Motion in a Straight Line

FIGURE 2.1 A fast-moving train passes a railroad crossing.
You can see that the train in Figure 2.1 is moving very fast by noticing its image is blurred compared with the stationary crossing signal and telephone pole. But can you tell if the train is speeding up, slowing down, or zipping by at constant speed? A photograph can convey an object’s speed because the object moves during the exposure time, but a photograph cannot show a change in speed, or acceleration. Yet acceleration is extremely important in physics, at least as important as speed itself.

In this chapter, we look at the terms used in physics to describe an object’s motion: displacement, velocity, and acceleration. We examine motion along a straight line (one-dimensional motion) in this chapter and motion on a curved path (motion in a plane, or two-dimensional motion) in the next chapter. One of the greatest advantages of physics is that its laws are universal, so the same general terms and ideas apply to a wide range of situations. Thus, we can use the same equations to describe the flight of a baseball and the lift-off of a rocket into space from Earth to Mars. In this chapter, we will use some of the problem-solving techniques discussed in Chapter 1 along with some new ones.

As you continue in this course, you will see that almost everything moves relative to other objects on some scale or other, whether it is a comet plunging through space at several kilometers per second or the atoms in a seemingly stationary object vibrating millions of times per second. The terms we introduce in this chapter will be part of your study for the rest of the course and afterward.

### 2.1 Introduction to Kinematics

The study of physics is divided into several large parts, one of which is mechanics. Mechanics, or the study of motion and its causes, is usually subdivided. In this chapter and the next, we examine the kinematics aspect of mechanics. Kinematics is the study of the motion of objects. These objects may be, for example, cars, baseballs, people, planets, or atoms. For now, we will set aside the question of what causes this motion. We will return to that question when we study forces.

We will also not consider rotation in this chapter, but concentrate on only translational motion (motion without rotation). Furthermore, we will neglect all internal structure of a moving object and consider it to be a point particle, or pointlike object. That is, to determine the equations of motion for an object, we imagine it to be located at a single point in space at each instant of time. What point of an object should we choose to represent its location? Initially, we will simply use the geometric center, the middle. (Chapter 8, on systems of particles and extended objects, will give a more precise definition for the point location of an object called the center of mass.)

### 2.2 Position Vector, Displacement Vector, and Distance

The simplest motion we can investigate is that of an object moving in a straight line. Examples of this motion include a person running the 100-m dash, a car driving on a straight segment of road, and a stone falling straight down off a cliff. In later chapters, we will consider
motion in two or more dimensions and will see that the same concepts that we derive for one-dimensional motion still apply.

If an object is located on a particular point on a line, we can denote this point with its **position vector**, as described in Section 1.6. Throughout this book, we use the symbol \( r \) to denote the position vector. Since we are working with motion in only one dimension in this chapter, the position vector has only one component. If the motion is in the horizontal direction, this one component is the \( x \)-component. (For motion in the vertical direction, we will use the \( y \)-component; see Section 2.7.) One number, the \( x \)-coordinate or \( x \)-component of the position vector (with a corresponding unit), uniquely specifies the position vector in one-dimensional motion. Some valid ways of writing a position are \( x = 4.3 \text{ m}, x = 7\text{ in.} \), and \( x = -2.04 \text{ km} \); it is understood that these specifications refer to the \( x \)-component of the position vector. Note that a position vector’s \( x \)-component can have a positive or a negative value, depending on the location of the point and the axis direction that we choose to be positive. The value of the \( x \)-component also depends on where we define the origin of the coordinate system—the zero of the straight line.

The position of an object can change as a function of time, \( t \); that is, the object can move. We can therefore formally write the position vector in function notation: \( \vec{r} = \vec{r}(t) \). In one dimension, this means that the \( x \)-component of the vector is a function of time, \( x = x(t) \). If we want to specify the position at some specific time \( t_1 \), we use the notation \( x_1 = x(t_1) \).

**Position Graphs**

Before we go any further, let’s graph an object’s position as a function of time. Figure 2.2a illustrates the principle involved by showing several frames of a video of a car driving down a road. The video frames were taken at time intervals of \( \frac{1}{4} \) second.

We are free to choose the origins of our time measurements and of our coordinate system. In this case, we choose the time of the second frame to be \( t = \frac{1}{4} \) s and the position of the center of the car in the second frame as \( x = 0 \). We can now draw our coordinate axes and graph the frames (Figure 2.2b). The position of the car as a function of time lies on a straight line. Again, keep in mind that we are representing the car by a single point.

When drawing graphs, it is customary to plot the independent variable—in this case, the time \( t \)—on the horizontal axis and to plot \( x \), which is called the dependent variable because its value depends on the value of \( t \), on the vertical axis. Figure 2.3 is a graph of the car’s position as a function of time drawn in this customary way. (Note that if Figure 2.2b were rotated 90° counterclockwise and the pictures of the car removed, the two graphs would be the same.)

**Displacement**

Now that we have specified the position vector, let’s go one step further and define displacement. **Displacement** is simply the difference between the final position vector, \( \vec{r}_2 = \vec{r}(t_2) \), at the end of a motion and the initial position vector, \( \vec{r}_1 = \vec{r}(t_1) \). We write the displacement vector as

\[
\Delta \vec{r} = \vec{r}_2 - \vec{r}_1. \tag{2.1}
\]

We use the notation \( \Delta \vec{r} \) for the displacement vector to indicate that it is a difference between two position vectors. Note that the displacement vector is independent of the location of the origin of the coordinate system. Why? Any shift of the coordinate system will add to the position vector \( \vec{r}_1 \), the same amount that it adds to the position vector \( \vec{r}_2 \); thus the difference between the position vectors, or \( \Delta \vec{r} \), will not change.

Just like the position vector, the displacement vector in one dimension has only an \( x \)-component, which is the difference between the \( x \)-components of the final and initial position vectors:

\[
\Delta x = x_2 - x_1. \tag{2.2}
\]

Also just like position vectors, displacement vectors can be positive or negative. In particular, the displacement vector \( \Delta \vec{r}_{ab} \) for going from point \( a \) to point \( b \) is exactly the negative of \( \Delta \vec{r}_{ba} \) going from point \( b \) to point \( a \):

\[
\Delta \vec{r}_{ba} = \vec{r}_b - \vec{r}_a = -(\vec{r}_a - \vec{r}_b) = -\Delta \vec{r}_{ba}. \tag{2.3}
\]
And it is probably obvious to you at this point that this relationship also holds for the \( x \)-component of the displacement vector, \( \Delta x_{ba} = x_b - x_a = -(x_a - x_b) = -\Delta x_{ab} \).

**Distance**

The **distance**, \( \ell \), that a moving object travels is the absolute value of the displacement vector:

\[
\ell = |\Delta r|.
\]

(2.4)

For one-dimensional motion, this distance is also the absolute value of the \( x \)-component of the displacement vector, \( \ell = |\Delta x| \). (For multidimensional motion, we calculate the length of the displacement vector as shown in Chapter 1). The distance is always greater than or equal to zero and is measured in the same units as position and displacement. However, distance is a scalar quantity, not a vector. If the displacement is not in a straight line or if it is not all in the same direction, the displacement must be broken up into segments that are approximately straight and unidirectional and then the distances for the various segments are added to get the total distance. The following solved problem illustrates the difference between distance and displacement.

**SOLVED PROBLEM 2.1 Trip Segments**

The distance between Des Moines and Iowa City is 170.5 km (106.0 miles) along Interstate 80, and as you can see from the map (Figure 2.4), the route is a straight line to a good approximation. Approximately halfway between the two cities, where I80 crosses highway US63, is the city of Malcom, 89.9 km (54.0 miles) from Des Moines.

![Figure 2.4](Route_I80_between_Des_Moines_and_Iowa_City.png)

**FIGURE 2.4** Route I80 between Des Moines and Iowa City.

**PROBLEM**

If we drive from Malcom to Des Moines and then go to Iowa City, what are the total distance and total displacement for this trip?

**SOLUTION**

**THINK**

Distance and displacement are not identical. If the trip consisted of one segment in one direction, the distance would just be the absolute value of the displacement, according to equation 2.4. However, this trip is composed of segments with a direction change, so we need to be careful. We’ll treat each segment individually and then add up the segments in the end.

**SKETCH**

Because I80 is almost a straight line, it is sufficient to draw a straight horizontal line and make this our coordinate axis. We enter the positions of the three cities as \( x_1 \) (Iowa City), \( x_M \) (Malcom), and \( x_D \) (Des Moines). We always have the freedom to define the origin of our coordinate system, so we elect to put it at Des Moines, thus setting \( x_D = 0 \). As is conventional, we define the positive direction to the right, in the eastward direction. See Figure 2.5.

We also draw arrows for the displacements of the two segments of the trip. We represent segment 1 from Malcom to Des Moines by...
2.2 Position Vector, Displacement Vector, and Distance

A red arrow, and segment 2 from Des Moines to Iowa City by a blue arrow. Finally, we draw a diagram for the total trip as the sum of the two trips.

**Research**

With our assignment of \(x_D = 0\), Des Moines is the origin of the coordinate system. According to the information given us, Malcom is then at \(x_M = +89.9 \text{ km}\) and Iowa City is at \(x_I = +170.5 \text{ km}\). Note that we write a plus sign in front of the numbers for \(x_M\) and \(x_I\) to remind us that these are components of position vectors and can have positive or negative values.

For the first segment, the displacement is given by

\[
\Delta x_1 = x_D - x_M.
\]

Thus the distance driven for this segment is

\[
\ell_1 = |\Delta x_1| = |x_D - x_M|.
\]

In the same way, the displacement and distance for the second segment are

\[
\Delta x_2 = x_I - x_D
\]

\[
\ell_2 = |\Delta x_2| = |x_I - x_D|.
\]

For the sum of the two segments, the total trip, we use simple addition to find the displacement,

\[
\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2,
\]

and the total distance,

\[
\ell_{\text{total}} = \ell_1 + \ell_2.
\]

**Simplify**

We can simplify the equation for the total displacement a little bit by inserting the expressions for the displacements for the two segments:

\[
\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2
\]

\[
= (x_D - x_M) + (x_I - x_D)
\]

\[
= x_I - x_M
\]

This is an interesting result—for the total displacement of the entire trip, it does not matter at all that we went to Des Moines. All that matters is where the trip started and where it ended. The total displacement is a result of a one-dimensional vector addition, as indicated in the bottom part of Figure 2.5 by the green arrow.

**Calculate**

Now we can insert the numbers for the positions of the three cities in our coordinate system. We then obtain for the net displacement in our trip

\[
\Delta x_{\text{total}} = x_I - x_M = (+170.5 \text{ km}) - (+89.9 \text{ km}) = +80.6 \text{ km}.
\]

For the total distance driven, we get

\[
\ell_{\text{total}} = |89.9 \text{ km}| + |170.5 \text{ km}| = 260.4 \text{ km}.
\]

(Remember, the distance between Des Moines and Malcom, or \(\Delta x_1\), and that between Des Moines and Iowa City, or \(\Delta x_2\), were given in the problem; so we do not have to calculate them again from the differences in the position vectors of the cities.)

**Round**

The numbers for the distances were initially given to a tenth of a kilometer. Since our entire calculation only amounted to adding or subtracting these numbers, it is not surprising that we end up with numbers that are also accurate to a tenth of a kilometer. No further rounding is needed.

*Continued—*
2.1 Self-Test Opportunity
Suppose we had chosen to put the origin of the coordinate system in Solved Problem 2.1 at Malcom instead of Des Moines. Would the final result of our calculation change? If yes, how? If no, why not?

DOUBLe-CHECK
As is customary, we first make sure that the units of our answer came out properly. Since we are looking for quantities with the dimension of length, it is comforting that our answers have the units of kilometers. At first sight, it may be surprising that the net displacement for the trip is only 80.6 km, much smaller than the total distance traveled. This is a good time to recall that the relationship between the absolute value of the displacement and the distance (equation 2.4) is valid only if the moving object does not change direction (but it did in this example).

This discrepancy is even more apparent for a round trip. In that case, the total distance driven is twice the distance between the two cities, but the total displacement is zero, because the starting point and the end point of the trip are identical.

This result is a general one: If the initial and final positions are the same, the total displacement is 0. As straightforward as this seems for the trip example, it is a potential pitfall in many exam questions. You need to remember that displacement is a vector, whereas distance is a positive scalar.

2.3 Velocity Vector, Average Velocity, and Speed
Just as distance (a scalar) and displacement (a vector) mean different things in physics, their rates of change with time are also different. Although the words “speed” and “velocity” are often used interchangeably in everyday speech, in physics “speed” refers to a scalar and “velocity” to a vector.

We define $v_x$, the $x$-component of the velocity vector, as the change in position (i.e., the displacement component) in a given time interval divided by that time interval, $\Delta x/\Delta t$. Velocity can change from moment to moment. The velocity calculated by taking the ratio of displacement per time interval is the average of the velocity over this time interval, or the $x$-component of the average velocity, $\bar{v}_x$:

$$\bar{v}_x = \frac{\Delta x}{\Delta t}. \tag{2.5}$$

Notation: A bar above a symbol is the notation for averaging over a finite time interval.

In calculus, a time derivative is obtained by taking a limit as the time interval approaches zero. We use the same concept here to define the instantaneous velocity, usually referred to simply as the velocity, as the time derivative of the displacement. For the $x$-component of the velocity vector, this implies

$$v_x = \lim_{\Delta t \to 0} \bar{v}_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \tag{2.6}$$

We can now introduce the velocity vector, $\vec{v}$, as the vector for which each component is the time derivative of the corresponding component of the position vector,

$$\vec{v} = \frac{d\vec{x}}{dt}, \tag{2.7}$$

with the understanding that the derivative operation applies to each of the components of the vector. In the one-dimensional case, this velocity vector $\vec{v}$ has only an $x$-component, $v_x$, and the velocity is equivalent to a single velocity component in the spatial $x$-direction.

Figure 2.6 presents three graphs of the position of an object with respect to time. Figure 2.6a shows that we can calculate the average velocity of the object by finding the change in position of the object between two points and dividing by the time it takes to go from $x_1$ to $x_2$. That is, the average velocity is given by the displacement, $\Delta x$, divided by the time interval, $\Delta t$, or $\bar{v}_x = \Delta x/\Delta t$. In Figure 2.6b, the average velocity, $\bar{v}_x = \Delta x/\Delta t$, is determined over a smaller time interval, $\Delta t$. In Figure 2.6c, the instantaneous velocity, $v(t_f) = \Delta x/\Delta t_{t_f}$, is represented by the slope of the blue line tangent to the red curve at $t = t_f$.

Velocity is a vector, pointing in the same direction as the vector of the infinitesimal displacement, $dx$. Because the position $x(t)$ and the displacement $\Delta x(t)$ are functions of
time, so is the velocity. Because the velocity vector is defined as the time derivative of the displacement vector, all the rules of differentiation introduced in calculus hold. If you need a refresher, consult Appendix A.

**Example 2.1 Time Dependence of Velocity**

**Problem**

During the time interval from 0.0 to 10.0 s, the position vector of a car on a road is given by \( x(t) = a + bt + ct^2 \), with \( a = 17.2 \text{ m} \), \( b = -10.1 \text{ m/s} \), and \( c = 1.10 \text{ m/s}^2 \). What is the car’s velocity as a function of time? What is the car’s average velocity during this interval?

**Solution**

According to the definition of velocity in equation 2.6, we simply take the time derivative of the position vector function to arrive at our solution:

\[
v_x = \frac{dx}{dt} = \frac{d}{dt}(a + bt + ct^2) = b + 2ct = -10.1 \text{ m/s} + 2 \cdot (1.10 \text{ m/s}^2) t.
\]

It is instructive to graph this solution. In Figure 2.7, the position as a function of time is shown in blue, and the velocity as a function of time is shown in red. Initially, the velocity has a value of \(-10.1 \text{ m/s}\), and at \( t = 10 \text{ s} \), the velocity has a value of \(+11.9 \text{ m/s}\).

Note that the velocity is initially negative, is zero at 4.59 s (indicated by the vertical dashed line in Figure 2.7), and then is positive after 4.59 s. At \( t = 4.59 \text{ s} \), the position graph \( x(t) \) shows an extremum (a minimum in this case), just as expected from calculus, since

\[
\frac{dx}{dt} = b + 2ct = 0 \Rightarrow t_0 = -\frac{b}{2c} = -\frac{-10.1 \text{ m/s}}{2 \cdot 1.10 \text{ m/s}^2} = 4.59 \text{ s}.
\]

From the definition of average velocity, we know that to determine the average velocity during a time interval, we need to subtract the position at the beginning of the interval from the position at the end of the interval. By inserting \( t = 0 \) and \( t = 10 \text{ s} \) into the equation for the position vector as a function of time, we obtain \( x(t = 0) = 17.2 \text{ m} \) and \( x(t = 10 \text{ s}) = 26.2 \text{ m} \). Therefore,

\[
\Delta x = x(t = 10) - x(t = 0) = 26.2 \text{ m} - 17.2 \text{ m} = 9.0 \text{ m}.
\]

We then obtain for the average velocity over this time interval:

\[
\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{9.0 \text{ m}}{10 \text{ s}} = 0.90 \text{ m/s}.
\]

The slope of the green dashed line in Figure 2.7 is the average velocity over this time interval.
Speed

**Speed** is the absolute value of the velocity vector. For a moving object, speed is always positive. “Speed” and “velocity” are used interchangeably in everyday contexts, but in physical terms they are very different. Velocity is a vector, which has a direction. For one-dimensional motion, the velocity vector can point either in the positive or negative direction; in other words, its component can have either sign. Speed is the absolute magnitude of the velocity vector and is thus a scalar quantity:

\[
\text{speed } v = |v| = |v_x|
\]  

(2.8)

The last part of this equation makes use of the fact that the velocity vector has only an \( x \)-component for one-dimensional motion.

In everyday experience, we recognize that speed can never be negative: Speed limits are always posted as positive numbers, and the radar of passing cars also always displays positive numbers (Figure 2.8).

Earlier, distance was defined as the absolute value of displacement for each straight-line segment in which the movement does not reverse direction (see the discussion following equation 2.4). The average speed when a distance \( \ell \) is traveled during a time interval \( \Delta t \) is

\[
\text{average speed } \bar{v} = \frac{\ell}{\Delta t}
\]  

(2.9)

---

**Example 2.2 Speed and Velocity**

Suppose a swimmer completes the first 50 m of the 100-m freestyle in 38.2 s. Once she reaches the far side of the 50-m-long pool, she turns around and swims back to the start in 42.5 s.

**Problem**

What are the swimmer’s average velocity and average speed for (a) the leg from the start to the far side of the pool, (b) the return leg, and (c) the total lap?

**Solution**

We start by defining our coordinate system, as shown in Figure 2.9. The positive \( x \)-axis points toward the bottom of the page.

(a) First leg of the swim:

The swimmer starts at \( x_1 = 0 \) and swims to \( x_2 = 50 \) m. It takes her \( \Delta t = 38.2 \) s to accomplish this leg. Her average velocity for leg 1 then, according to our definition, is

\[
\bar{v}_{x1} = \frac{x_2 - x_1}{\Delta t} = \frac{50 \text{ m} - 0 \text{ m}}{38.2 \text{ s}} = \frac{50 \text{ m}}{38.2 \text{ s}} = 1.31 \text{ m/s}.
\]

Her average speed is the distance divided by time interval, which, in this case, is the same as the absolute value of her average velocity, or \( |\bar{v}_{x1}| = 1.31 \text{ m/s} \).

(b) Second leg of the swim:

We use the same coordinate system for leg 2 as for leg 1. This choice means that the swimmer starts at \( x_3 = 50 \) m and finishes at \( x_4 = 0 \), and it takes \( \Delta t = 42.5 \) s to do so. Her average velocity for this leg is

\[
\bar{v}_{x2} = \frac{x_4 - x_3}{\Delta t} = \frac{0 \text{ m} - 50 \text{ m}}{42.5 \text{ s}} = \frac{-50 \text{ m}}{42.5 \text{ s}} = -1.18 \text{ m/s}.
\]

Note the negative sign for the average velocity for this leg. The average speed is again the absolute magnitude of the average velocity, or \( |\bar{v}_{x2}| = 1.18 \text{ m/s} = 1.18 \text{ m/s} \).

(c) The entire lap:

We can find the average velocity in two ways, demonstrating that they result in the same answer. First, because the swimmer started at \( x_1 = 0 \) and finished at \( x_4 = 0 \), the difference is 0. Thus, the net displacement is 0, and consequently the average velocity is also 0.
We can also find the average velocity for the whole lap by taking the time-weighted sum of the components of the average velocities of the individual legs:

\[
\bar{v}_x = \frac{v_{x1} \cdot \Delta t_1 + v_{x2} \cdot \Delta t_2}{\Delta t_1 + \Delta t_2} = \frac{(1.31 \text{ m/s})(38.2 \text{ s}) + (-1.18 \text{ m/s})(42.5 \text{ s})}{(38.2 \text{ s}) + (42.5 \text{ s})} = 0.
\]

What do we find for the average speed? The average speed, according to our definition, is the total distance divided by the total time. The total distance is 100 m and the total time is 38.2 s plus 42.5 s, or 80.7 s. Thus,

\[
\bar{v} = \frac{\text{total distance}}{\text{total time}} = \frac{100 \text{ m}}{80.7 \text{ s}} = 1.24 \text{ m/s}.
\]

We can also use the time-weighted sum of the average speeds, leading to the same result. Note that the average speed for the entire lap is between that for leg 1 and that for leg 2. It is not exactly halfway between these two values, but is closer to the lower value because the swimmer spent more time completing leg 2.

### 2.4. Acceleration Vector

Just as the average velocity is defined as the displacement per time interval, the \(x\)-component of average acceleration is defined as the velocity change per time interval:

\[
a_x = \frac{\Delta v_x}{\Delta t}.
\]  
\[(2.10)\]

Similarly, the \(x\)-component of the instantaneous acceleration is defined as the limit of the average acceleration as the time interval approaches 0:

\[
a_x = \lim_{\Delta t \to 0} \bar{a}_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}.
\]  
\[(2.11)\]

We can now define the acceleration vector as

\[
\vec{a} = \frac{dv}{dt},
\]  
\[(2.12)\]

where again the derivative operation is understood to act component-wise, just as in the definition of the velocity vector.

Figure 2.10 illustrates this relationship among velocity, time interval, average acceleration, and instantaneous acceleration as the limit of the average acceleration (for a decreasing time interval). In Figure 2.10a, the average acceleration is given by the velocity change, \(\Delta v_x\), divided by the time interval \(\Delta t_1\): \(\bar{a}_x = \Delta v_x / \Delta t_1\). In Figure 2.10b, the average acceleration is determined over a smaller time interval, \(\Delta t_2\). In Figure 2.10c, the instantaneous acceleration, \(a(t_3) = dv_x/dt\big|_{t=t_3}\), is represented by the slope of the blue line tangent to the red curve at \(t = t_3\). Figure 2.10 looks very similar to Figure 2.6, and this is not a coincidence. The similarity emphasizes that the mathematical operations and physical relationships that connect the velocity and acceleration vectors are the same as those that connect the position and velocity vectors.

**Figure 2.10** Instantaneous acceleration as the limit of the ratio of velocity change to time interval: (a) average acceleration over a large time interval; (b) average acceleration over a smaller time interval; and (c) instantaneous acceleration in the limit as the time interval goes to zero.
2.1 In-Class Exercise

When you’re driving a car along a straight road, you may be traveling in the positive or negative direction and you may have a positive acceleration or a negative acceleration. Match the following combinations of velocity and acceleration with the list of outcomes.

a) positive velocity, positive acceleration
b) positive velocity, negative acceleration
c) negative velocity, positive acceleration
d) negative velocity, negative acceleration

1) slowing down in positive direction
2) speeding up in negative direction
3) speeding up in positive direction
4) slowing down in negative direction

The acceleration is the time derivative of the velocity, and the velocity is the time derivative of the displacement. The acceleration is therefore the second derivative of the displacement:

\[ a_x = \frac{d}{dt} v_x = \frac{d}{dt} \left( \frac{d}{dt} x \right) = \frac{d^2}{dt^2} x. \]  

(2.13)

There is no word in everyday language for the absolute value of the acceleration.

Note that we often refer to the deceleration of an object as a decrease in the speed the object over time, which corresponds to acceleration in the opposite direction of the motion of the object.

In one-dimensional motion, an acceleration, which is a change in velocity, necessarily entails a change in the magnitude of the velocity—that is, the speed. However, in the next chapter, we will consider motion in more than one spatial dimension, where the velocity vector can also change its direction, not just its magnitude. In Chapter 9, we will examine motion in a circle with constant speed; in that case, there is a constant acceleration that keeps the object on a circular path but leaves the speed constant.

As the in-class exercise shows, even in one dimension, a positive acceleration does not necessarily mean speeding up and a negative acceleration does not mean that it must be slowing down. Rather, the combination of velocity and acceleration determines the motion. If the velocity and acceleration are in the same direction, the object moves faster; if they are in opposite directions, it slows down. We will examine this relationship further in the next chapter.

2.5 Computer Solutions and Difference Formulas

In some situations, the acceleration changes as a function of time, but the exact functional form is not known beforehand. However, we can still calculate velocity and acceleration even if the position is known only at certain points in time. The following example illustrates this procedure.

**Example 2.3 World Record for the 100-m Dash**

In the 1991 Track and Field World Championships in Tokyo, Japan, Carl Lewis of the United States set a new world record in the 100-m dash. Figure 2.11 lists the times at which he arrived at the 10-m mark, the 20-m mark, and so on, as well as values for his average velocity and average acceleration, calculated from the formulas in equations 2.5 and 2.10. From Figure 2.11, it is clear that after about 3 s, Lewis reached an approximately constant average velocity between 11 and 12 m/s.

Figure 2.11 also indicates how the values for average velocity and average acceleration were obtained. Take, for example, the upper two green boxes, which contain the times and positions for two measurements. From these we get \( \Delta t = 2.96 \text{ s} - 1.88 \text{ s} = 1.08 \text{ s} \) and \( \Delta x = 20 \text{ m} - 10 \text{ m} = 10 \text{ m} \). The average velocity in this time interval is then \( \bar{v} = \Delta x/\Delta t = 10 \text{ m} / 1.08 \text{ s} = 9.26 \text{ m/s} \). We have rounded this result to three significant digits, because the times were given to that accuracy. The accuracy for the distances can be assumed to be even better, because these data were extracted from video analysis that showed the times at which Lewis crossed marks on the ground.

In Figure 2.11 the calculated average velocity is positioned halfway between the lines for time and distance, indicating that it is a good approximation for the instantaneous velocity in the middle of the time interval.

The average velocities for other time intervals were obtained in the same way. Using the numbers in the second and third green boxes in Figure 2.11, we obtained an average velocity of 10.87 m/s for the time interval from 2.96 s to 3.88 s. With two velocity values, we can use the difference formula for the acceleration to calculate the average

**Figure 2.11** Time, position, average velocity, and average acceleration during Carl Lewis’s world record 100-m dash.
acceleration. Here we assume that the instantaneous velocity at a time corresponding to halfway between the first two green boxes (2.42 s) is equal to the average velocity during the interval between the first two green boxes, or 9.26 m/s. Similarly, we take the instantaneous velocity at 3.42 s (midway between the second and third boxes) to be 10.87 m/s. Then, the average acceleration between 2.42 s and 3.42 s is

\[ \bar{a} = \frac{\Delta v}{\Delta t} = \frac{(10.87 \text{ m/s} - 9.26 \text{ m/s})}{(3.42 \text{ s} - 2.42 \text{ s})} = 1.61 \text{ m/s}^2. \]

From the entries in Figure 2.11 obtained in this way, we can see that Lewis did most of his accelerating between the start of the race and the 30-m mark, where he attained his maximum velocity between 11 and 12 m/s. He then ran with about that velocity until he reached the finish line. This result is clearer in a graphical display of his position versus time during the race (Figure 2.12a). The red dots represent the data points from Figure 2.11, and the green straight line represents a constant velocity of 11.58 m/s. In Figure 2.12b, Lewis’s velocity is plotted as a function of time. The green line again represents a constant velocity of 11.58 m/s, fitted to the last six points, where Lewis is no longer accelerating but running at a constant velocity.

The type of numerical analysis that treats average velocities and accelerations as approximations for the instantaneous values of these quantities is very common in all types of scientific and engineering applications. It is indispensable in situations where the precise functional dependencies on time are not known and researchers must rely on numerical approximations for derivatives obtained via the difference formulas. Most practical solutions of scientific and engineering problems found with the aid of computers make use of difference formulas such as those introduced here.

The entire field of numerical analysis is devoted to finding better numerical approximations that will enable more precise and faster computer calculations and simulations of natural processes. Difference formulas similar to those introduced here are as important for the everyday work of scientists and engineers as the calculus-based analytic expressions. This importance is a consequence of the computer revolution in science and technology, which, however, does not make the contents of the textbook any less important. In order to devise a valid solution to an engineering or science problem, you have to understand the basic underlying physical principles, no matter what calculation techniques you use. This fact is well recognized by cutting-edge movie animators and creators of special digital effects, who have to take basic physics classes to ensure that the products of their computer simulations look realistic to the audience.
2.6 Finding Displacement and Velocity from Acceleration

The fact that integration is the inverse operation to differentiation is known as the Fundamental Theorem of Calculus. It allows us to reverse the differentiation process leading from displacement to velocity to acceleration and instead integrate the equation for velocity (2.6) to obtain displacement and the equation for acceleration (2.13) to obtain velocity. Let’s start with the equation for the $x$-component of the velocity:

\[
v_x(t) = \frac{dx(t)}{dt} \Rightarrow \int_{t_0}^{t} v_x(t') dt' = \int_{t_0}^{t} \frac{dx(t')}{dt'} dt' = x(t) - x(t_0) \Rightarrow \]

\[
x(t) = x_0 + \int_{t_0}^{t} v_x(t') dt'. \quad (2.14)
\]

**Notation:** Here again we have used the convention that $x(t_0) = x_0$, the initial position. Further, we have used the notation $t'$ in the definite integrals in equation 2.14. This prime notation reminds us that the integration variable is a dummy variable, which serves to identify the physical quantity we want to integrate. Throughout this book, we will reserve the prime notation for dummy integration variables in definite integrals. (Note that some books use a prime to denote a spatial derivative, but to avoid possible confusion, this book will not do so.)

In the same way, we integrate equation 2.13 for the $x$-component of the acceleration to obtain an expression for the $x$-component of the velocity:

\[
a_x(t) = \frac{dv_x(t)}{dt} \Rightarrow \int_{t_0}^{t} a_x(t') dt' = \int_{t_0}^{t} \frac{dv_x(t')}{dt'} dt' = v_x(t) - v_x(t_0) \Rightarrow \]

\[
v_x(t) = v_{x0} + \int_{t_0}^{t} a_x(t') dt'. \quad (2.15)
\]

Here $v_x(t_0) = v_{x0}$ is the initial velocity component in the $x$-direction. Just as the derivative operation is understood to act component-wise, integration follows the same convention, so we can write the integral relationships for the vectors from those for the components in equations 2.14 and 2.15. Formally, we then have

\[
\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{v}(t') dt' \quad (2.16)
\]

and

\[
\vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t') dt'. \quad (2.17)
\]

This result means that for any given time dependence of the acceleration vector, we can calculate the velocity vector, provided we are given the initial value of the velocity vector. We can also calculate the displacement vector, if we know its initial value and the time dependence of the velocity vector.

In calculus, you probably learned that the geometrical interpretation of the definite integral is an area under a curve. This is true for equations 2.14 and 2.15. We can interpret the area under the curve of $v_x(t)$ between $t_0$ and $t$ as the difference in the position between these two times, as shown in Figure 2.13a. Figure 2.13b shows that the area under the curve of $a_x(t)$ in the time interval between $t_0$ and $t$ is the velocity difference between these two times.

**Figure 2.13** Geometrical interpretation of the integrals of (a) velocity and (b) acceleration with respect to time.
2.7 Motion with Constant Acceleration

In many physical situations, the acceleration experienced by an object is approximately, or perhaps even exactly, constant. We can derive useful equations for these special cases of motion with constant acceleration. If the acceleration, $a_x$, is a constant, then the time integral used to obtain the velocity in equation 2.15 results in

$$v_x(t) = v_{x0} + a_x \int_0^t dt$$

(2.18)

where we have taken the lower limit of the integral to be $t_0 = 0$ for simplicity. This means that the velocity is a linear function of time.

$$x = x_0 + \int_0^t v_x(t) dt = x_0 + \int_0^t (v_{x0} + a_x t) dt$$

$$= x_0 + v_{x0} \int_0^t dt + a_x \int_0^t t \ dt$$

$$= x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2.$$  

(2.19)

Thus, with a constant acceleration, the velocity is always a linear function of time, and the position is a quadratic function of time. Three other useful equations can be derived using equations 2.18 and 2.19 as a starting point. After listing these three equations, we’ll work through their derivations.

The average velocity in the time interval from 0 to $t$ is the average of the velocities at the beginning and end of the time interval:

$$\overline{v}_x = \frac{1}{2} (v_{x0} + v_x).$$  

(2.20)

The average velocity from equation 2.20 leads to an alternative way to express the position:

$$x = x_0 + \overline{v}_x t.$$  

(2.21)

Finally, we can write an equation for the square of the velocity that does not contain the time explicitly:

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0).$$  

(2.22)

**DERIVATION 2.1**

Mathematically, to obtain the time average of a quantity over a certain interval $\Delta t$, we have to integrate this quantity over the time interval and then divide by the time interval:

$$\overline{v}_x = \frac{1}{\Delta t} \int_0^{\Delta t} v_x(t') dt' = \frac{1}{\Delta t} \int_0^{\Delta t} (v_{x0} + at') dt'$$

$$= \frac{v_{x0}}{\Delta t} \int_0^{\Delta t} dt' + \frac{a}{\Delta t} \int_0^{\Delta t} t' dt' = v_{x0} + \frac{1}{2} a t$$

$$= \frac{1}{2} v_{x0} + \frac{1}{2} (v_{x0} + at)$$

$$= \frac{1}{2} (v_{x0} + v_x).$$

This averaging procedure for the time interval $t_0$ to $t$ is illustrated in Figure 2.14. You can see that the area of the trapezoid formed by the blue line representing $v(t)$ and the two vertical lines at $t_0$ and $t$ is equal to the area of the square formed by the horizontal line to $\overline{v}_x$ and the two vertical lines. The base line for both areas is the horizontal $t$-axis. It is more apparent that these two areas are equal if you note that the yellow triangle (part of the square) and the orange triangle (part of the trapezoid) are equal in size. [Algebraically, the
area of the square is $\overline{v}_x(t - t_0)$, and the area of the trapezoid is $\frac{1}{2}(v_{x0} + v_x)(t - t_0)$. Setting these two areas equal to each other gives us equation 2.20 again.

To derive the equation for the position, we take $t_0 = 0$ and use the expression $\overline{v}_x = v_{x0} + \frac{1}{2}a_xt$ and multiply both sides by the time:

$$v_x = v_{x0} + \frac{1}{2}a_xt$$

$$\Rightarrow v_x t = v_{x0}t + \frac{1}{2}a_xt^2.$$  

Now we compare this result to the expression we already obtained for $x$ (equation 2.19) and find:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2 = x_0 + \overline{v}_xt.$$  

For the derivation of equation 2.22 for the square of the velocity, we solve $v_x = v_{x0} + a_xt$ for the time, getting

$$t = \frac{v_x - v_{x0}}{a_x}.$$  

We then substitute into the expression for the position, which is equation 2.19:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$$

$$= x_0 + v_{x0}\left(\frac{v_x - v_{x0}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_x - v_{x0}}{a_x}\right)^2$$

$$= x_0 + \frac{v_xv_{x0} - v_{x0}^2}{a_x} + \frac{1}{2} \left( \frac{v_x^2 - v_{x0}^2}{a_x} - 2v_xv_{x0} \right).$$

Next, we subtract $x_0$ from both sides of the equation and then multiply by $a_x$:

$$a_x(x - x_0) = v_xv_{x0} - v_{x0}^2 + \frac{1}{2}(v_x^2 + v_{x0}^2 - 2v_xv_{x0})$$

$$\Rightarrow a_x(x - x_0) = \frac{1}{2}v_x^2 - \frac{1}{2}v_{x0}^2$$

$$\Rightarrow v_x^2 = v_{x0}^2 + 2a_x(x - x_0).$$

Here are the five kinematical equations we have obtained for the special case of motion with constant acceleration (where initial time when $x = x_0$, $v = v_0$ has been chosen to be 0):

(i)  \hspace{1cm} x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2

(ii) \hspace{1cm} x = x_0 + \overline{v}_xt

(iii) \hspace{1cm} v_x = v_{x0} + a_xt

(iv) \hspace{1cm} \overline{v}_x = \frac{1}{2}(v_x + v_{x0})

(v) \hspace{1cm} v_x^2 = v_{x0}^2 + 2a_x(x - x_0)

These five equations allow us to solve many kinds of problems for motion in one dimension with constant acceleration. However, remember that if the acceleration is not constant, these equations will not give the correct solutions.

Many real-life problems involve motion along a straight line with constant acceleration. In these situations, equations 2.23 provide the template for answering any question about the motion. The following solved problem and example will illustrate how useful these kinematical equations are. However, keep in mind that physics is not simply about finding an appropriate equation and plugging in numbers, but is instead about understanding concepts. Only if you understand the underlying ideas will you be able to extrapolate from specific examples to becoming skilled at solving more general problems.

**SOLVED PROBLEM 2.2 Airplane Takeoff**

As an airplane rolls down a runway to reach takeoff speed, it is accelerated by its jet engines. On one particular flight, one of the authors of this book measured the acceleration produced by the plane's jet engines. Figure 2.15 shows the measurements.
You can see that the assumption of constant acceleration is not quite correct in this case. However, an average acceleration of \( a_x = 4.3 \text{ m/s}^2 \) over the 18.4 s (measured with a stopwatch) it took the airplane to take off is a good approximation.

**PROBLEM**

Assuming a constant acceleration of \( a_x = 4.3 \text{ m/s}^2 \) starting from rest, what is the airplane's takeoff velocity after 18.4 s? How far down the runway has the plane moved by the time it takes off?

**SOLUTION**

**THINK**

An airplane moving along a runway prior to takeoff is a nearly perfect example of one-dimensional accelerated motion. Because we are assuming constant acceleration, we know that the velocity increases linearly with time, and the displacement increases as the second power of time. Since the plane starts from rest, the initial value of the velocity is 0. As usual, we can define the origin of our coordinate system at any location; it is convenient to locate it at the point of the plane's standing start.

**SKETCH**

The sketch in Figure 2.16 shows how we expect the velocity and displacement to increase for this case of constant acceleration, where the initial conditions are set at \( v_{x0} = 0 \) and \( x_0 = 0 \). Note that no scales have been placed on the axes, because displacement, velocity, and acceleration are measured in different units. Thus, the points at which the three curves intersect are completely arbitrary.

**RESEARCH**

Finding the takeoff velocity is actually a straightforward application of equation 2.23(iii):

\[
\begin{align*}
v_x &= v_{x0} + a_x t.
\end{align*}
\]

Similarly, the distance down the runway that the airplane moves before taking off can be obtained from equation 2.23(i):

\[
\begin{align*}
x &= x_0 + v_{x0} t + \frac{1}{2} a_x t^2.
\end{align*}
\]

**SIMPLIFY**

The airplane accelerates from a standing start, so the initial velocity is \( v_{x0} = 0 \), and by our choice of coordinate system origin, we have set \( x_0 = 0 \). Therefore, the equations for takeoff velocity and distance simplify to

\[
\begin{align*}
v_x &= a_x t, \\
x &= \frac{1}{2} a_x t^2.
\end{align*}
\]

Continued—
CALCULATE
The only thing left to do is to put in the numbers:

\[ v_x = (4.3 \text{ m/s}^2)(18.4 \text{ s}) = 79.12 \text{ m/s} \]
\[ x = \frac{1}{2} (4.3 \text{ m/s}^2)(18.4 \text{ s})^2 = 727.904 \text{ m}. \]

ROUND
The acceleration was specified to two significant digits, and the time to three. Multiplying these two numbers must result in an answer that has two significant digits. Our final answers are therefore

\[ v_x = 79 \text{ m/s} \]
\[ x = 7.3 \cdot 10^2 \text{ m}. \]

Note that the measured time to takeoff of 18.4 s was probably not actually that precise. If you have ever tried to determine the moment at which a plane starts to accelerate down the runway, you will have noticed that it is almost impossible to determine that point in time with an accuracy to 0.1 s.

DOUBLE-CHECK
As this book has repeatedly stressed, the most straightforward check of any answer to a physics problem is to make sure the units fit the situation. This is the case here, because we obtained displacement in units of meters and velocity in units of meters per second.

Solved problems in the rest of this book may sometimes skip this simple test; however, if you want to do a quick check for algebraic errors in your calculations, it can be valuable to first look at the units of the answer.

Now let’s see if our answers have the appropriate orders of magnitude. A takeoff displacement of 730 m (~0.5 mi) is reasonable, because it is on the order of the length of an airport runway. A takeoff velocity of \( v_x = 79 \text{ m/s} \) translates into

\[ (79 \text{ m/s})(1 \text{ mi/1609 m})(3600 \text{ s/1 h}) \approx 180 \text{ mph}. \]

This answer also appears to be in the right ballpark.

ALTERNATIVE SOLUTION
Many problems in physics can be solved in several ways, because it is often possible to use more than one relationship between the known and unknown quantities. In this case, once we obtained the final velocity, we could use this information and solve kinematical equation 2.23(v) for \( x \). This results in

\[ v_x^2 = v_0^2 + 2a_x (x - x_0) \Rightarrow \]
\[ x = x_0 + \frac{v_x^2 - v_0^2}{2a_x} = 0 + \frac{(79 \text{ m/s})^2}{2(4.3 \text{ m/s}^2)} = 7.3 \cdot 10^2 \text{ m}. \]

Thus, we arrive at the same answer for the distance in a different way, giving us additional confidence that our solution makes sense.

Solved Problem 2.2 was a fairly easy one; solving it amounted to little more than plugging in numbers. Nevertheless, it shows that the kinematical equations we derived can be applied to real-world situations and lead to answers that have physical meaning. The following short example, this time from motor sports, addresses the same concepts of velocity and acceleration, but in a slightly different light.

EXAMPLE 2.4 Top Fuel Racing

Accelerating from rest, a top fuel race car (Figure 2.17) can reach 333.2 mph (= 148.9 m/s), a record established in 2003, at the end of a quarter mile (= 402.3 m). For this example, we will assume constant acceleration.
Problem 1
What is the value of the race car’s constant acceleration?

Solution 1
Since the initial and final values of the velocity are given and the distance is known, we are looking for a relationship between these three quantities and the acceleration, the unknown. In this case, it is most convenient to use kinematics equation 2.23(v) and solve for the acceleration, $a_x$:
\[
v_f^2 = v_i^2 + 2a_x(x - x_0) \Rightarrow a_x = \frac{v_f^2 - v_i^2}{2(x - x_0)} = \frac{(148.9 \text{ m/s})^2}{2(402.3 \text{ m})} = 27.6 \text{ m/s}^2.
\]

Problem 2
How long does it take the race car to complete a quarter-mile run from a standing start?

Solution 2
Because the final velocity is 148.9 m/s, the average velocity is [using equation 2.23(iv)]:
\[
\bar{v}_x = \frac{1}{2}(148.9 \text{ m/s} + 0) = 74.45 \text{ m/s}.
\]
Relating this average velocity to the displacement and time using equation 2.23(ii), we obtain:
\[
x = x_0 + \bar{v}_x t \Rightarrow t = \frac{x - x_0}{\bar{v}_x} = \frac{402.3 \text{ m}}{74.45 \text{ m/s}} = 5.40 \text{ s}.
\]
Note that we could have obtained the same result by using kinematics equation 2.23(iii), because we already calculated the acceleration in Solution 1.

If you are a fan of top fuel racing, however, you know that the real record time for the quarter mile is slightly lower than 4.5 s. The reason our calculated answer is somewhat higher is that our assumption of constant acceleration is not quite correct. The acceleration of the car at the beginning of the race is actually higher than the value we calculated above, and the actual acceleration is lower than our value toward the end of the race.

Free Fall
The acceleration due to the gravitational force is constant, to a good approximation, near the surface of Earth. If this statement is true, it must have observable consequences. Let’s assume it is true and work out the consequences for motion of objects under the influence of a gravitational attraction to Earth. Then we’ll compare our results with experimental observations and see if constant acceleration due to gravity makes sense.

The acceleration due to gravity near the surface of the Earth has the value $g = 9.81 \text{ m/s}^2$. We call the vertical axis the $y$-axis and define the positive direction as up. Then the acceleration vector $\vec{a}$ has only a nonzero $y$-component, which is given by
\[
a_y = -g. \tag{2.24}
\]
This situation is a specific application of motion with constant acceleration, which we discussed earlier in this section. We modify equations 2.23 by substituting for acceleration from equation 2.24. We also use $y$ instead of $x$ to indicate that the displacement takes place in the $y$-direction. We obtain:
\[
\begin{align*}
(i) & \quad y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \\
(ii) & \quad y = y_0 + \bar{v}_y t \\
(iii) & \quad v_y = v_{y0} - gt \\
(iv) & \quad \bar{v}_y = \frac{1}{2}(v_y + v_{y0}) \\
(v) & \quad v_y^2 = v_{y0}^2 - 2g(y - y_0)
\end{align*}
\tag{2.25}
\]
Motion under the sole influence of a gravitational acceleration is called free fall, and equations 2.25 allow us to solve problems for objects in free fall.
Now let’s consider an experiment that tested the assumption of constant gravitational acceleration. The authors went to the top of a building of height 12.7 m and dropped a computer from rest ($v_{y0} = 0$) under controlled conditions. The computer’s fall was recorded by a digital video camera. Because the camera records at 30 frames per second, we know the time information. Figure 2.18 displays 14 frames, equally spaced in time, from this experiment, with the time after release marked on the horizontal axis for each frame. The yellow curve superimposed on the frames has the form

$$y(t) = 12.7 - \frac{1}{2}(9.81 m/s^2)t^2,$$

which is what we expect for initial conditions $y_0 = 12.7 m$, $v_{y0} = 0$ and the assumption of a constant acceleration, $a_y = -9.81 m/s^2$. As you can see, the computer’s fall follows this curve almost perfectly. This agreement is, of course, not a conclusive proof, but it is a strong indication that the gravitational acceleration is constant near the surface of Earth, and that it has the stated value.

In addition, the value of the gravitational acceleration is the same for all objects. This is by no means a trivial statement. Objects of different sizes and masses, if released from the same height, should hit the ground at the same time. Is this consistent with our everyday experience? Well, not quite! In a common lecture demonstration, a feather and a coin are dropped from the same height. It is easy to observe that the coin reaches the floor first, while the feather slowly floats down. This difference is due to air resistance. If this experiment is done in an evacuated glass tube, the coin and the feather fall at the same rate. We will return to air resistance in Chapter 4, but for now we can conclude that the gravitational acceleration near the surface of Earth is constant, has the absolute value of $g = 9.81 m/s^2$, and is the same for all objects provided we can neglect air resistance. In Chapter 4, we will examine the conditions under which the assumption of zero air resistance is justified.

To help you understand the answer to the in-class exercise, consider throwing a ball straight up, as illustrated in Figure 2.19. In Figure 2.19a, the ball is thrown upward with a velocity $v_1$, as shown in Figure 2.19. The ball reaches a maximum height of $h$. What is the ratio of the speed of the ball, $v_2$, at $y = h/2$ in Figure 2.19b, to the initial upward speed of the ball, $v_1$, at $y = 0$ in Figure 2.19a?

a) $v_2/v_1 = 0$

b) $v_2/v_1 = 0.50$

c) $v_2/v_1 = 0.71$

d) $v_2/v_1 = 0.75$

e) $v_2/v_1 = 0.90$

![Figure 2.18](image_url)

**FIGURE 2.18** Free-fall experiment: dropping a computer off the top of a building.
EXAMPLE 2.5 Reaction Time

It takes time for a person to react to any external stimulus. For example, at the beginning of a 100-m dash in a track-and-field meet, a gun is fired by the starter. A slight time delay occurs before the runners come out of the starting blocks, due to their finite reaction time. In fact, it counts as a false start if a runner leaves the blocks less than 0.1 s after the gun is fired. Any shorter time indicates that the runner has “jumped the gun.”

There is a simple test, shown in Figure 2.20, that you can perform to determine your reaction time. Your partner holds a meter stick, and you get ready to catch it when your partner releases it, as shown in the left frame of the figure. From the distance \( h \) that the meter stick falls after it is released until you grab it (shown in the right frame), you can determine your reaction time.

PROBLEM
If the meter stick falls 0.20 m before you catch it, what is your reaction time?

SOLUTION
This situation is a free-fall scenario. For these problems, the solution invariably comes from one of equations 2.25. The problem we want to solve here involves the time as an unknown.

We are given the displacement, \( h = y_0 - y \). We also know that the initial velocity of the meter stick is zero because it is released from rest. We can use kinematical equation 2.25(i): \( y = y_0 + v_0 t - \frac{1}{2} g t^2 \). With \( h = y_0 - y \) and \( v_{0,y} = 0 \), this equation becomes

\[
y = y_0 - \frac{1}{2} g t^2
\]

\[
\Rightarrow h = \frac{1}{2} g t^2
\]

\[
\Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 0.20 \text{ m}}{9.81 \text{ m/s}^2}} = 0.20 \text{ s.}
\]

Your reaction time was 0.20 s. This reaction time is typical. For comparison, when Usain Bolt established a world record of 9.69 s for the 100-m dash in August 2008, his reaction time was measured to be 0.165 s.

Let’s consider one more free-fall scenario, this time with two moving objects.
**2.4 In-Class Exercise**

If the reaction time of person B determined with the meter stick method is twice as long as that of person A, then the displacement $h_B$ measured for person B relative to the displacement $h_A$ for person A is

a) $h_B = 2h_A$

b) $h_B = \frac{1}{2}h_A$

c) $h_B = \sqrt{2}h_A$

d) $h_B = 4h_A$

e) $h_B = \sqrt{\frac{1}{2}}h_A$

**SOLVED PROBLEM 2.3 Melon Drop**

Suppose you decide to drop a melon from rest from the first observation platform of the Eiffel Tower. The initial height $h$ from which the melon is released is 58.3 m above the head of your French friend Pierre, who is standing on the ground right below you. At the same instant you release the melon, Pierre shoots an arrow straight up with an initial velocity of 25.1 m/s. (Of course, Pierre makes sure the area around him is cleared and gets out of the way quickly after he shoots his arrow.)

**PROBLEM**

(a) How long after you drop the melon will the arrow hit it? (b) At what height above Pierre's head does this collision occur?

**SOLUTION**

**THINK**

At first sight, this problem looks complicated. We will solve it using the full set of steps and then examine a shortcut we could have taken. Obviously, the dropped melon is in a free fall. However, because the arrow is shot straight up, the arrow is also in free fall, only with an upward initial velocity.

**SKETCH**

We set up our coordinate system with the $y$-axis pointing vertically up, as is conventional, and we locate the origin of the coordinate system at Pierre's head (Figure 2.21). Thus, the arrow is released from an initial position $y = 0$, and the melon from $y = h$.

**RESEARCH**

We use the subscript $m$ for the melon and $a$ for the arrow. We start from the general free-fall equation, $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$, and use the initial conditions given for the melon ($v_{y0} = 0$, $y_0 = h = 58.3$ m) and for the arrow ($v_{a0} = v_{a0} = 25.1$ m/s, $y_{a0} = 0$) to set up the two equations of free-fall motion:

$$y_m(t) = h - \frac{1}{2}gt^2$$

$$y_a(t) = v_{a0}t - \frac{1}{2}gt^2.$$  

The key insight is that at $t_c$, the moment when the melon and arrow collide, their coordinates are identical:

$$y_a(t_c) = y_m(t_c).$$

**SIMPLIFY**

Inserting $t_c$ into the two equations of motion and setting them equal results in

$$h - \frac{1}{2}gt^2_c = v_{a0}t_c - \frac{1}{2}gt^2_c \Rightarrow$$

$$h = v_{a0}t_c \Rightarrow$$

$$t_c = \frac{h}{v_{a0}}.$$  

We can now insert this value for the time of collision in either of the two free-fall equations and obtain the height above Pierre's head at which the collision occurs. We select the equation for the melon:

$$y_m(t_c) = h - \frac{1}{2}gt^2_c.$$  

**CALCULATE**

(a) All that is left to do is to insert the numbers given for the height of release of the melon and the initial velocity of the arrow, which results in

$$t_c = \frac{58.3 \text{ m}}{25.1 \text{ m/s}} = 2.32271 \text{ s}$$

for the time of impact.
(b) Using the number we obtained for the time, we find the position at which the collision occurs:

\[ y_m(t_c) = 58.3 \text{ m} - \frac{1}{2}(9.81 \text{ m/s}^2)(2.32271 \text{ s})^2 = 31.8376 \text{ m}. \]

**ROUND**

Since the initial values of the release height and the arrow velocity were given to three significant figures, we have to limit our final answers to three digits. Thus, the arrow will hit the melon after 2.32 s, and this will occur at a position 31.8 m above Pierre’s head.

**DOUBLE-CHECK**

Could we have obtained the answers in an easier way? Yes, if we had realized that both melon and arrow fall under the influence of the same gravitational acceleration, and thus their free-fall motion does not influence the distance between them. This means that the time it takes them to meet is simply the initial distance between them divided by their initial velocity difference. With this realization, we could have written \( t = \frac{h}{v_{a0}} \) right away and been done. However, thinking in terms of relative motion in this way takes some practice, and we will return to it in more detail in the next chapter.

Figure 2.22 shows the complete graph of the positions of arrow and melon as functions of time. The dashed portions of both graphs indicate where the arrow and melon would have gone, had they not collided.

**ADDITIONAL QUESTION**

What are the velocities of melon and arrow at the moment of the collision?

**SOLUTION**

We obtain the velocity by taking the time derivative of the position. For arrow and melon, we get

\[ y_m(t) = h - \frac{1}{2}gt^2 \Rightarrow v_m(t) = \frac{dy_m(t)}{dt} = -gt \]

\[ y_a(t) = v_{a0}t - \frac{1}{2}gt^2 \Rightarrow v_a(t) = \frac{dy_a(t)}{dt} = v_{a0} - gt. \]

Now, inserting the time of the collision, 2.32 s, will produce the answers. Note that, unlike the positions of the arrow and the melon, the velocities of the two objects are not the same right before contact!

\[ v_m(t_c) = -(9.81 \text{ m/s}^2)(2.32 \text{ s}) = -22.8 \text{ m/s} \]

\[ v_a(t_c) = (25.1 \text{ m/s}) - (9.81 \text{ m/s}^2)(2.32 \text{ s}) = 2.34 \text{ m/s}. \]

Furthermore, you should note that the difference between the two velocities is still 25.1 m/s, just as it was at the beginning of the trajectories.

We finish this problem by plotting (Figure 2.23) the velocities as a function of time. You can see that the arrow starts out with a velocity that is 25.1 m/s greater than that of the melon. As time progresses, arrow and melon experience the same change in velocity under the influence of gravity, meaning that their velocities maintain the initial difference.

**FIGURE 2.23** Velocities of the arrow (red curve) and melon (green curve) as a function of time.

---

### 2.5 In-Class Exercise

If the melon in Solved Problem 2.3 is thrown straight up with an initial velocity of 5 m/s at the same time that the arrow is shot upward, how long does it take before the collision occurs?

- a) 2.32 s
- b) 2.90 s
- c) 1.94 s
- d) They do not collide before the melon hits the ground.

### 2.3 Self-Test Opportunity

As you can see from the answer to Solved Problem 2.3, the velocity of the arrow is only 2.34 m/s when it hits the melon. This means that by the time the arrow hits the melon, its initial velocity has dropped substantially because of the effect of gravity. Suppose that the initial velocity of the arrow was smaller by 5.0 m/s. What would change? Would the arrow still hit the melon?
Chapter 2 Motion in a Straight Line

2.8 Reducing Motion in More Than One Dimension to One Dimension

Motion in one spatial dimension is not all there is to kinematics. We can also investigate more general cases, in which objects move in two or three spatial dimensions. We will do this in the next few chapters. However, in some cases, motion in more than one dimension can be reduced to one-dimensional motion. Let’s consider a very interesting case of motion in two dimensions for which each segment can be described by motion in a straight line.

Example 2.6 Aquathlon

The triathlon is a sporting competition that was invented by the San Diego Track Club in the 1970s and became an event in the Sydney Olympic Games in 2000. Typically, it consists of a 1.5-km swim, followed by a 40-km bike race, and finishing with a 10-km foot race. To be competitive, athletes must be able to swim the 1.5-km distance in less than 20 minutes, do the 40-km bike race in less than 70 minutes, and run the 10-km race in less than 35 minutes.

However, for this example, we’ll consider a competition where thinking is rewarded in addition to athletic prowess. The competition consists of only two legs: a swim followed by a run. (This competition is sometimes called an aquathlon.)

Athletes start a distance \( b = 1.5 \) km from shore, and the finish line is a distance \( a = 3 \) km to the left along the shoreline (Figure 2.24). Suppose you can swim with a speed of \( v_1 = 3.5 \) km/h and can run across the sand with a speed of \( v_2 = 14 \) km/h.

Problem

What angle \( \theta \) will result in the shortest finish time under these conditions?

Solution

Clearly, the dotted red line marks the shortest distance between start and finish. This distance is \( \sqrt{a^2 + b^2} = \sqrt{1.5^2 + 3^2} \) km = 3.354 km. Because this entire path is in water, the time it takes to complete the race in this way is

\[
t_{\text{red}} = \frac{\sqrt{a^2 + b^2}}{v_1} = \frac{3.354 \text{ km}}{3.5 \text{ km/h}} = 0.958 \text{ h}.
\]

Because you can run faster than you can swim, we can also try the approach indicated by the dotted blue line: swim straight to shore and then run. This takes

\[
t_{\text{blue}} = \frac{b}{v_1} + \frac{a}{v_2} = \frac{1.5 \text{ km}}{3.5 \text{ km/h}} + \frac{3 \text{ km}}{14 \text{ km/h}} = 0.643 \text{ h}.
\]

Thus, the blue path is better than the red one. But is it the best? To answer this question, we need to search the angle interval from 0 (blue path) to \( \tan^{-1}(3/1.5) \) = 63.43° (red path). Let’s consider the green path, with an arbitrary angle \( \theta \) with respect to the straight line to shore (that is, the normal to the shoreline). On the green path, you have to swim a distance of \( \sqrt{a_2^2 + b^2} \) and then run a distance of \( a_2 \), as indicated in Figure 2.24. The total time for this path is

\[
t = \frac{\sqrt{a_2^2 + b^2}}{v_1} + \frac{a_2}{v_2}.
\]

To find the minimum time, we can express the time in terms of the distance \( a_1 \) only, take the derivative of that time with respect to that distance, set that derivative equal to zero, and solve for the distance. Using \( a_1 \), we can then calculate the angle \( \theta \) that the athlete must swim. We can express the distance \( a_2 \) in terms of the given distance \( a \) and \( a_1 \):

\[
a_2 = a - a_1.
\]
We can then express the time to complete the race in terms of $a_1$:

$$t(a_1) = \frac{\sqrt{a_1^2 + b^2}}{v_1} + \frac{a - a_1}{v_2}. \tag{2.26}$$

Taking the derivative with respect to $a_1$ and setting that result equal to zero gives us

$$\frac{dt(a_1)}{da_1} = \frac{a_1}{v_1 \sqrt{a_1^2 + b^2}} - \frac{1}{v_2} = 0.$$ 

Rearranging yields

$$\frac{a_1}{v_1 \sqrt{a_1^2 + b^2}} = \frac{1}{v_2},$$

which we can rewrite as

$$\frac{v_1 \sqrt{a_1^2 + b^2}}{a_1} = v_2.$$ 

Squaring both sides and rearranging terms produces

$$v_1^2 \left( a_1^2 + b^2 \right) = a_1^2 v_1^2 \Rightarrow a_1^2 v_1^2 + b^2 v_1^2 = a_1^2 v_2^2 \Rightarrow b^2 v_1^2 = a_1^2 \left( v_2^2 - v_1^2 \right).$$

Solving for $a_1$ gives us

$$a_1 = \frac{b v_1}{\sqrt{v_2^2 - v_1^2}}.$$ 

In Figure 2.24, we can see that $\tan \theta = a_1/b$, so we can write

$$\tan \theta = \frac{a_1}{b} = \frac{\sqrt{v_2^2 - v_1^2}}{b} = \frac{v_1}{\sqrt{v_2^2 - v_1^2}}.$$ 

We can simplify this result by looking at the triangle for the angle $\theta$ in Figure 2.25. The Pythagorean theorem tells us that the hypotenuse of the triangle is

$$\sqrt{v_1^2 + v_2^2 - v_1^2} = v_2.$$ 

We can then write

$$\sin \theta = \frac{v_1}{v_2}.$$ 

This result is very interesting because the distances $a$ and $b$ do not appear in it at all! Instead the sine of the optimum angle is simply the ratio of the speeds in water and on land. For the given values of the two speeds, this angle is

$$\theta_m = \sin^{-1} \frac{3.5}{14} = 14.48^\circ.$$ 

Inserting $a_1 = b \tan \theta_m$ into equation 2.26, we find $t(\theta_m) = 0.629$ h. This time is approximately 49 seconds faster than swimming straight to shore and then running (blue path).

Strictly speaking, we have not quite shown that this angle results in the minimum time. To accomplish this, we also need to show that the second derivative of the time with respect to the angle is larger than zero. However, since we did find one extremum, and since its value is smaller than those of the boundaries, we know that this extremum is a true minimum.

Continued—
Finally, Figure 2.26 plots the time, in hours, needed to complete the race for all angles between 0° and 63.43°, indicated by the green curve. This plot is obtained by substituting \( a_1 = b \tan \theta \) into equation 2.26, which gives us

\[
t(\theta) = \sqrt{\frac{(b \tan \theta)^2 + b^2}{v_1}} + \frac{a - b \tan \theta}{v_2} = \frac{b \sec \theta}{v_1} + \frac{a - b \tan \theta}{v_2},
\]

using the identity \( \tan^2 \theta + 1 = \sec^2 \theta \). A vertical red line marks the maximum angle, corresponding to a straight-line swim from start to finish. The vertical blue line marks the optimum angle that we calculated, and the horizontal blue line marks the duration of the race for this angle.

Having completed Example 2.6, we can address a more complicated question: If the finish line is not at the shoreline, but at a perpendicular distance \( b \) away from the shoreline, as shown in Figure 2.27, what are the angles \( \theta_1 \) and \( \theta_2 \) that a competitor needs to select to achieve the minimum time?

We proceed in a fashion very similar to our approach in Example 2.6. However, now we have to realize that the time depends on two angles, \( \theta_1 \) and \( \theta_2 \). These two angles are not independent of each other. We can better see the relationship between \( \theta_1 \) and \( \theta_2 \) by changing the orientation of the lower triangle in Figure 2.27, as shown in Figure 2.28.

Now we see that the two right triangles \( a_1bc_1 \) and \( a_2bc_2 \) have a common side \( b \), which helps us relate the two angles to each other. We can express the time to complete the race as

\[
t = \frac{\sqrt{a_1^2 + b^2}}{v_1} + \frac{\sqrt{a_2^2 + b^2}}{v_2}.
\]

Again realizing that \( a_2 = a - a_1 \), we can write

\[
t(a_1) = \frac{\sqrt{a_1^2 + b^2}}{v_1} + \frac{\sqrt{(a - a_1)^2 + b^2}}{v_2}.
\]

Taking the derivative of the time to complete the race with respect to \( a_1 \) and setting that result equal to zero, we obtain

\[
\frac{dt}{da_1} = \frac{a_1}{v_1 \sqrt{a_1^2 + b^2}} - \frac{a - a_1}{v_2 \sqrt{(a - a_1)^2 + b^2}} = 0.
\]

We can rearrange this equation to get

\[
\frac{a_1}{\sqrt{a_1^2 + b^2}} = \frac{a - a_1}{\sqrt{(a - a_1)^2 + b^2}} = \frac{a_2}{\sqrt{a_2^2 + b^2}}.
\]

Looking at Figure 2.28 and referring to our previous result from Figure 2.25, we can see that

\[
\sin \theta_1 = \frac{a_1}{\sqrt{a_1^2 + b^2}}
\]

and

\[
\sin \theta_2 = \frac{a_2}{\sqrt{a_2^2 + b^2}}.
\]

We can insert these two results into the previous equation and finally find the shortest time to finish the race we require

\[
\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2}.
\]

(2.27)
What We Have Learned

We can now see that our previous result, where we forced the running to take place along the beach, is a special case of this more general result equation 2.27, with \( \theta_2 = 90^\circ \).

Just as for that special case, we find that the relationship between the angles does not depend on the values of the displacements \( a \) and \( b \), but depends only on the speeds with which the competitor can move in the water and on land. The angles are still related to \( a \) and \( b \), by the overall constraint that the competitor has to get from the start to the finish. However, for the path of minimum time, the change in direction at the boundary between water and land, as expressed by the two angles \( \theta_1 \) and \( \theta_2 \), is determined exclusively by the ratio of the speeds \( v_1 \) and \( v_2 \).

The initial condition specified that the perpendicular distance \( b \) from the starting point to the shoreline be the same as the perpendicular distance between the shoreline and the finish. We did this to keep the algebra relatively brief. However, in the final formula you see that there are no more references to \( b \); it has canceled out. Thus, equation 2.27 is even valid in the case that the two perpendicular distances have different values. The ratio of angles for the path of minimum time is exclusively determined by the two speeds in the different media.

Interestingly, we will encounter the same relationship between two angles and two speeds when we study light changing direction at the interface between two media through which light moves with different speeds. In Chapter 32, we will see that light also moves along the path of minimum time and that the result obtained in equation 2.27 is known as Snell's Law.

Finally, we make an observation that may appear trivial, but is not: If a competitor started at the point marked “Finish” in Figure 2.27 and ended up at the point marked “Start,” he or she would have to take exactly the same path as the one we just calculated for the reverse direction. Snell's Law holds in both directions.

**WHAT We HAVe LeArNeD | EXAM STUDY GUIDE**

- \( x \) is the \( x \)-component of the position vector.
  - Displacement is the change in position: \( \Delta x = x_2 - x_1 \).
- Distance is the absolute value of displacement, \( \ell = |\Delta x| \), and is a positive scalar for motion in one direction.
- The average velocity of an object in a given time interval is given by \( \bar{v}_x = \frac{\Delta x}{\Delta t} \).
- The \( x \)-component of (instantaneous) velocity vector is the derivative of the \( x \)-component of position vector as a function of time, \( v_x = \frac{dx}{dt} \).
- Speed is the absolute value of the velocity: \( v = |v_x| \).
- The \( x \)-component of (instantaneous) acceleration vector is the derivative of the \( x \)-component of the velocity vector as a function of time, \( a_x = \frac{dv_x}{dt} \).

- For constant accelerations, five kinematical equations describe motion in one dimension:
  1. \( x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2 \)
  2. \( v_x = v_{x0} + a_xt \)
  3. \( v_x^2 = v_{x0}^2 + 2a_x(x-x_0) \)
  4. \( v_x^2 = v_{x0}^2 + 2a_x(x-x_0) \)

- For situations involving free fall (constant acceleration), we replace the acceleration \( a \) with \(-g\) and \( x \) with \( y \) to obtain:
  1. \( y = y_0 + v_{y0}t - \frac{1}{2}gt^2 \)
  2. \( v_y = v_{y0} - gt \)
  3. \( v_y^2 = v_{y0}^2 - 2gt \)
  4. \( v_y^2 = v_{y0}^2 - 2gt \)
  5. \( v_y^2 = v_{y0}^2 - 2gt(y-y_0) \)

**KEY TERMS**

| mechanics, p. 36 | displacement, p. 37 | instantaneous velocity, p. 40 | instantaneous acceleration, p. 43 |
| kinematics, p. 36 | distance, p. 38 | speed, p. 42 | free fall, p. 51 |
| position vector, p. 37 | average velocity, p. 40 | average acceleration, p. 43 |
NEW SYMBOLS

\[ \Delta x = x_2 - x_1, \text{ displacement in one dimension} \]

\[ \bar{v}_x = \frac{\Delta x}{\Delta t}, \text{ average velocity in one dimension in a time interval } \Delta t \]

\[ v_x = \frac{dx}{dt}, \text{ instantaneous velocity in one dimension} \]

\[ \bar{a}_x = \frac{\Delta v}{\Delta t}, \text{ average acceleration in one dimension in a time interval } \Delta t \]

\[ a_x = \frac{dv_x}{dt}, \text{ instantaneous acceleration in one dimension} \]

ANSWERS TO SELF-TEST OPPORTUNITIES

2.1 The result would not change, because a shift in the origin of the coordinate system has no influence on the net displacements or distances.

2.2 This method is more precise for longer reaction times, because the slope of the curve decreases as a function of the height, \( h \). (In the graph, the slope at 0.1 s is indicated by the blue line, and that at 0.3 s by the green line.) So, a given uncertainty, \( \Delta h \), in measuring the height results in a smaller uncertainty, \( \Delta t \), in the value for the reaction time for longer reaction times.

2.3 The arrow would still hit the melon, but at the time of collision the arrow would already have negative velocity and thus be moving downward again. Thus, the melon would catch the arrow as it was falling. The collision would occur a bit later, after \( t = \frac{58.3 \text{ m}}{(20.1 \text{ m/s})} = 2.90 \text{ s} \). The altitude of the collision would be quite a bit lower, at \( y_m(t_c) = 58.3 \text{ m} - \frac{1}{2}(9.81 \text{ m/s}^2)(2.90 \text{ s})^2 = 1.70 \text{ m} \) above Pierre’s head.

PROBLEM-SOLVING PRACTICE

SOLVED PROBLEM 2.4 Racing with a Head Start

Cheri has a new Dodge Charger with a Hemi engine and has challenged Vince, who owns a tuned VW GTI, to a race at a local track. Vince knows that Cheris Charger is rated to go from 0 to 60 mph in 5.3 s, whereas his VW needs 7.0 s. Vince asks for a head start and Cheri agrees to give him exactly 1.0 s.

PROBLEM

How far down the track is Vince before Cheri gets to start the race? At what time does Cheri catch Vince? How far away from the start are they when this happens? (Assume constant acceleration for each car during the race.)

SOLUTION

THINK

This race is a good example of one-dimensional motion with constant acceleration. The temptation is to look over the kinematical equations 2.23 and see which one we can apply. However, we have to be a bit more careful here, because the time delay between Vince’s start and Cheri’s start adds a complication. In fact, if you try to solve this problem using the kinematical equations directly, you will not get the right answer. Instead, this problem requires some careful definition of the time coordinates for each car.

SKETCH

For our sketch, we plot time on the horizontal axis and position on the vertical axis. Both cars move with constant acceleration from a standing start, so we expect simple parabolas for their paths in this diagram.
Since Cheri’s car has the greater acceleration, her parabola (blue curve in Figure 2.29) has the larger curvature and thus the steeper rise. Therefore, it is clear that Cheri will catch Vince at some point, but it is not yet clear where this point is.

**Research**

We set up the problem quantitatively. We call the time delay before Cheri can start \( \Delta t \), and we use the indices (subscripts) \( C \) for Cheri’s Charger and \( V \) for Vince’s VW. We put the coordinate system’s origin at the starting line. Thus, both cars have initial position \( x_C(t = 0) = x_V(t = 0) = 0 \). Since both cars are at rest at the start, their initial velocities are zero. The equation of motion for Vince’s VW is

\[
x_V(t) = \frac{1}{2} a_V t^2.
\]

Here we use the symbol \( a_V \) for the acceleration of the VW. We can calculate its value from the 0 to 60 mph time given by the problem statement, but we will postpone this step until it is time to put in the numbers.

To obtain the equation of motion for Cheri’s Charger we have to be careful, because she is forced to wait \( \Delta t \) after Vince has taken off. We can account for this delay with a shifted time: \( t' = t - \Delta t \). Once \( t \) reaches the value of \( \Delta t \), the time \( t' \) has the value 0 and then Cheri can take off. So her Charger’s equation of motion is

\[
x_C(t') = \frac{1}{2} a_C (t' - t_\Delta t)^2 \quad \text{for} \quad t \geq \Delta t.
\]

Just like \( a_V \), the constant acceleration \( a_C \) for Cheri’s Charger will be evaluated below.

**Simplify**

When Cheri catches Vince, their coordinates will have the same value. We will call the time at which this happens \( t_\text{s} \), and the coordinate where it happens \( x_\text{s} = x(t_\text{s}) \). Because the two coordinates are the same, we have

\[
x_\text{s} = \frac{1}{2} a_V t_\text{s}^2 = \frac{1}{2} a_C (t_\text{s} - \Delta t)^2.
\]

We can solve this equation for \( t_\text{s} \) by dividing out the common factor of \( \frac{1}{2} \) and then taking the square root of both sides of the equation:

\[
\sqrt{a_V} t_\text{s} = \sqrt{a_C} (t_\text{s} - \Delta t) \Rightarrow
t_\text{s} = \sqrt{a_C - a_V} \frac{\Delta t}{\sqrt{a_C - a_V}}.
\]

Why do we use the positive root and discard the negative root here? The negative root would lead to a physically impossible solution: We are interested in the time that the two cars meet after they have left the start, not in a negative value that would imply a time before they left.

**Calculate**

Now we can get a numerical answer for each of the questions that were asked. First, let’s figure out the values of the cars’ constant accelerations from the 0 to 60 mph specifications given. We use \( a = (v_f - v_i)/t \) and get

\[
a_V = \frac{60 \text{ mph}}{7.0 \text{ s}} = \frac{26.8167 \text{ m/s}}{7.0 \text{ s}} = 3.83095 \text{ m/s}^2
\]

\[
a_C = \frac{60 \text{ mph}}{5.3 \text{ s}} = \frac{26.8167 \text{ m/s}}{5.3 \text{ s}} = 5.05975 \text{ m/s}^2.
\]

Again, we postpone rounding our results until we have completed all steps in our calculations. However, with the values for the accelerations, we can immediately calculate how far Vince travels during the time \( \Delta t = 1.0 \text{ s} \):

\[
x_V(1.0 \text{ s}) = \frac{1}{2} (3.83095 \text{ m/s}^2)(1.0 \text{ s})^2 = 1.91548 \text{ m}.
\]

Continued—
Now we can calculate the time when Cheri catches up to Vince:

\[
t_{\text{Cheri}} = \frac{\Delta t \sqrt{a_v}}{\sqrt{a_V} - a_v} = \frac{(1.0 \text{ s})\sqrt{5.05975 \text{ m/s}^2}}{\sqrt{5.05975 \text{ m/s}^2} - \sqrt{3.83095 \text{ m/s}^2}} = 7.70055 \text{ s}.
\]

At this time, both cars have traveled to the same point. Thus, we can insert this value into either equation of motion to find this position:

\[
x_{\text{Cheri}} = \frac{1}{2}a_v t_{\text{Cheri}}^2 = \frac{1}{2}(3.83095 \text{ m/s}^2)(7.70055 \text{ s})^2 = 113.585 \text{ m}.
\]

**ROUND**
The initial data were specified only to a precision of two significant digits. Rounding our results to the same precision, we finally arrive at our answers: Vince receives a head start of 1.9 m, and Cheri will catch him after 7.7 s. At that time, they will be \(1.1 \times 10^2\) m into their race.

**DOUBLE-CHECK**
It may seem strange to you that Vince’s car was able to move only 1.9 m, or approximately half the length of the car, during the first second. Have we done anything wrong in our calculation? The answer is no; from a standing start, cars move only a comparatively short distance during the first second of acceleration. The following solved problem contains visible proof for this statement.

Figure 2.30 shows a plot of the equations of motion for both cars, this time with the proper units.

**SOLVED PROBLEM 2.5 Accelerating Car**

**PROBLEM**
You are given the image sequence shown in Figure 2.31, and you are told that there is a time interval of 0.333 s between successive frames. Can you determine how fast this car (2007 Ford Escape Hybrid, length 174.9 in) was accelerating from rest? Also, can you give an estimate for the time it takes this car to go from 0 to 60 mph?

**SOLUTION**

**THINK**

Acceleration is measured in dimensions of length per time per time. To find a number for the value of the acceleration, we need to know the time and length scales in Figure 2.31. The time scale is straightforward because we were given the information that 0.333 s passes between successive frames. We can get the length scale from the specified vehicle dimensions. For example, if we focus on the length of the car and compare it to the overall width of the frame, we can find the distance the car covered between the first and the last frame (which are 3.000 s apart).

**SKETCH**

We draw vertical yellow lines over Figure 2.31, as shown in Figure 2.32. We put the center of the car at the line between the front and rear windows (the exact location is irrelevant, as long as we are consistent). Now we can use a ruler and measure the perpendicular distance between the two yellow lines, indicated by the yellow double-headed arrow in the figure. We can also measure the length of the car, as indicated by the red double-headed arrow.

**RESEARCH**

Dividing the measured length of the yellow double-headed arrow by that of the red one gives us a ratio of 3.474. Since the two vertical yellow lines mark the position of the center of the car at 0.000 s and 3.000 s, we know that the car covered a distance of 3.474 car lengths in this time interval. The car length was given as 174.9 in = 4.442 m. Thus, the
total distance covered is \( d = 3.474 \cdot \ell_{\text{car}} = 3.474 \cdot 4.442 \text{ m} = 15.4315 \text{ m} \) (remember that we round to the proper number of significant digits at the end).

**Simplify**

We have two choices on how to proceed. The first one is more complicated: We could measure the position of the car in each frame and then use difference formulas like those in Example 2.3. The other and much faster way to proceed is to assume constant acceleration and then use the measurements of the car’s positions in only the first and last frames. We’ll use the second way, but in the end we’ll need to double-check that our assumption of constant acceleration is justified.

For a constant acceleration from a standing start, we simply have

\[
x = x_0 + \frac{1}{2}at^2 \quad \Rightarrow
\]

\[
d = x - x_0 = \frac{1}{2}at^2 \quad \Rightarrow
\]

\[
a = \frac{2d}{t^2}.
\]

This is the acceleration we want to find. Once we have the acceleration, we can give an estimate for the time from 0 to 60 mph by using \( v_x = v_{x0} + at \quad \Rightarrow \quad t = (v_x - v_{x0})/a \). A standing start means \( v_{x0} = 0 \), and so we have

\[
t(0-60 \text{ mph}) = \frac{60 \text{ mph}}{a}.
\]

**Calculate**

We insert the numbers for the acceleration:

\[
a = \frac{2d}{t^2} = \frac{2(15.4315 \text{ m})}{(3.000 \text{ s})^2} = 3.42922 \text{ m/s}^2.
\]

Then, for the time from 0 to 60 mph, we obtain

\[
t(0-60 \text{ mph}) = \frac{60 \text{ mph}}{a} = \frac{(60 \text{ mph})(1609 \text{ m/mi})(1 \text{ h}/3600 \text{ s})}{3.42922 \text{ m/s}^2} = 7.82004 \text{ s}.
\]

**Round**

The length of the car sets our length scale, and it was given to four significant figures. The time was given to three significant figures. Are we entitled to quote our results to three significant digits? The answer is no, because we also performed measurements on Figure 2.32, which are probably accurate to only two digits at best. In addition, you can see field-of-vision and lens distortions in the image sequence: In the first few frames, you see a little bit of the front of the car, and in the last few frames, a little bit of the back. Taking all this into account, our results should be quoted to two significant figures. So our final answer for the acceleration is

\[
a = 3.4 \text{ m/s}^2.
\]

For the time from 0 to 60 mph, we give

\[
t(0-60 \text{ mph}) = 7.8 \text{ s}.
\]

**Double-Check**

The numbers we have found for the acceleration and the time from 0 to 60 mph are fairly typical for cars or small SUVs; see also Solved Problem 2.4. Thus, we have confidence that we are not off by orders of magnitude.

What we must also double-check, however, is the assumption of constant acceleration. For constant acceleration from a standing start, the points \( x(t) \) should fall on a parabola \( x(t) = \frac{1}{2}at^2 \). Therefore, if we plot \( x \) on the horizontal axis and \( t \) on the vertical axis as in Figure 2.33, the points \( t(x) \) should follow a square root dependence: \( t(x) = \sqrt{2x/a} \).

This functional dependence is shown by the red curve in Figure 2.33. We can see that the same point of the car is hit by the curve in every frame, giving us confidence that the assumption of constant acceleration is reasonable.
MULTIPLE-CHOICE QUESTIONS

2.1 Two athletes jump straight up. Upon leaving the ground, Adam has half the initial speed of Bob. Compared to Adam, Bob jumps
a) 0.50 times as high. d) three times as high.
b) 1.41 times as high. e) four times as high.
c) twice as high.

2.2 Two athletes jump straight up. Upon leaving the ground, Adam has half the initial speed of Bob. Compared to Adam, Bob is in the air
a) 0.50 times as long. d) three times as long.
b) 1.41 times as long. e) four times as long.
c) twice as long.

2.3 A car is traveling due west at 20.0 m/s. Find the velocity of the car after 3.00 s if its acceleration is 1.0 m/s² due west. Assume the acceleration remains constant.
A) 17.0 m/s west c) 23.0 m/s west e) 11.0 m/s south
B) 17.0 m/s east d) 23.0 m/s east

2.4 A car is traveling due west at 20.0 m/s. Find the velocity of the car after 37.00 s if its constant acceleration is 1.0 m/s² due east. Assume the acceleration remains constant.
A) 17.0 m/s west c) 23.0 m/s west e) 11.0 m/s south
B) 17.0 m/s east d) 23.0 m/s east

2.5 An electron, starting from rest and moving with a constant acceleration, travels 1.0 cm in 2.0 ms. What is the magnitude of this acceleration?
A) 25 km/s² c) 15 km/s² e) 5.0 km/s²
B) 20 km/s² d) 10 km/s²

QUESTIONS

2.11 Consider three ice skaters: Anna moves in the positive x-direction without reversing. Bertha moves in the negative x-direction without reversing. Christine moves in the positive x-direction and then reverses the direction of her motion. For which of these skaters is the magnitude of the average velocity smaller than the average speed over some time interval?

2.12 You toss a small ball vertically up in the air. How are the velocity and acceleration vectors of the ball oriented with respect to one another during the ball's flight up and down?

2.13 After you apply the brakes, the acceleration of your car is in the opposite direction to its velocity. If the acceleration of your car remains constant, describe the motion of your car.

2.14 Two cars are traveling at the same speed, and the drivers hit the brakes at the same time. The deceleration of one car is double that of the other. By what factor does the time required for that car to come to a stop compare with that for the other car?

2.15 If the acceleration of an object is zero and its velocity is nonzero, what can you say about the motion of the object? Sketch velocity versus time and acceleration versus time graphs for your explanation.

2.16 Can an object's acceleration be in the opposite direction to its motion? Explain.

2.17 You and a friend are standing at the edge of a snow-covered cliff. At the same time, you both drop a snowball over the edge of the cliff. Your snowball is twice as heavy as your friend's. Neglect air resistance. (a) Which snowball will hit the ground first? (b) Which snowball will have the greater speed?

2.18 You and a friend are standing at the edge of a snow-covered cliff. At the same time, you throw a snowball straight upward with a speed of 8.0 m/s over the edge of the cliff and your friend throws a snowball straight downward over the edge of the cliff with the same speed. Your snowball is twice as heavy as your friend's. Neglecting air resistance, which snowball will hit the ground first, and which will have the greater speed?
2.19 A car is slowing down and comes to a complete stop. The figure shows an image sequence of this process. The time between successive frames is 0.333 s, and the car is the same as the one in Solved Problem 2.5. Assuming constant acceleration, what is its value? Can you give some estimate of the error in your answer? How well justified is the assumption of constant acceleration?

2.20 A car moves along a road with a constant velocity. Starting at time \( t = 2.5 \) s, the driver accelerates with constant acceleration. The resulting position of the car as a function of time is shown by the blue curve in the figure.

![Graph showing car's position over time.]

a) What is the value of the constant velocity of the car before 2.5 s? (Hint: The dashed blue line is the path the car would take in the absence of the acceleration.)
b) What is the velocity of the car at \( t = 7.5 \) s? Use a graphical technique (i.e., draw a slope).
c) What is the value of the constant acceleration?

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**PROBLEMS**

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

**Section 2.2**

2.25 A car travels north at 30 m/s for 10 min. It then travels south at 40 m/s for 20 min. What are the total distance the car travels and its displacement?

2.26 You ride your bike along a straight line from your house to a store 1000 m away. On your way back, you stop at a friend's house which is halfway between your house and the store.

a) What is your displacement?
b) What is the total distance traveled? After talking to your friend, you continue to your house. When you arrive back at your house,
c) What is your displacement?
d) What is the distance you have traveled?

**Section 2.3**

2.27 Running along a rectangular track 50 m \( \times \) 40 m you complete one lap in 100 s. What is your average velocity for the lap?

2.28 An electron moves in the positive x-direction a distance of 2.42 m in \( 2.91 \cdot 10^{-6} \) s, bounces off a moving proton, and then moves in the opposite direction a distance of 1.69 m in \( 3.43 \cdot 10^{-8} \) s.

a) What is the average velocity of the electron over the entire time interval?
b) What is the average speed of the electron over the entire time interval?

2.29 The graph describes the position of a particle in one dimension as a function of time. Answer the following questions.

a) In which time interval does the particle have its maximum speed? What is that speed?
b) What is the average velocity in the time interval between \(-5\) s and \(+5\) s?
c) What is the average speed in the time interval between \(-5\) s and \(+5\) s?

Continued—
d) What is the ratio of the velocity in the interval between 2 s and 3 s to that in the interval between 3 s and 4 s?

2.30 The position of a particle moving along the x-axis is given by \( x = (11 + 14t - 2.0t^2) \), where \( t \) is in seconds and \( x \) is in meters. What is the average velocity during the time interval from \( t = 1.0 \) s to \( t = 4.0 \) s?

2.31 The position of a particle moving along the x-axis is given by \( x = 3.0t^2 - 2.0t^3 \), where \( x \) is in meters and \( t \) is in seconds. What is the position of the particle when it achieves its maximum speed in the positive x-direction?

2.32 The rate of continental drift is on the order of 10 mm/yr. Approximately how long did it take North America and Europe to reach their current separation of about 3000 mi?

2.33 You and a friend are driving to the beach during spring break. You travel 16.0 km east and then 80.0 km south in a total time of 40 minutes. (a) What is the average speed of the trip? (b) What is the particle's velocity zero?

2.34 The trajectory of an object is given by the equation

\[
x(t) = (4.35 \text{ m}) + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2
\]

a) For which time \( t \) is the displacement \( x(t) \) at its maximum?

b) What is this maximum value?

Section 2.4

2.35 A bank robber in a getaway car approaches an intersection at a speed of 45 mph. Just as he passes the intersection, he realizes that he needed to turn. So he steps on the brakes, comes to a complete stop, and then accelerates driving straight backward. He reaches a speed of 22.5 mph moving backward. Altogether his deceleration and re-acceleration in the opposite direction take 12.4 s. What is the average acceleration during this time?

2.36 A car is traveling west at 22.0 m/s. After 10.0 s, its velocity is 17.0 m/s in the same direction. Find the magnitude and direction of the car's average acceleration.

2.37 Your friend's car starts from rest and travels 0.500 km in 10.0 s. What is the magnitude of the constant acceleration required to do this?

2.38 A fellow student found in the performance data for his new car the velocity-versus-time graph shown in the figure.

- Find the average acceleration of the car during each of the segments I, II, and III.
- What is the total distance traveled by the car from \( t = 0 \) s to \( t = 24 \) s?

2.39 The velocity of a particle moving along the x-axis is given, for \( t > 0 \), by \( v_x = (50.0t - 2.0t^2) \) m/s, where \( t \) is in seconds. What is the acceleration of the particle when (after \( t = 0 \)) it achieves its maximum displacement in the positive x-direction?

2.40 The 2007 world record for the men's 100-m dash was 9.77 s. The third-place runner crossed the finish line in 10.07 s. When the winner crossed the finish line, how far was the third-place runner behind him?

a) Compute an answer that assumes that each runner ran at his average speed for the entire race.

b) Compute another answer that uses the result of Example 2.5, that a world-class sprinter runs at a speed of 12 m/s after an initial acceleration phase. If both runners in this race reach this speed, how far behind is the third-place runner when the winner finishes?

2.41 The position of an object as a function of time is given as \( x = At^3 + Bt^2 + Ct + D \). The constants are \( A = 2.1 \text{ m/s}^3 \), \( B = 1.0 \text{ m/s}^2 \), \( C = -4.1 \text{ m/s} \), and \( D = 3 \text{ m} \).

a) What is the velocity of the object at \( t = 10.0 \) s?

b) At what time(s) is the object at rest?

c) What is the acceleration of the object at \( t = 0.50 \) s?

d) Plot the acceleration as a function of time for the time interval from \( t = -10.0 \) s to \( t = 10.0 \) s.

Section 2.5

- 2.42 An F-14 Tomcat fighter jet is taking off from the deck of the USS Nimitz aircraft carrier with the assistance of a steam-powered catapult. The jet's location along the flight deck is measured at intervals of 0.20 s. These measurements are tabulated as follows:

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>0.00</th>
<th>0.20</th>
<th>0.40</th>
<th>0.60</th>
<th>0.80</th>
<th>1.00</th>
<th>1.20</th>
<th>1.40</th>
<th>1.60</th>
<th>1.80</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) (m)</td>
<td>0.0</td>
<td>0.7</td>
<td>3.0</td>
<td>6.6</td>
<td>11.8</td>
<td>18.3</td>
<td>26.6</td>
<td>36.2</td>
<td>47.3</td>
<td>59.9</td>
<td>73.9</td>
</tr>
</tbody>
</table>

Use difference formulas to calculate the jet's average velocity and average acceleration for each time interval. After completing this analysis, can you say if the F-14 Tomcat accelerated with approximately constant acceleration?

Section 2.6

2.43 A particle starts from rest at \( x = 0 \) and moves for 20 s with an acceleration of \(+2.0 \text{ cm/s}^2 \). For the next 40 s, the acceleration of the particle is \(-4.0 \text{ cm/s}^2 \). What is the position of the particle at the end of this motion?
2.44 A car moving in the x-direction has an acceleration $a_x$ that varies with time as shown in the figure. At the moment $t = 0$ s, the car is located at $x = 12$ m and has a velocity of 6 m/s in the positive x-direction. What is the velocity of the car at $t = 5.0$ s?

2.45 The velocity as a function of time for a car on an amusement park ride is given as $v = At^2 + Bt$ with constants $A = 2.0$ m/s$^3$ and $B = 1.0$ m/s$^2$. If the car starts at the origin, what is its position at $t = 3.0$ s?

2.46 An object starts from rest and has an acceleration given by $a = Bt^2 - \frac{1}{2}Ct$, where $B = 2.0$ m/s$^4$ and $C = -4.0$ m/s$^3$.
   a) What is the object’s velocity after 5.0 s?
   b) How far has the object moved after $t = 5.0$ s?

2.47 A car is moving along the x-axis and its velocity, $v_x$, varies with time as shown in the figure. If $x_0 = 2.0$ m at $t_0 = 2.0$ s, what is the position of the car at $t = 10.0$ s?

2.48 A car is moving along the x-axis and its velocity, $v_x$, varies with time as shown in the figure. What is the displacement, $\Delta x$, of the car from $t = 4$ s to $t = 9$ s?

2.49 A motorcycle starts from rest and accelerates as shown in the figure. Determine (a) the motorcycle’s speed at $t = 4.00$ s and at $t = 14.0$ s, and (b) the distance traveled in the first 14.0 s.

Section 2.7

2.50 How much time does it take for a car to accelerate from a standing start to 22.2 m/s if the acceleration is constant and the car covers 243 m during the acceleration?

2.51 A car slows down from a speed of 31.0 m/s to a speed of 12.0 m/s over a distance of 380. m.
   a) How long does this take, assuming constant acceleration?
   b) What is the value of this acceleration?

2.52 A runner of mass 57.5 kg starts from rest and accelerates with a constant acceleration of 1.25 m/s$^2$ until she reaches a velocity of 6.3 m/s. She then continues running with this constant velocity.
   a) How far has she run after 59.7 s?
   b) What is the velocity of the runner at this point?

2.53 A fighter jet lands on the deck of an aircraft carrier. It touches down with a speed of 70.4 m/s and comes to a complete stop over a distance of 197.4 m. If this process happens with constant deceleration, what is the speed of the jet 44.2 m before its final stopping location?

2.54 A bullet is fired through a board 10.0 cm thick, with a line of motion perpendicular to the face of the board. If the bullet enters with a speed of 400 m/s and emerges with a speed of 200 m/s, what is its acceleration as it passes through the board?

2.55 A car starts from rest and accelerates at 10.0 m/s$^2$. How far does it travel in 2.00 s?

2.56 An airplane starts from rest and accelerates at 12.1 m/s$^2$. What is its speed at the end of a 500-m runway?

2.57 Starting from rest, a boat increases its speed to 5.00 m/s with constant acceleration.
   a) What is the boat’s average speed?
   b) If it takes the boat 4.00 s to reach this speed, how far has it traveled?
2.58 A ball is tossed vertically upward with an initial speed of 26.4 m/s. How long does it take before the ball is back on the ground?
2.59 A stone is thrown upward, from ground level, with an initial velocity of 10.0 m/s.
   a) What is the velocity of the stone after 0.50 s?
   b) How high above ground level is the stone after 0.50 s?
2.60 A stone is thrown downward with an initial velocity of 10.0 m/s. The acceleration of the stone is constant and has the value of the free-fall acceleration, 9.81 m/s². What is the velocity of the stone after 0.500 s?
2.61 A ball is thrown directly downward, with an initial speed of 10.0 m/s, from a height of 50.0 m. After what time interval does the ball strike the ground?
2.62 An object is thrown vertically upward and has a speed of 20 m/s when it reaches two thirds of its maximum height above the launch point. Determine its maximum height.
2.63 What is the velocity at the midpoint point of a ball able to reach a height $y$ when thrown with an initial velocity $v_0$?
2.64 Runner 1 is standing still on a straight running track. Runner 2 passes him, running with a constant speed of 5.1 m/s. Just as runner 2 passes, runner 1 accelerates with a constant acceleration of 0.89 m/s². How far down the track does runner 1 catch up with runner 2?
2.65 A girl is riding her bicycle. When she gets to a corner, she stops to get a drink from her water bottle. At that time, a friend passes by her, traveling at a constant speed of 8.0 m/s.
   a) After 20 s, the girl gets back on her bike and travels with a constant acceleration of 2.2 m/s². How long does it take for her to catch up with her friend?
   b) If the girl had been on her bike and rolling along at a speed of 1.2 m/s when her friend passed, what constant acceleration would she need to catch up with her friend in the same amount of time?
2.66 A speeding motorcyclist is traveling at a constant speed of 36.0 m/s when he passes a police car parked on the side of the road. The radar, positioned in the police car’s rear window, measures the speed of the motorcycle. At the instant the motorcycle passes the police car, the police officer starts to chase the motorcyclist with a constant acceleration of 4.0 m/s².
   a) How long will it take the police officer to catch the motorcyclist?
   b) What is the speed of the police car when it catches up to the motorcycle?
   c) How far will the police car be from its original position?
2.67 Two train cars are on a straight, horizontal track. One car starts at rest and is put in motion with a constant acceleration of 2.0 m/s². This car moves toward a second car that is 30 m away and moving at a constant speed of 4.0 m/s.
   a) Where will the cars collide?
   b) How long will it take for the cars to collide?
2.68 The planet Mercury has a mass that is 5% of that of Earth, and its gravitational acceleration is $g_{mercury} = 3.7 \text{ m/s}^2$.
   a) How long does it take for a rock that is dropped from a height of 1.75 m to hit the ground on Mercury?
   b) How does this time compare to the time it takes the same rock to reach the ground on Earth, if dropped from the same height?
   c) From what height would you have to drop the rock on Earth so that the fall-time on both planets is the same?
2.69 Bill Jones has a bad night in his bowling league. When he gets home, he drops his bowling ball in disgust out the window of his apartment, from a height of 63.17 m above the ground. John Smith sees the bowling ball pass by his window when it is 40.95 m above the ground. How much time passes from the time when John Smith sees the bowling ball pass his window to when it hits the ground?
2.70 Picture yourself in the castle of Helm’s Deep from the Lord of the Rings. You are on top of the castle wall and are dropping rocks on assorted monsters that are 18.35 m below you. Just when you release a rock, an archer located exactly below you shoots an arrow straight up toward you with an initial velocity of 47.4 m/s. The arrow hits the rock in midair. How long after you release the rock does this happen?
2.71 An object is thrown vertically and has an upward velocity of 25 m/s when it reaches one fourth of its maximum height above its launch point. What is the initial (launch) speed of the object?
2.72 In a fancy hotel, the back of the elevator is made of glass so that you can enjoy a lovely view on your ride. The elevator travels at an average speed of 1.75 m/s. A boy on the 15th floor, 80.0 m above the ground level, drops a rock at the same instant the elevator starts its ascent from the 1st to the 5th floor. Assume the elevator travels at its average speed for the entire trip and neglect the dimensions of the elevator.
   a) How long after it was dropped do you see the rock?
   b) How long does it take for the rock to reach ground level?
2.73 You drop a water balloon straight down from your dormitory window 80.0 m above your friend’s head. At 2.00 s after you drop the balloon, not realizing it has water in it your friend fires a dart from a gun, which is at the same height as his head, directly upward toward the balloon with an initial velocity of 20.0 m/s.
   a) How long after you drop the balloon will the dart burst the balloon?
   b) How long after the dart hits the balloon will your friend have to move out of the way of the falling water? Assume the balloon breaks instantaneously at the touch of the dart.

Additional Problems
2.74 A runner of mass 56.1 kg starts from rest and accelerates with a constant acceleration of 1.23 m/s² until she reaches a velocity of 5.10 m/s. She then continues running at this constant velocity. How long does the runner take to travel 173 m?
2.75 A jet touches down on a runway with a speed of 142.4 mph. After 12.4 s, the jet comes to a complete stop. Assuming constant acceleration of the jet, how far down the runway from where it touched down does the jet stand?

2.76 On the graph of position as a function of time, mark the points where the velocity is zero, and the points where the acceleration is zero.

2.77 An object is thrown upward with a speed of 28.0 m/s. How long does it take it to reach its maximum height?

2.78 An object is thrown upward with a speed of 28.0 m/s. How high above the projection point is it after 1.00 s?

2.79 An object is thrown upward with a speed of 28 m/s. What maximum height above the projection point does it reach?

2.80 The minimum distance necessary for a car to brake to a stop from a speed of 100.0 km/h is 40 m on a dry pavement. What is the minimum distance necessary for this car to brake to a stop from a speed of 130.0 km/h on dry pavement?

2.81 A car moving at 60 km/h comes to a stop in $t = 4.0$ s. Assume uniform deceleration.
   a) How far does the car travel while stopping?
   b) What is its deceleration?

2.82 You are driving at 29.1 m/s when the truck ahead of you comes to a halt 200.0 m away from your bumper. Your brakes are in poor condition and you decelerate at a constant rate of 2.4 m/s².
   a) How close do you come to the bumper of the truck?
   b) How long does it take you to come to a stop?

2.83 A train traveling at 40.0 m/s is headed straight toward another train, which is at rest on the same track. The moving train decelerates at 6.0 m/s², and the stationary train is 100.0 m away. How far from the stationary train will the moving train be when it comes to a stop?

2.84 A car traveling at 25.0 m/s applies the brakes and decelerates uniformly at a rate of 1.2 m/s².
   a) How far does it travel in 3.0 s?
   b) What is its velocity at the end of this time interval?
   c) How long does it take for the car to come to a stop?
   d) What distance does the car travel before coming to a stop?

2.85 The fastest speed in NASCAR racing history was 212.809 mph (reached by Bill Elliott in 1987 at Talladega). If the race car decelerated from that speed at a rate of 8.0 m/s², how far would it travel before coming to a stop?

2.86 You are flying on a commercial airline on your way from Houston, Texas, to Oklahoma City, Oklahoma. Your pilot announces that the plane is directly over Austin, Texas, traveling at a constant speed of 245 mph, and will be flying directly over Dallas, Texas, 362 km away. How long will it be before you are directly over Dallas, Texas?

2.87 The position of a race car on a straight track is given as $x = at^3 + bt^2 + c$, where $a = 2.0$ m/s³, $b = 2.0$ m/s², and $c = 3.0$ m.
   a) What is the car's position between $t = 4.0$ s and $t = 9.0$ s?
   b) What is the average speed between $t = 4.0$ s and $t = 9.0$ s?

2.88 A girl is standing at the edge of a cliff 100 m above the ground. She reaches out over the edge of the cliff and throws a rock straight upward with a speed 8.0 m/s.
   a) How long does it take the rock to hit the ground?
   b) What is the speed of the rock the instant before it hits the ground?

2.89 A double speed trap is set up on a freeway. One police cruiser is hidden behind a billboard, and another is some distance away under a bridge. As a sedan passes by the first cruiser, its speed is measured to be 105.9 mph. Since the driver has a radar detector, he is alerted to the fact that his speed has been measured, and he tries to slow his car down gradually without stepping on the brakes and alerting the police that he knew he was going too fast. Just taking the foot off the gas leads to a constant deceleration. Exactly 7.05 s later the sedan passes the second police cruiser. Now its speed is measured to be only 67.1 mph, just below the local freeway speed limit.
   a) What is the value of the deceleration?
   b) How far apart are the two cruisers?

2.90 During a test run on an airport runway, a new race car reaches a speed of 258.4 mph from a standing start. The car accelerates with constant acceleration and reaches this speed mark at a distance of 612.5 m from where it started. What was its speed after one-fourth, one-half, and three-fourths of this distance?

2.91 The vertical position of a ball suspended by a rubber band is given by the equation

$$ y(t) = (3.8 \text{ m}) \sin(0.46 \frac{t}{s} - 0.31) - (0.2 \text{ m/s})t + 5.0 \text{ m} $$

   a) What are the equations for velocity and acceleration for this ball?
   b) For what times between 0 and 30 s is the acceleration zero?
2.95 The Bellagio Hotel in Las Vegas, Nevada, is well known for its Musical Fountains, which use 192 HyperShooters to fire water hundreds of feet into the air to the rhythm of music. One of the HyperShooters fires water straight upward to a height of 240 ft.

a) What is the initial speed of the water?

b) What is the speed of the water when it is at half this height on its way down?

c) How long will it take for the water to fall back to its original height from half its maximum height?

2.96 You are trying to improve your shooting skills by shooting at a can on top of a fence post. You miss the can, and the bullet, moving at 200 m/s, is embedded 1.5 cm into the post when it comes to a stop. If constant acceleration is assumed, how long does it take for the bullet to stop?

2.97 You drive with a constant speed of 13.5 m/s for 30.0 s. You then accelerate for 10.0 s to a speed of 22.0 m/s. You then slow to a stop in 10.0 s. How far have you traveled?

2.98 A ball is dropped from the roof of a building. It hits the ground and it is caught at its original height 5.0 s later. A) What was the speed of the ball just before it hits the ground?

b) How tall was the building? You are watching from a window 2.5 m above the ground. The window opening is 1.2 m from the top to the bottom.

c) At what time after the ball was dropped did you first see the ball in the window?

2.92 The position of a particle moving along the x-axis varies with time according to the expression \( x = 4t^2 \), where \( x \) is in meters and \( t \) is in seconds. Evaluate the particle's position

a) at \( t = 2.00 \) s.

b) at \( 2.00 \) s + \( \Delta t \).

c) Evaluate the limit of \( \Delta x / \Delta t \) as \( \Delta t \) approaches zero, to find the velocity at \( t = 2.00 \) s.

2.93 In 2005, Hurricane Rita hit several states in the southern United States. In the panic to escape her wrath, thousands of people tried to flee Houston, Texas by car. One car full of college students traveling to Tyler, Texas, 199 miles north of Houston, moved at an average speed of 3.0 m/s for one-fourth of the time, then at 4.5 m/s for another one-fourth of the time, and at 6.0 m/s for the remainder of the trip.

a) How long did it take the students to reach their destination?

b) Sketch a graph of position versus time for the trip.

2.94 A ball is thrown straight upward in the air at a speed of 15.0 m/s. Ignore air resistance.

a) What is the maximum height the ball will reach?

b) What is the speed of the ball when it reaches 5.00 m?

c) How long will it take to reach 5.00 m above its initial position on the way up?

d) How long will it take to reach 5.00 m above its initial position on its way down?