Several problems in Section 8-7 are related to the general type of problems called linear programming problems. Linear programming is a mathematical process that has been developed to help management in decision making, and it has become one of the most widely used and best known tools of management science and industrial engineering. We will use an intuitive graphical approach based on the techniques discussed in Section 8-7 to illustrate this process for problems involving two variables.

The American mathematician George B. Dantzig (1914— ) formulated the first linear programming problem in 1947 and introduced a solution technique, called the simplex method, that does not rely on graphing and is readily adaptable to computer solutions. Today, it is quite common to use a computer to solve applied linear programming problems involving thousands of variables and thousands of inequalities.

**A Linear Programming Problem**

We begin our discussion with an example that will lead to a general procedure for solving linear programming problems in two variables.

### Example 1: Production Scheduling

A manufacturer of fiberglass camper tops for pickup trucks makes a compact model and a regular model. Each compact top requires 5 hours from the fabricating department and 2 hours from the finishing department. Each regular top requires 4 hours from the fabricating department and 3 hours from the finishing department. The maximum labor-hours available per week in the fabricating department and the finishing department are 200 and 108, respectively. If the company makes a profit of $40 on each compact top and $50 on each regular top, how many tops of each type should be manufactured each week to maximize the total weekly profit, assuming all tops can be sold? What is the maximum profit?

**Solution**

This is an example of a linear programming problem. To see relationships more clearly, we summarize the manufacturing requirements, objectives, and restrictions in the table:

<table>
<thead>
<tr>
<th></th>
<th>Compact Model</th>
<th>Regular Model</th>
<th>Maximum Labor-Hours Available per Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabricating</td>
<td>5</td>
<td>4</td>
<td>200</td>
</tr>
<tr>
<td>Finishing</td>
<td>2</td>
<td>3</td>
<td>108</td>
</tr>
<tr>
<td>Profit per top</td>
<td>$40</td>
<td>$50</td>
<td></td>
</tr>
</tbody>
</table>
We now proceed to formulate a mathematical model for the problem and then to solve it using graphical methods.

**Objective Function**

The objective of management is to decide how many of each camper top model should be produced each week to maximize profit. Let

\[
\begin{align*}
  x &= \text{Number of compact tops produced per week} \\
  y &= \text{Number of regular tops produced per week}
\end{align*}
\]

Decision variables

The following function gives the total profit \( P \) for \( x \) compact tops and \( y \) regular tops manufactured each week:

\[
P = 40x + 50y \quad \text{Objective function}
\]

Mathematically, management needs to decide on values for the decision variables \((x \text{ and } y)\) that achieve its objective, that is, maximizing the objective function (profit) \( P = 40x + 50y \). It appears that the profit can be made as large as we like by manufacturing more and more tops—or can it?

**Constraints**

Any manufacturing company, no matter how large or small, has manufacturing limits imposed by available resources, plant capacity, demand, and so forth. These limits are referred to as problem constraints.

Fabricating department constraint:

\[
\begin{align*}
(\text{Weekly fabricating time for } x \text{ compact tops}) & + (\text{Weekly fabricating time for } y \text{ regular tops}) \\ & \leq (\text{Maximum labor-hours available per week}) \\
5x & + 4y \\ & \leq 200
\end{align*}
\]

Finishing department constraint:

\[
\begin{align*}
(\text{Weekly finishing time for } x \text{ compact tops}) & + (\text{Weekly finishing time for } y \text{ regular tops}) \\ & \leq (\text{Maximum labor-hours available per week}) \\
2x & + 3y \\ & \leq 108
\end{align*}
\]

Nonnegative constraints: It is not possible to manufacture a negative number of tops; thus, we have the nonnegative constraints

\[
\begin{align*}
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

which we usually write in the form

\[
x, y \geq 0
\]

**Mathematical Model**

We now have a mathematical model for the problem under consideration:
Maximize  \[ P = 40x + 50y \]  \hspace{1cm} \text{Objective function}
Subject to \begin{align*}
5x + 4y &\leq 200 \\
2x + 3y &\leq 108
\end{align*}
\hspace{1cm} \text{Problem constraints}
\begin{align*}
x, y &\geq 0
\end{align*}
\hspace{1cm} \text{Nonnegative constraints}

**Graphic Solution**

Solving the system of linear inequality constraints graphically, as in Section 8-7, we obtain the feasible region for production schedules, as shown in Figure 1.

By choosing a production schedule \((x, y)\) from the feasible region, a profit can be determined using the objective function \(P = 40x + 50y\). For example, if \(x = 24\) and \(y = 10\), then the profit for the week is

\[ P = 40(24) + 50(10) = 1,460 \]

Or if \(x = 15\) and \(y = 20\), then the profit for the week is

\[ P = 40(15) + 50(20) = 1,600 \]

The question is, out of all possible production schedules \((x, y)\) from the feasible region, which schedule(s) produces the maximum profit? Such a schedule, if it exists, is called an optimal solution to the problem because it produces the maximum value of the objective function and is in the feasible region. It is not practical to use point-by-point checking to find the optimal solution. Even if we consider only points with integer coordinates, there are over 800 such points in the feasible region for this problem. Instead, we use the theory that has been developed to solve linear programming problems. Using advanced techniques, it can be shown that:

If the feasible region is bounded, then one or more of the corner points of the feasible region is an optimal solution to the problem.

The maximum value of the objective function is unique; however, there can be more than one feasible production schedule that will produce this unique value. We will have more to say about this later in this section.
Since the feasible region for this problem is bounded, at least one of the corner points, \((0, 0), (0, 36), (24, 20), \) or \((40, 0)\), is an optimal solution. To find which one, we evaluate \(P = 40x + 50y\) at each corner point and choose the corner point that produces the largest value of \(P\). It is convenient to organize these calculations in a table, as shown in the margin.

Examining the values in the table, we see that the maximum value of \(P\) at a corner point is \(P = 1,960\) at \(x = 24\) and \(y = 20\). Since the maximum value of \(P\) over the entire feasible region must always occur at a corner point, we conclude that the maximum profit is $1,960 when 24 compact tops and 20 regular tops are produced each week.

We now convert the surfboard problem discussed in Section 8-7 into a linear programming problem. A manufacturer of surfboards makes a standard model and a competition model. Each standard board requires 6 labor-hours for fabricating and 1 labor-hour for finishing. Each competition board requires 8 labor-hours for fabricating and 3 labor-hours for finishing. The maximum labor-hours available per week in the fabricating and finishing departments are 120 and 30, respectively. If the company makes a profit of $40 on each standard board and $75 on each competition board, how many boards of each type should be manufactured each week to maximize the total weekly profit?

(A) Identify the decision variables.
(B) Write the objective function \(P\).
(C) Write the problem constraints and the nonnegative constraints.
(D) Graph the feasible region, identify the corner points, and evaluate \(P\) at each corner point.
(E) How many boards of each type should be manufactured each week to maximize the profit? What is the maximum profit?

Refer to Example 1. If we assign the profit function \(P\) in \(P = 40x + 50y\) a particular value and plot the resulting equation in the coordinate system shown in Figure 1, we obtain a constant-profit line (isoprofit line). Every point in the feasible region on this line represents a production schedule that will produce the same profit. Figure 2 shows the constant-profit lines for \(P = \$1,000\) and \(P = \$1,500\).
A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear objective function of the form

\[ z = ax + by \]

where the decision variables \( x \) and \( y \) are subject to problem constraints in the form of linear inequalities and to nonnegative constraints \( x, y \geq 0 \). The set of points satisfying both the problem constraints and the nonnegative constraints is called the feasible region for the problem. Any point in the feasible region that produces the optimal value of the objective function over the feasible region is called an optimal solution.

Theorem 1 is fundamental to the solving of linear programming problems.

**Fundamental Theorem of Linear Programming**

Let \( S \) be the feasible region for a linear programming problem, and let \( z = ax + by \) be the objective function. If \( S \) is bounded, then \( z \) has both a maximum and a minimum value on \( S \) and each of these occurs at a corner point of \( S \). If \( S \) is unbounded, then a maximum or minimum value of \( z \) on \( S \) may not exist. However, if either does exist, then it must occur at a corner point of \( S \).

We will not consider any problems with unbounded feasible regions in this brief introduction. If a feasible region is bounded, then Theorem 1 provides the basis for the following simple procedure for solving the associated linear programming problem:
Before considering additional applications, we use this procedure to solve a linear programming problem where the model has already been determined.

### Example 2

#### Solving a Linear Programming Problem

Minimize and maximize \( z = 5x + 15y \)

Subject to:

\[
\begin{align*}
x + 3y & \leq 60 \\
x + y & \geq 10 \\
x - y & \leq 10 \\
x, y & \geq 0
\end{align*}
\]

**Solution**

This problem is a combination of two linear programming problems—a minimization problem and a maximization problem. Since the feasible region is the same for both problems, we can solve these problems together. To begin, we graph the feasible region \( S \), as shown in Figure 3, and find the coordinates of each corner point.

![Figure 3](image-url)

Next, we evaluate the objective function at each corner point, with the results given in the table:
Examining the values in the table, we see that the minimum value of $z$ on the feasible region $S$ is 100 at (5, 5). Thus, (5, 5) is the optimal solution to the minimization problem. The maximum value of $z$ on the feasible region $S$ is 300, which occurs at (0, 20) and at (15, 15). Thus, the maximization problem has multiple optimal solutions. In general,

**If two corner points are both optimal solutions of the same type (both produce the same maximum value or both produce the same minimum value) to a linear programming problem, then any point on the line segment joining the two corner points is also an optimal solution of that type.**

It can be shown that this is the only time that an optimal value occurs at more than one point.

### Matched Problem 2

Minimize and maximize $z = 10x + 5y$

Subject to

- $2x + y \geq 40$
- $3x + y \leq 150$
- $2x - y \geq 0$
- $x, y \geq 0$

### Application

Now we consider another application where we must first find the mathematical model and then find its solution.

#### Example 3

<table>
<thead>
<tr>
<th>Pounds Per Cubic Yard</th>
<th>Mix A</th>
<th>Mix B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Potash</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>Phosphoric acid</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

**Agriculture**

A farmer can use two types of plant food, mix $A$ and mix $B$. The amounts (in pounds) of nitrogen, phosphoric acid, and potash in a cubic yard of each mix are given in the table. Tests performed on the soil in a large field indicate that the field needs at least 840 pounds of potash and at least 350 pounds of nitrogen. The tests also indicate that no more than 630 pounds of phosphoric acid should be added to the field. A cubic yard of mix $A$ costs $7, and a cubic yard of mix $B$ costs $9. How many cubic yards of each mix should the farmer add to the field in order to supply the necessary nutrients at minimal cost?
Let

\[ x = \text{Number of cubic yards of mix } A \text{ added to the field} \]
\[ y = \text{Number of cubic yards of mix } B \text{ added to the field} \]

We form the linear objective function

\[ C = 7x + 9y \]

which gives the cost of adding \( x \) cubic yards of mix \( A \) and \( y \) cubic yards of mix \( B \) to the field. Using the data in the table and proceeding as in Example 1, we formulate the mathematical model for the problem:

Minimize \( C = 7x + 9y \)
Subject to
\[ \begin{align*}
10x + 5y & \geq 350 \\
8x + 24y & \geq 840 \\
9x + 6y & \leq 630
\end{align*} \]
\[ x, y \geq 0 \]

Solving the system of constraint inequalities graphically, we obtain the feasible region \( S \) shown in Figure 4, and then we find the coordinates of each corner point.

<table>
<thead>
<tr>
<th>Corner Point ((x, y))</th>
<th>Objective Function (C = 7x + 9y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 105)</td>
<td>945</td>
</tr>
<tr>
<td>(0, 70)</td>
<td>630</td>
</tr>
<tr>
<td>(21, 28)</td>
<td>399 (^\text{Minimum value of } C)</td>
</tr>
<tr>
<td>(60, 15)</td>
<td>555</td>
</tr>
</tbody>
</table>

Next, we evaluate the objective function at each corner point, as shown in the table in the margin.

The optimal value is \( C = 399 \) at the corner point \((21, 28)\). Thus, the farmer should add 21 cubic yards of mix \( A \) and 28 cubic yards of mix \( B \) at a cost of $399. This will result in adding the following nutrients to the field:

Nitrogen: \( 10(21) + 5(28) = 350 \) pounds
Potash: \( 8(21) + 24(28) = 840 \) pounds
Phosphoric acid: \( 9(21) + 6(28) = 357 \) pounds

All the nutritional requirements are satisfied.
Repeat Example 3 if the tests indicate that the field needs at least 400 pounds of nitrogen with all other conditions remaining the same.

Answers to Matched Problems

1. (A) \(x = \) Number of standard boards manufactured each week
   \(y = \) Number of competition boards manufactured each week
   (B) \(P = 40x + 75y\)
   (C) \(6x + 8y \leq 120\)
   \(x + 3y \leq 30\)
   \(x, y \geq 0\)  
   Fabricating constraint
   Finishing constraint
   Nonnegative constraints

   (D)  
   (E) 12 standard boards and 6 competition boards for a maximum profit of $930

2. Max \(z = 600\) at (30, 60); min \(z = 200\) at (10, 20) and (20, 0) (multiple optimal solutions)

3. 27 cubic yards of mix \(A\), 26 cubic yards of mix \(B\); min \(C = 423\)

Exercise 8-8

In Problems 1–4, find the maximum value of each objective function over the feasible region \(S\) shown in the figure below.

1. \(z = x + y\)  
2. \(z = 4x + y\)  
3. \(z = 3x + 7y\)  
4. \(z = 9x + 3y\)  

In Problems 5–8, find the minimum value of each objective function over the feasible region \(T\) shown in the figure below.

5. \(z = 7x + 4y\)  
6. \(z = 7x + 9y\)  
7. \(z = 3x + 8y\)  
8. \(z = 5x + 4y\)
In Problems 9–22, solve the linear programming problems.

9. Maximize \( z = 3x + 2y \) 
   Subject to 
   \[ x + 2y \leq 10 \]
   \[ 3x + y \leq 15 \]
   \[ x, y \geq 0 \]

10. Maximize \( z = 4x + 5y \) 
    Subject to 
    \[ 2x + y \leq 12 \]
    \[ x + 3y \leq 21 \]
    \[ x, y \geq 0 \]

11. Minimize \( z = 3x + 4y \) 
    Subject to 
    \[ 2x + y \leq 8 \]
    \[ x + 2y \leq 10 \]
    \[ x, y \geq 0 \]

12. Minimize \( z = 2x + y \) 
    Subject to 
    \[ 4x + 3y \leq 24 \]
    \[ 4x + y \leq 16 \]
    \[ x, y \geq 0 \]

13. Maximize \( z = 3x + 4y \) 
    Subject to 
    \[ x + 2y \leq 24 \]
    \[ x + y \leq 14 \]
    \[ 2x + y \leq 24 \]
    \[ x, y \geq 0 \]

14. Maximize \( z = 5x + 3y \) 
    Subject to 
    \[ 3x + y \leq 24 \]
    \[ x + y \leq 10 \]
    \[ x + 3y \leq 24 \]
    \[ x, y \geq 0 \]

15. Minimize \( z = 5x + 6y \) 
    Subject to 
    \[ x + 4y \geq 20 \]
    \[ 4x + y \geq 20 \]
    \[ x + y \geq 20 \]
    \[ x, y \geq 0 \]

16. Minimize \( z = x + 2y \) 
    Subject to 
    \[ 2x + 3y \geq 30 \]
    \[ 3x + 2y \geq 30 \]
    \[ x + y \leq 15 \]
    \[ x, y \geq 0 \]

17. Minimize and maximize \( z = 25x + 50y \) 
    Subject to 
    \[ x + 2y \leq 120 \]
    \[ x + y \leq 60 \]
    \[ x - 2y \leq 0 \]
    \[ x, y \geq 0 \]

18. Minimize and maximize \( z = 15x + 30y \) 
    Subject to 
    \[ x + 2y \geq 100 \]
    \[ 2x - y \leq 0 \]
    \[ 2x + y \leq 200 \]
    \[ x, y \geq 0 \]

19. Minimize and maximize \( z = 25x + 15y \) 
    Subject to 
    \[ 4x + 5y \leq 100 \]
    \[ 3x + 4y \leq 240 \]
    \[ x \leq 60 \]
    \[ y \leq 45 \]
    \[ x, y \geq 0 \]

20. Minimize and maximize \( z = 25x + 30y \) 
    Subject to 
    \[ 2x + 3y \leq 120 \]
    \[ 3x + 2y \leq 360 \]
    \[ x \leq 80 \]
    \[ y \leq 120 \]
    \[ x, y \geq 0 \]

21. Maximize \( P = 525x_1 + 478x_2 \) 
    Subject to 
    \[ 275x_1 + 322x_2 \leq 3,381 \]
    \[ 350x_1 + 340x_2 \leq 3,762 \]
    \[ 425x_1 + 306x_2 \leq 4,114 \]
    \[ x_1, x_2 \geq 0 \]

22. Maximize \( P = 300x_1 + 460x_2 \) 
    Subject to 
    \[ 245x_1 + 452x_2 \leq 4,181 \]
    \[ 290x_1 + 379x_2 \leq 3,888 \]
    \[ 390x_1 + 299x_2 \leq 4,407 \]
    \[ x_1, x_2 \geq 0 \]

23. The corner points for the feasible region determined by the problem constraints 
    \[ 2x + y \leq 10 \]
    \[ x + 3y \leq 15 \]
    \[ x, y \geq 0 \]
    are \( O = (0, 0), \) \( A = (5, 0), \) \( B = (3, 4), \) and \( C = (0, 5). \) If \( z = ax + by \) 
    and \( a, b > 0, \) determine conditions on \( a \) and \( b \) that ensure that the maximum value of \( z \) occurs
    (A) Only at \( A \)
    (B) Only at \( B \)
    (C) Only at \( C \)
    (D) At both \( A \) and \( B \)
    (E) At both \( B \) and \( C \)

24. The corner points for the feasible region determined by the problem constraints 
    \[ x + y \geq 4 \]
    \[ x + 2y \geq 6 \]
    \[ 2x + 3y \leq 12 \]
    \[ x, y \leq 0 \]
    are \( A = (6, 0), \) \( B = (2, 2), \) and \( C = (0, 4). \) If \( z = ax + by \) 
    and \( a, b > 0, \) determine conditions on \( a \) and \( b \) that ensure that the minimum value of \( z \) occurs
    (A) Only at \( A \)  
    (B) Only at \( B \)
    (C) Only at \( C \)
    (D) At both \( A \) and \( B \)
    (E) At both \( B \) and \( C \)
25. Resource Allocation. A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The relevant manufacturing data is given in the table.

(A) If the profit on a trick ski is $40 and the profit on a slalom ski is $30, how many of each type of ski should be manufactured each day to realize a maximum profit? What is the maximum profit?

(B) Discuss the effect on the production schedule and the maximum profit if the profit on a slalom ski decreases to $25 and all other data remains the same.

(C) Discuss the effect on the production schedule and the maximum profit if the profit on a slalom ski increases to $45 and all other data remains the same.

<table>
<thead>
<tr>
<th>Trick Ski [labor-hours per ski]</th>
<th>Slalom Ski [labor-hours per ski]</th>
<th>Maximum Labor-Hours Available per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabricating department</td>
<td>6</td>
<td>108</td>
</tr>
<tr>
<td>Finishing department</td>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

26. Psychology. In an experiment on conditioning, a psychologist uses two types of Skinner boxes with mice and rats. The amount of time (in minutes) each mouse and each rat spends in each box per day is given in the table. What is the maximum total number of mice and rats that can be used in this experiment? How many mice and how many rats produce this maximum?

<table>
<thead>
<tr>
<th>Mice [minutes]</th>
<th>Rats [minutes]</th>
<th>Max. Time Available [minutes]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skinner box A</td>
<td>10</td>
<td>800</td>
</tr>
<tr>
<td>Skinner box B</td>
<td>20</td>
<td>640</td>
</tr>
</tbody>
</table>

27. Purchasing. A trucking firm wants to purchase a maximum of 15 new trucks that will provide at least 36 tons of additional shipping capacity. A model A truck holds 2 tons and costs $15,000. A model B truck holds 3 tons and costs $24,000. How many trucks of each model should the company purchase to provide the additional shipping capacity at minimal cost? What is the minimal cost?

28. Transportation. The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs $1,200 to rent. Each van can transport 8 students, requires 1 chaperone, and costs $100 to rent. The officers want to be able to accommodate at least 400 students with no more than 36 chaperones. How many vehicles of each type should they rent in order to minimize the transportation costs? What are the minimal transportation costs?

29. Resource Allocation. A furniture company manufactures dining room tables and chairs. Each table requires 8 hours from the assembly department and 2 hours from the finishing department and contributes a profit of $90. Each chair requires 2 hours from the assembly department and 1 hour from the finishing department and contributes a profit of $25. The maximum labor-hours available each day in the assembly and finishing departments are 400 and 120, respectively.

(A) How many tables and how many chairs should be manufactured each day to maximize the daily profit? What is the maximum daily profit?

(B) Discuss the effect on the production schedule and the maximum profit if the marketing department of the company decides that the number of chairs produced should be at least four times the number of tables produced.

30. Resource Allocation. An electronics firm manufactures two types of personal computers, a desktop model and a portable model. The production of a desktop computer requires a capital expenditure of $400 and 40 hours of labor. The production of a portable computer requires a capital expenditure of $250 and 30 hours of labor. The firm has $20,000 capital and 2,160 labor-hours available for production of desktop and portable computers.

(A) What is the maximum number of computers the company is capable of producing?

(B) If each desktop computer contributes a profit of $320 and each portable contributes a profit of $220, how much profit will the company make by producing the maximum number of computers determined in part A? Is this the maximum profit? If not, what is the maximum profit?

31. Pollution Control. Because of new federal regulations on pollution, a chemical plant introduced a new process to supplement or replace an older process used in the production of a particular chemical. The older process emitted 20 grams of sulfur dioxide and 40 grams of particulate matter into the atmosphere for each gallon of chemical produced. The new process emits 5 grams of sulfur dioxide and 20 grams of particulate matter for each gallon produced. The company makes a profit of 60¢ per gallon and 20¢ per gallon on the old and new processes, respectively.

(A) If the regulations allow the plant to emit no more than 16,000 grams of sulfur dioxide and 30,000 grams of particulate matter daily, how many gallons of the chemical should be produced by each process to maximize daily profit? What is the maximum daily profit?
(B) Discuss the effect on the production schedule and the maximum profit if the regulations restrict emissions of sulfur dioxide to 11,500 grams daily and all other data remains unchanged.

(C) Discuss the effect on the production schedule and the maximum profit if the regulations restrict emissions of sulfur dioxide to 7,200 grams daily and all other data remains unchanged.

** 32. Sociology.** A city council voted to conduct a study on inner-city community problems. A nearby university was contacted to provide a maximum of 40 sociologists and research assistants. Allocation of time and cost per week are given in the table.

(A) How many sociologists and research assistants should be hired to meet the weekly labor-hour requirements and minimize the weekly cost? What is the weekly cost?

(B) Discuss the effect on the solution in part A if the council decides that they should not hire more sociologists than research assistants and all other data remains unchanged.

<table>
<thead>
<tr>
<th></th>
<th>Sociologist [labor-hours]</th>
<th>Research Assistant [labor-hours]</th>
<th>Minimum Labor-Hours Needed per Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fieldwork</td>
<td>10</td>
<td>30</td>
<td>280</td>
</tr>
<tr>
<td>Research center</td>
<td>30</td>
<td>10</td>
<td>360</td>
</tr>
<tr>
<td>Cost per week</td>
<td>$500</td>
<td>$300</td>
<td></td>
</tr>
</tbody>
</table>

** 33. Plant Nutrition.** A fruit grower can use two types of fertilizer in her orange grove, brand A and brand B. The amounts (in pounds) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each mix are given in the table. Tests indicate that the grove needs at least 480 pounds of phosphoric acid, at least 540 pounds of potash, and at most 620 pounds of chlorine. If the grower always uses a combination of bags of brand A and brand B that will satisfy the constraints of phosphoric acid, potash, and chlorine, discuss the effect that this will have on the amount of nitrogen added to the field.

<table>
<thead>
<tr>
<th></th>
<th>Brand A</th>
<th>Brand B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Phosphoric acid</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Potash</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Chlorine</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

** 34. Diet.** A dietitian in a hospital is to arrange a special diet composed of two foods, M and N. Each ounce of food M contains 16 units of calcium, 5 units of iron, 6 units of cholesterol, and 8 units of vitamin A. Each ounce of food N contains 4 units of calcium, 25 units of iron, 4 units of cholesterol, and 4 units of vitamin A. The diet requires at least 320 units of calcium, at least 575 units of iron, and at most 300 units of cholesterol. If the dietitian always selects a combination of foods M and N that will satisfy the constraints for calcium, iron, and cholesterol, discuss the effects that this will have on the amount of vitamin A in the diet.