = 0, 1, 2, \ldots, 9 \) and estimate the size of the cut-out squares that will make the volume maximum. What is the estimated volume?

(B) Graph \( y = V(x) \) and approximate to two decimal places the size of the cut-out squares that will make the volume maximum. Approximate the maximum volume to two decimal places also.
78. **Manufacturing.** A box with a hinged lid is to be made out of a piece of cardboard that measures 20 by 40 inches. Six squares, \( x \) inches on a side, will be cut from each corner and the middle, and then the ends and sides will be folded up to form the box and its lid (see the figure). The volume of the box is given by

\[
V(x) = 0.5x(40 - 3x)(20 - 2x) \quad 0 \leq x \leq 10
\]

(A) Examine the values of the volume function \( V(x) \) for \( x = 0, 1, 2, \ldots, 10 \) and estimate the size of the cut-out squares that will make the volume maximum. What is the estimated volume?

(B) Graph \( y = V(x) \) and approximate to two decimal places the size of the cut-out squares that will make the volume maximum. Approximate the maximum volume to two decimal places also.

79. **Construction.** A freshwater pipe is to be run from a source on the edge of a lake to a small resort community on an island 8 miles offshore, as indicated in the figure. It costs $10,000 per mile to lay the pipe on land and $16,000 per mile to lay the pipe in the lake. The total cost \( C(x) \) in thousands of dollars of laying the pipe is given by

\[
C(x) = 10(20 - x) + 16\sqrt{x^2 + 64} \quad 0 \leq x \leq 20
\]

(A) Examine the values of the cost function \( C(x) \) (rounded to the nearest thousand dollars) for \( x = 0, 5, 10, 15, \) and 20 and estimate to the nearest mile the length of the land portion of the pipe that will make the production costs minimum. What is the estimated cost?

(B) Graph \( y = C(x) \) and use zoom and trace procedures to approximate to one decimal place the length of the land portion of the trip that will make the time minimum. Approximate the minimum time to the nearest minute also.

80. **Transportation.** The construction company laying the freshwater pipe in Problem 79 uses an amphibious vehicle to travel down the beach and then out to the island. The vehicle travels at 30 miles per hour on land and 7.5 miles per hour in water. The total time \( T(x) \) in minutes for a trip from the freshwater source to the island is given by

\[
T(x) = 2(20 - x) + 8\sqrt{x^2 + 64} \quad 0 \leq x \leq 20
\]

(A) Examine the values of the time function \( T(x) \) (rounded to the nearest minute) for \( x = 0, 5, 10, 15, \) and 20 and estimate the length of the land portion of the trip that will make the time minimum. What is the estimated time?

(B) Graph \( y = T(x) \) and use zoom and trace procedures to approximate to one decimal place the length of the land portion of the trip that will make the time minimum. Approximate the minimum time to the nearest minute also.

81. **Tire Mileage.** An automobile tire manufacturer collected the data in the table relating tire pressure \( x \), in pounds per square inch, and mileage in thousands of miles. A mathematical model for this data is given by

\[
f(x) = -0.518x^2 + 33.3x - 481
\]

(A) Complete the following table. Round values of \( f(x) \) to one decimal place.

<table>
<thead>
<tr>
<th>( x )</th>
<th>28</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage</td>
<td>45</td>
<td>52</td>
<td>55</td>
<td>51</td>
<td>47</td>
</tr>
</tbody>
</table>

(B) Sketch by hand the graph of \( f \) and the mileage data on the same axes.

(C) Use values of the modeling function rounded to two decimal places to estimate the mileage for a tire pressure of 31 lb/in.². For 35 lb/in.².
(D) Write a brief description of the relationship between tire pressure and mileage, using the terms increasing, decreasing, local maximum, and local minimum where appropriate.

82. **Automobile Production.** The table lists General Motor’s total U.S. vehicle production in millions of units from 1989 to 1993.

<table>
<thead>
<tr>
<th>Year</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>4.7</td>
</tr>
<tr>
<td>90</td>
<td>4.1</td>
</tr>
<tr>
<td>91</td>
<td>3.5</td>
</tr>
<tr>
<td>92</td>
<td>3.7</td>
</tr>
<tr>
<td>93</td>
<td>5.0</td>
</tr>
</tbody>
</table>

A mathematical model for GM’s production data is given by

\[ f(x) = 0.33x^2 - 1.3x + 4.8 \]

where \( x = 0 \) corresponds to 1989.

(A) Complete the following table. Round values of \( x \) to one decimal place.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>4.7</td>
<td>4.1</td>
<td>3.5</td>
<td>3.7</td>
<td>5.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

(B) Sketch by hand the graph of \( f \) and the production data on the same axes.

(C) Use values of the modeling function \( f \) rounded to two decimal places to estimate the production in 1994. In 1995.

(D) Write a brief verbal description of GM’s production from 1989 to 1993, using increasing, decreasing, local maximum, and local minimum terminology where appropriate.

---

**Section 1-5 Functions: Graphs and Transformations**

- A Beginning Library of Elementary Functions
- Vertical and Horizontal Shifts
- Reflections, Expansions, and Contractions
- Even and Odd Functions

The functions

\[ g(x) = x^2 + 2 \quad h(x) = (x + 2)^2 \quad k(x) = 2x^2 \]

can be expressed in terms of the function \( f(x) = x^2 \) as follows:

\[ g(x) = f(x) + 2 \quad h(x) = f(x + 2) \quad k(x) = 2f(x) \]

In this section we will see that the graphs of functions \( g, h, \) and \( k \) are closely related to the graph of function \( f. \) Insight gained by understanding these relationships will help us analyze and interpret the graphs of many different functions.

**A Beginning Library of Elementary Functions**

As we progress through this book, we will encounter a number of basic functions that we will want to add to our library of elementary functions. Figure 1 shows six basic functions that you will encounter frequently. You should know the definition, domain, and range of each of these functions, and be able to recognize their graphs. You should graph each basic function in Figure 1 on your graphing utility.

Most graphing utilities allow you to define a number of functions, usually denoted by \( y_1, y_2, y_3, \ldots \) You can graph all of these functions simultaneously, or