Figure 21.1 (a) A spark due to static electricity occurs between a person's hand and a metal surface when pushing an elevator button. (b) and (c) Similar sparks are generated when the person holds a metal object like a car key or a coin, but are painless because the spark forms between the metal surface and the metal object.
Electricity and magnetism together make up electromagnetism, one of the four fundamental forces of nature.

There are two kinds of electric charge, positive and negative. Like charges repel, and unlike charges attract.

Electric charge is quantized, meaning that it occurs only in integral multiples of a smallest elementary quantity. Electric charge is also conserved.

Most materials around us are electrically neutral. The electron is an elementary particle, and its charge is the smallest observable quantity of electric charge.

Many people think of static electricity as the annoying spark that occurs when they reach for a metal object like a doorknob on a dry day, after they have been walking on a carpet (Figure 21.1). In fact, many electronics manufacturers place small metal plates on equipment so that users can discharge any spark on the plate and not damage the more sensitive parts of the equipment. However, static electricity is more than just an occasional annoyance; it is the starting point for any study of electricity and magnetism, forces that have changed human society as radically as anything since the discovery of fire or the wheel.

In this chapter, we examine the properties of electric charge. A moving electric charge gives rise to a separate phenomenon, called magnetism, which is covered in later chapters. Here we look at charged objects that are not moving—hence the term electrostatics.

All objects have charge, since charged particles make up atoms and molecules. We often don’t notice the effects of electrical charge because most objects are electrically neutral. The forces that hold atoms together and that keep objects separate even when they’re in contact, are all electric in nature.

Perhaps no mystery puzzled ancient civilizations more than electricity, primarily in the form of lightning strikes (Figure 21.2). The destructive force inherent in lightning, which could set objects on fire and kill people and animals, seemed godlike. The ancient Greeks, for example, believed Zeus, father of the gods, had the ability to throw lightning bolts. The Germanic tribes ascribed this power to the god Thor and the Romans to the god Jupiter. Characteristically, the ability to cause lightning belonged to the god at the top (or near the top) of the hierarchy.

The ancient Greeks knew that if you rubbed a piece of amber with a piece of cloth, you could attract small, light objects with the amber. We now know that rubbing amber with a cloth transfers negatively charged particles called electrons from the cloth to the amber. (The words electron and electricity derive from the Greek word for amber.) Lightning also consists of a flow of electrons. The early Greeks and others also knew about naturally occurring magnetic objects called lodestones, which were found in deposits of magnetite, a mineral consisting of iron oxide. These objects were used to construct compasses as early as 300 BC.

The relationship between electricity and magnetism was not understood until the middle of the 19th century. The following chapters will reveal how electricity and magnetism can be unified into a common framework called electromagnetism. However, unification of forces does not stop there. During the early part of the 20th century, two more fundamental forces were discovered: the weak force, which operates in beta decay (in which an electron and a neutrino are spontaneously emitted from certain types of nuclei), and the strong force, which acts inside
the atomic nucleus. We’ll study these forces in more detail in Chapter 39 on particle physics. Currently, the electromagnetic and weak forces are viewed as two aspects of the electroweak force (Figure 21.3). For the phenomena discussed in the following chapters, this electroweak unification has no influence; it becomes important in the highest-energy particle collisions. Because the energy scale for the electroweak unification is so high, most textbooks continue to speak of four fundamental forces: gravitational, electromagnetic, weak, and strong.

Today, a large number of physicists believe that the electroweak force and the strong force can also be unified, that is, described in a common framework. Several theories propose ways to accomplish this, but so far experimental evidence is missing. Interestingly, the force that has been known longer than any of the other fundamental forces, gravity, seems to be hardest to shoehorn into a unified framework with the other fundamental forces. Quantum gravity, supersymmetry, and string theory are current foci of cutting-edge physics research in which theorists are attempting to construct this grand unification and discover the (hubristically named) Theory of Everything. They are mainly guided by symmetry principles and the conviction that nature must be elegant and simple.

We’ll return to these considerations in Chapters 39 and 40. In this chapter, we consider electric charge, how materials react to electric charge, static electricity, and the forces resulting from electric charges. **Electrostatics** covers situations where charges stay in place and do not move.

### 21.2 Electric Charge

Let’s look a little deeper into the cause of the electric sparks that you occasionally receive on a dry winter day if you walk across a carpet and then touch a metal doorknob. (Electrostatic sparks have even ignited gas fumes while someone is filling the tank at a gas station. This is not an urban legend; a few of these cases have been caught on gas station surveillance cameras.) The process that causes this sparking is called **charging**. Charging consists of the transfer of negatively charged particles, called **electrons**, from the atoms and molecules of the material of the carpet to the soles of your shoes. This charge can move relatively easily through your body, including your hands. The built-up electric charge discharges through the metal of the doorknob, creating a spark.

The two types of electric charge found in nature are **positive charge** and **negative charge**. Normally, objects around us do not seem to be charged; instead, they are electrically neutral. Neutral objects contain roughly equal numbers of positive and negative charges that largely cancel each other. Only when positive and negative charges are not balanced do we observe the effects of electric charge.
If you rub a glass rod with a cloth, the glass rod becomes charged and the cloth acquires a charge of the opposite sign. If you rub a plastic rod with fur, the rod and fur also become oppositely charged. If you bring two charged glass rods together, they repel each other. Similarly, if you bring two charged plastic rods together, they also repel each other. However, a charged glass rod and a charged plastic rod will attract each other. This difference arises because the glass rod and the plastic rod have opposite charge. This observation leads us to the

**Law of Electric Charges**

Like charges repel and opposite charges attract.

The unit of electric charge is the **coulomb** (C), named after the French physicist Charles-Auguste de Coulomb (1736–1806). The coulomb is defined in terms of the SI unit for current, the ampere (A), named after another French physicist, André-Marie Ampère (1775–1836). Neither the ampere nor the coulomb can be derived in terms of the other SI units: meter, kilogram, and second. Instead, the ampere is another fundamental SI unit. For this reason, the SI system of units is sometimes called MKSA (meter-kilogram-second-ampere) system. The charge unit is defined as

\[ 1 \text{ C} = 1 \text{ A s}. \]  

The total electric charge of an isolated system is conserved. This law is the fourth conservation law we have encountered so far, the first three being the conservation laws for total energy, momentum, and angular momentum. Conservation laws are a common thread that runs throughout all of physics and thus throughout this book as well.
It is important to note that there is a conservation law for charge, but not for mass. We'll see later in this book that mass and energy are not independent of each other. What is sometimes described in introductory chemistry as conservation of mass is not an exact conservation law, but only an approximation used to keep track of the number of atoms in chemical reactions. (It is a good approximation to a large number of significant figures but not an exact law, like charge conservation.) Conservation of charge applies to all systems, from the macroscopic system of plastic rod and fur down to systems of subatomic particles.

**Elementary Charge**

Electric charge occurs only in integral multiples of a minimum size. This is expressed by saying that charge is quantized. The smallest observable unit of electric charge is the charge of the electron, which is $-1.602 \times 10^{-19}$ C (as defined in equation 21.3).

The fact that electric charge is quantized was verified in an ingenious experiment carried out in 1910 by American physicist Robert A. Millikan (1868–1953) and known as the Millikan oil drop experiment (Figure 21.4). In this experiment, oil drops were sprayed into a chamber where electrons were knocked out of the drops by some form of radiation, usually X-rays. The resulting positively charged drops were allowed to fall between two electrically charged plates. Adjusting the charge of the plates caused the drops to stop falling and allowed their charge to be measured. What Millikan observed was that charge was quantized rather than continuous. (A quantitative analysis of this experiment will be presented in Chapter 23 on electric potential.) That is, this experiment and its subsequent refinements established that charge comes only in integer multiples of the charge of an electron. In everyday experiences with electricity, we do not notice that charge is quantized because most electrical phenomena involve huge numbers of electrons.

In Chapter 13, we discussed the fact that matter is composed of atoms and that an atom consists of a nucleus containing charged protons and neutral neutrons. A schematic drawing of a carbon atom is shown in Figure 21.5. A carbon atom has six protons and (usually) six neutrons in its nucleus. This nucleus is surrounded by six electrons. Note that this drawing is not to scale. In the actual atom, the distance of the electrons from the nucleus is much larger (by a factor on the order of 10,000) than the size of the nucleus. In addition, the electrons are shown in circular orbits, which is also not quite correct. In Chapter 38, we'll see that the locations of electrons in the atom can be characterized only by probability distributions.

As mentioned earlier, a proton has a positive charge with a magnitude that is exactly equal to the magnitude of the negative charge of an electron. In a neutral atom, the number of negatively charged electrons is equal to the number of positively charged protons. The mass of the electron is much smaller than the mass of the proton or the neutron. Therefore, most of the mass of an atom resides in the nucleus. Electrons can be removed from atoms relatively easily. For this reason, electrons are typically the carriers of electricity, rather than protons or atomic nuclei.

The electron is a fundamental particle and has no substructure: It is a point particle with zero radius (at least, according to current understanding). However, high-energy probes have been used to look inside the proton. A proton is composed of charged particles called quarks, held together by uncharged particles called gluons. Quarks have a charge of $\pm \frac{2}{3}e$ or $\pm \frac{1}{3}e$ times the charge of the electron. These fractionally charged particles cannot exist independently and have never been observed directly, despite numerous extensive searches. Just like the charge of an electron, the charges of quarks are intrinsic properties of these elementary particles.

A proton is composed of two up quarks (each with charge $+\frac{2}{3}e$) and one down quark (with charge $-\frac{1}{3}e$), giving the proton a charge of $q_p = (2)(+\frac{2}{3}e) + (1)(-\frac{1}{3}e) = +e$ as illustrated
in Figure 21.6a. The electrically neutral neutron (hence the name!) is composed of an up quark and two down quarks, as shown in Figure 21.6b, so its charge is \( q_n = (1)(+\frac{2}{3}e) + (2)(-\frac{1}{3}e) = 0 \). In Chapter 39, we’ll see that there are other, much more massive, quarks named strange, charm, bottom, and top, which have the same charges as the up and down quarks. There are also much more massive electron-like particles named muon and tau. But the basic fact remains that all of the matter in everyday experience is made up of electrons (with electrical charge \(-e\)), up and down quarks (with electrical charges \(+\frac{2}{3}e\) and \(-\frac{1}{3}e\), respectively), and gluons (zero charge).

It is remarkable that the charges of the quarks inside a proton add up to exactly the same magnitude as the charge of the electron. This fact is still a puzzle, pointing to some deep symmetry in nature that is not yet understood.

Because all macroscopic objects are made of atoms, which in turn are made of electrons and atomic nuclei consisting of protons and neutrons, the charge, \( q \), of any object can be expressed in terms of the sum of the number of protons, \( N_p \), minus the sum of the number of electrons, \( N_e \), that make up the object:

\[
q = e \left( N_p - N_e \right). \tag{21.5}
\]

**EXAMPLE 21.1 Net Charge**

**Problem**

If we wanted a block of iron of mass 3.25 kg to acquire a positive charge of 0.100 C, what fraction of the electrons would we have to remove?

**Solution**

Iron has mass number 56. Therefore, the number of iron atoms in the 3.25-kg block is

\[
N_{\text{atom}} = \frac{(3.25 \text{ kg})(6.022 \cdot 10^{23} \text{ atoms/mole})}{0.0560 \text{ kg/mole}} = 3.495 \cdot 10^{25} \text{ atoms.}
\]

Note that we have used Avogadro’s number, \( 6.022 \cdot 10^{23} \), and the definition of the mole, which specifies that the mass of 1 mole of a substance in grams is just the mass number of the substance—in this case, 56.

Because the atomic number of iron is 26, which equals the number of protons or electrons in an iron atom, the total number of electrons in the 3.25-kg block is:

\[
N_e = 26N_{\text{atom}} = (26)(3.495 \cdot 10^{25}) = 9.09 \cdot 10^{26} \text{ electrons.}
\]

We use equation 21.5 to find the number of electrons, \( N_{\Delta e} \), that we would have to remove. Because the number of electrons equals the number of protons in the original uncharged object, the difference in the number of protons and electrons is the number of removed electrons, \( N_{\Delta e} \):

\[
q = e \cdot N_{\Delta e} \Rightarrow N_{\Delta e} = \frac{q}{e} = \frac{0.100 \text{ C}}{1.602 \cdot 10^{-19} \text{ C}} = 6.24 \cdot 10^{17}.
\]

Finally, we obtain the fraction of electrons we would have to remove:

\[
\frac{N_{\Delta e}}{N_e} = \frac{6.24 \cdot 10^{17}}{9.09 \cdot 10^{26}} = 6.87 \cdot 10^{-10}.
\]

We would have to remove fewer than one in a billion electrons from the iron block in order to put the sizable positive charge of 0.100 C on it.

**21.3 Insulators, Conductors, Semiconductors, and Superconductors**

Materials that conduct electricity well are called **conductors**. Materials that do not conduct electricity are called **insulators**. (Of course, there are good and poor conductors and good and poor insulators, depending on the properties of the specific materials.)
The electronic structure of a material refers to the way in which electrons are bound to nuclei, as we’ll discuss in later chapters. For now, we are interested in the relative propensity of the atoms of a material to either give up or acquire electrons. For insulators, no free movement of electrons occurs because the material has no loosely bound electrons that can escape from its atoms and thereby move freely throughout the material. Even when external charge is placed on an insulator, this external charge cannot move appreciably. Typical insulators are glass, plastic, and cloth.

On the other hand, materials that are conductors have an electronic structure that allows the free movement of some electrons. The positive charges of the atoms of a conducting material do not move, since they reside in the heavy nuclei. Typical solid conductors are metals. Copper, for example, is a very good conductor used in electrical wiring.

Fluids and organic tissue can also serve as conductors. Pure distilled water is not a very good conductor. However, dissolving common table salt (NaCl), for example, in water improves its conductivity tremendously, because the positively charged sodium ions (Na+) and negatively charged chlorine ions (Cl–) can move within the water to conduct electricity. In liquids, unlike solids, positive as well as negative charge carriers are mobile. Organic tissue is not a very good conductor, but it conducts electricity well enough to make large currents dangerous to us. (We’ll learn more about electrical current in Chapter 26, where these terms, which are in everyday use, will be defined precisely.)

**Semiconductors**
A class of materials called semiconductors can change from being an insulator to being a conductor and back to an insulator again. Semiconductors were discovered only a little more than 50 years ago but are the backbone of the entire computer and consumer electronics industries. The first widespread use of semiconductors was in transistors (Figure 21.7a); modern computer chips (Figure 21.7b) perform the functions of millions of transistors. Computers and basically all modern consumer electronics products and devices (televisions, cameras, video game players, cell phones, etc.) would be impossible without semiconductors. Gordon Moore, cofounder of Intel, famously stated that due to advancing technology, the power of the average computer’s CPU (central processing unit) doubles every 18 months, which is an empirical average over the last 5 decades. This doubling phenomenon is known as Moore’s Law. Physicists have been and will undoubtedly continue to be the driving force behind this process of scientific discovery, invention, and improvement.

Semiconductors are of two kinds: intrinsic and extrinsic. Examples of intrinsic semiconductors are chemically pure crystals of gallium arsenide, germanium, or, especially, silicon. Engineers produce extrinsic semiconductors by doping, which is the addition of minute amounts (typically 1 part in 10^6) of other materials that can act as electron donors or electron receptors. Semiconductors doped with electron donors are called n-type (n stands for “negative charge”). If the doping substance acts as an electron receptor, the hole left behind by an electron that attaches to a receptor can also travel through the semiconductor and acts as an effective positive charge carrier. These semiconductors are consequently called p-type (p stand for “positive charge”). Thus, unlike normal solid conductors in which only negative charges move, semiconductors have movement of negative or positive charges (which are really electron holes, that is, missing electrons).

**Superconductors**
Superconductors are materials that have zero resistance to the conduction of electricity, as opposed to normal conductors, which conduct electricity well but with some losses. Materials are superconducting only at very low temperatures. A typical superconductor is a niobium-titanium alloy that must be kept near the temperature of liquid helium (4.2 K) to retain its superconducting properties. During the last 20 years, new materials called high-T_c superconductors (T_c stands for “critical temperature,” which is the maximum temperature that allows superconductivity) have been developed. These are superconducting at liquid-nitrogen temperature (77.3 K). Materials that are superconductors at room temperature (300 K) have not yet been found, but they would be extremely useful. Research directed
toward developing such materials and on theoretically explaining what physical phenomena cause high-$T_c$ superconductivity is currently in progress.

The topics of conductivity, superconductivity, and semiconductors will be discussed in more quantitative detail in the following chapters.

### 21.4 Electrostatic Charging

Giving a static charge to an object is a process known as **electrostatic charging**. Electrostatic charging can be understood through a series of simple experiments. A power supply serves as a ready source of positive and negative charge. The battery in your car is a similar power supply; it uses chemical reactions to create a separation between positive and negative charge. Several insulating paddles can be charged with positive or negative charge from the power supply. In addition, a conducting connection is made to the Earth. The Earth is a nearly infinite reservoir of charge, capable of effectively neutralizing electrically charged objects in contact with it. This taking away of charge is called **grounding**, and an electrical connection to the Earth is called a **ground**.

An **electroscope** is a device that gives an observable response when it is charged. You can build a relatively simple electroscope by using two strips of very thin metal foil that are attached at one end and are allowed to hang straight down adjacent to each other from an isolating frame. Kitchen aluminum foil is not suitable, because it is too thick, but hobby shops sell thinner metal foils. For the isolating frame, you can use a Styrofoam coffee cup turned sideways, for example.

The lecture-demonstration-quality electroscope shown in Figure 21.8 has two conductors that in their neutral position are touching and oriented in a vertical direction. One of the conductors is hinged at its midpoint so that it will move away from the fixed conductor if a charge appears on the electroscope. These two conductors are in contact with a conducting ball on top of the electroscope, which allows charge to be applied or removed easily.

An uncharged electroscope is shown in Figure 21.9a. The power supply is used to give a negative charge to one of the insulating paddles. When the paddle is brought near the ball of the electroscope, as shown in Figure 21.9b, the electrons in the conducting ball of the electroscope are repelled, which produces a net negative charge on the conductors of the electroscope. This negative charge causes the movable conductor to rotate because the stationary conductor also has negative charge and repels it. Because the paddle did not touch the ball, the charge on the movable conductors is **induced**. If the charged paddle is then taken away, as illustrated in Figure 21.9c, the induced charge reduces to zero, and the movable conductor returns to its original position, because the total charge on the electroscope did not change in the process.

If the same process is carried out with a positively charged paddle, the electrons in the conductors are attracted to the paddle and flow into the conducting ball. This leaves a net positive charge on the conductors, causing the movable conducting arm to rotate again. Note that the net charge of the electroscope is zero in both cases and that the motion of the conductor indicates only that the paddle is charged. When the positively charged paddle

---

**Figure 21.8** A typical electroscope used in lecture demonstrations.

**Figure 21.9** Inducing a charge:
(a) An uncharged electroscope. (b) A negatively charged paddle is brought near the electroscope. (c) The negatively charged paddle is taken away.
Electrostatic Charging

is removed, the movable conductor again returns to its original position. It is important to note that we cannot determine the sign of this charge!

On the other hand, if a negatively charged insulating paddle touches the ball of the electroscope, as shown in Figure 21.10b, electrons will flow from the paddle to the conductor, producing a net negative charge. When the paddle is removed, the charge remains and the movable arm remains rotated, as shown in Figure 21.10c. Similarly, if a positively charged insulating paddle touches the ball of the uncharged electroscope, the electroscope transfers electrons to the positively charged paddle and becomes positively charged. Again, both a positively charged paddle and a negatively charged paddle have the same effect on the electroscope, and we have no way of determining whether the paddles are positively charged or negatively charged. This process is called charging by contact.

The two different kinds of charge can be demonstrated by first touching a negatively charged paddle to the electroscope, producing a rotation of the movable arm, as shown in Figure 21.10. If a positively charged paddle is then brought into contact with the electroscope, the movable arm returns to the uncharged position. The charge is neutralized (assuming both paddles originally had the same absolute value of charge). Thus, there are two kinds of charge. However, because charges are manifestations of mobile electrons, a negative charge is an excess of electrons and a positive charge is a deficit of electrons.

The electroscope can be given a charge without touching it with the charged paddle, as shown in Figure 21.11. The uncharged electroscope is shown in Figure 21.11a. A negatively charged paddle is brought close to the ball of the electroscope but not touching it, as shown in Figure 21.11b. In Figure 21.11c, the electroscope is connected to a ground. Then, while the charged paddle is still close to but not touching the ball of the electroscope, the ground
connection is removed in Figure 21.11d. Now, when the paddle is moved away from the electroscope in Figure 21.11e, the electroscope is still positively charged (but with a smaller deflection than in Figure 21.11b). The same process also works with a positively charged paddle. This process is called charging by induction and yields an electroscope charge that has the opposite sign from the charge on the paddle.

## 21.5 Electrostatic Force—Coulomb’s Law

The law of electric charges is evidence of a force between any two charges at rest. Experiments show that for the electrostatic force exerted by a charge $q_1$ on a charge $q_2$, $F_{2 \rightarrow 1}$, the force on $q_1$ points toward $q_2$ if the charges have opposite signs and away from $q_2$ if the charges have like signs (Figure 21.12). This force on one charge due to another charge always lies on a line between the two charges. Coulomb’s Law gives the magnitude of this force as

$$F = k \frac{|q_1 q_2|}{r^2},$$

where $q_1$ and $q_2$ are electric charges, $r = |\vec{r}_1 - \vec{r}_2|$ is the distance between them, and

$$k = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

is Coulomb’s constant. You can see that one Coulomb is a very large charge. If two charges of 1 C each were at a distance of 1 m apart, the magnitude of the force they would exert on each other would be 8.99 billion N. For comparison, this force equals the weight of 450 fully loaded space shuttles!

The relationship between Coulomb’s constant and another constant, $\varepsilon_0$, called the electric permittivity of free space, is

$$k = \frac{1}{4 \pi \varepsilon_0},$$

Consequently, the value of $\varepsilon_0$ is

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}.$$  

(21.9)

An alternative way of writing equation 21.6 is then

$$F = \frac{1}{4 \pi \varepsilon_0} \frac{|q_1 q_2|}{r^2}.$$  

(21.10)

As you’ll see in the next few chapters, some equations in electrostatics are more convenient to write with $k$, while others are more easily written in terms of $1/(4 \pi \varepsilon_0)$.

Note that the charges in equations 21.6 and 21.10 can be positive or negative, so the product of the charges can also be positive or negative. Since opposite charges attract and like charges repel, a negative value for the product $q_1 q_2$ signifies attraction and a positive value means repulsion.

Finally, Coulomb’s Law for the force due to charge 2 on charge 1 can be written in vector form:

$$\vec{F}_{2 \rightarrow 1} = -k \frac{q_1 q_2}{r^3} (\hat{r}_1 - \hat{r}_2) = -k \frac{q_1 q_2}{r^2} \hat{r}_{21}. $$ 

(21.11)

In this equation, $\hat{r}_{21}$ is a unit vector pointing from $q_2$ to $q_1$ (see Figure 21.13). The negative sign indicates that the force is repulsive if both charges are positive or both charges are negative. In that case, $\vec{F}_{2 \rightarrow 1}$ points away from charge 2, as depicted in Figure 21.13a. On the other hand, if one of the charges is positive and the other negative, then $\vec{F}_{2 \rightarrow 1}$ points toward charge 2, as shown in Figure 21.13b.

**21.3 In-Class Exercise**

You place two charges a distance $r$ apart. Then you double each charge and double the distance between the charges. How does the force between the two charges change?

- a) The new force is twice as large.
- b) The new force is half as large.
- c) The new force is four times as large.
- d) The new force is four times smaller.
- e) The new force is the same.
Electrostatic Force—Coulomb’s Law

If charge 2 exerts the force $\vec{F}_{2\rightarrow 1}$ on charge 1, then the force $\vec{F}_{1\rightarrow 2}$ that charge 1 exerts on charge 2 is simply obtained from Newton’s Third Law (see Chapter 4): $\vec{F}_{1\rightarrow 2} = -\vec{F}_{2\rightarrow 1}$.

Superposition Principle

So far in this chapter, we have been dealing with two charges. Now let’s consider three point charges, $q_1$, $q_2$, and $q_3$, at positions $x_1$, $x_2$, and $x_3$, respectively, as shown in Figure 21.14. The force exerted by charge 1 on charge 3, $\vec{F}_{1\rightarrow 3}$, is given by

$$\vec{F}_{1\rightarrow 3} = -\frac{kq_1q_3}{(x_3 - x_1)^2}\hat{x}.$$

The force exerted by charge 2 on charge 3 is

$$\vec{F}_{2\rightarrow 3} = -\frac{kq_2q_3}{(x_3 - x_2)^2}\hat{x}.$$

The force that charge 1 exerts on charge 3 is not affected by the presence of charge 2. The force that charge 2 exerts on charge 3 is not affected by the presence of charge 1. In addition, the forces exerted by charge 1 and charge 2 on charge 3 add vectorially to produce a net force on charge 3:

$$\vec{F}_{\text{net}\rightarrow 3} = \vec{F}_{1\rightarrow 3} + \vec{F}_{2\rightarrow 3}.$$

This superposition of forces is completely analogous to that described in Chapter 4 for forces such as gravity and friction.

**Example 21.2 Electrostatic Force inside the Atom**

**Problem 1**
What is the magnitude of the electrostatic force that the two protons inside the nucleus of a helium atom exert on each other?

**Solution 1**
The two protons and two neutrons in the nucleus of the helium atom are held together by the strong force; the electrostatic force is pushing the protons apart. The charge of each proton is $e = 1.602 \times 10^{-19}$ C.

**Figure 21.13** Electrostatic force vectors, which two charges exert on each other: (a) two charges of like sign; (b) two charges of opposite sign.

**21.4 In-Class Exercise**

What do the forces acting on the charge $q_3$ in Figure 21.14 indicate about the signs of the three charges?

a) All three charges must be positive.
b) All three charges must be negative.
c) Charge $q_3$ must be zero.
d) Charges $q_1$ and $q_2$ must have opposite signs.
e) Charges $q_1$ and $q_2$ must have the same sign, and $q_3$ must have the opposite sign.

**21.5 In-Class Exercise**

Assuming that the lengths of the vectors in Figure 21.14 are proportional to the magnitudes of the forces they represent, what do they indicate about the magnitudes of the charges $q_1$ and $q_2$? (Hint: The distance between $x_1$ and $x_2$ is the same as the distance between $x_2$ and $x_3$.)

a) $|q_1| < |q_2|$
b) $|q_1| = |q_2|$
c) $|q_1| > |q_2|$
d) The answer cannot be determined from the information given in the figure.
proton is \( q_p = +e \). A distance of approximately \( r = 2 \cdot 10^{-15} \) m separates the two protons. Using Coulomb’s Law, we can find the force:

\[
F = k \frac{|q_p q_p|}{r^2} = \frac{8.99 \cdot 10^9 \text{ N m}^2}{\text{C}^2} \left( \frac{+1.6 \cdot 10^{-19} \text{ C}}{2 \cdot 10^{-15} \text{ m}} \right)^2 = 58 \text{ N}.
\]

Therefore, the two protons in the atomic nucleus of a helium atom are being pushed apart with a force of 58 N (approximately the weight of a small dog). Considering the size of the nucleus, this is an astonishingly large force. Why do atomic nuclei not simply explode? The answer is that an even stronger force, the aptly named strong force, keeps them together.

**Problem 2**

What is the magnitude of the electrostatic force between a gold nucleus and an electron of the gold atom in an orbit with radius \( 4.88 \cdot 10^{-12} \) m?

**Solution 2**

The negatively charged electron and the positively charged gold nucleus attract each other with a force whose magnitude is

\[
F = k \frac{|q_e q_N|}{r^2},
\]

where the charge of the electron is \( q_e = -e \) and the charge of the gold nucleus is \( q_N = +79e \). The force between the electron and the nucleus is then

\[
F = k \frac{|q_e q_N|}{r^2} = \frac{8.99 \cdot 10^9 \text{ N m}^2}{\text{C}^2} \left( \frac{1.60 \cdot 10^{-19} \text{ C}}{79} \left( \frac{1.60 \cdot 10^{-19} \text{ C}}{4.88 \cdot 10^{-12} \text{ m}} \right) \right) = 7.63 \cdot 10^{-4} \text{ N}.
\]

Thus, the magnitude of the electrostatic force exerted on an electron in a gold atom by the nucleus is about 100,000 times less than that between protons inside a nucleus.

**Note:** The gold nucleus has a mass that is approximately 400,000 times that of the electron. But the force the gold nucleus exerts on the electron has exactly the same magnitude as the force that the electron exerts on the gold nucleus. You may say that this is obvious from Newton’s Third Law (see Chapter 4), which is true. But it is worth emphasizing that this basic law holds for electrostatic forces as well.

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**Example 21.3** Equilibrium Position

![Figure 21.15](image.png)

**Problem**

Two charged particles are placed as shown in Figure 21.15: \( q_1 = 0.15 \mu \text{C} \) is located at the origin, and \( q_2 = 0.35 \mu \text{C} \) is located on the positive \( x \)-axis at \( x_2 = 0.40 \) m. Where should a third charged particle, \( q_3 \), be placed to be at an equilibrium point (the forces on it sum to zero)?

**Solution**

Let’s first determine where not to put the third charge. If the third charge is placed anywhere off the \( x \)-axis, there will always be a force component pointing toward or away
from the $x$-axis. Thus, we can find an equilibrium point (a point where the forces sum to zero) only on the $x$-axis. The $x$-axis can be divided into three different segments: $x \leq x_1 = 0$, $x_1 < x < x_2$, and $x_2 \leq x$. For $x \leq x_1 = 0$, the force vectors from both $q_1$ and $q_2$ acting on $q_3$ will point in the positive direction if the charge is negative and in the negative direction if the charge is positive. Because we are looking for a location where the two forces cancel, the segment $x \leq x_1 = 0$ can be excluded. A similar argument excludes $x \geq x_2$.

In the remaining segment of the $x$-axis, $x_1 < x < x_2$, the forces from $q_1$ and $q_2$, on $q_3$ point in opposite directions. We look for the location, $x_3$, where the absolute magnitudes of both forces are equal and the forces thus sum to zero. We express the equality of the two forces as

$$|\vec{F}_{1-3}| = |\vec{F}_{2-3}|,$$

which we can rewrite as

$$k \frac{|q_1 q_3|}{(x_3 - x_1)^2} = k \frac{|q_1 q_2|}{(x_2 - x_3)^2}.$$

We now see that the magnitude and sign of the third charge do not matter because that charge cancels out, as does the constant $k$, giving us

$$\frac{q_1}{(x_3 - x_1)^2} = \frac{q_2}{(x_2 - x_3)^2}$$

or

$$q_1 (x_2 - x_3)^2 = q_2 (x_3 - x_1)^2. \quad (i)$$

Taking the square root of both sides and solving for $x_3$, we find

$$\sqrt{q_1} (x_2 - x_3) = \sqrt{q_2} (x_3 - x_1),$$

or

$$x_3 = \frac{\sqrt{q_1} x_2 + \sqrt{q_2} x_1}{\sqrt{q_1} + \sqrt{q_2}}.$$

We can take the square root of both sides of equation $\text{(i)}$ because $x_1 < x_3 < x_2$, and so both of the roots, $x_2 - x_3$ and $x_3 - x_1$, are assured to be positive.

Inserting the numbers given in the problem statement, we obtain

$$x_3 = \frac{\sqrt{0.15 \mu C} x_2 + \sqrt{0.35 \mu C} x_1}{\sqrt{0.15 \mu C} + \sqrt{0.35 \mu C}} = \frac{\sqrt{0.15 \mu C} (0.4 \text{ m})}{\sqrt{0.15 \mu C} + \sqrt{0.35 \mu C}} = 0.16 \text{ m}.$$

This result makes sense because we expect the equilibrium point to reside closer to the smaller charge.

**Solved Problem 21.1 Charged Balls**

**PROBLEM**

Two identical charged balls hang from the ceiling by insulated ropes of equal length, $l = 1.50 \text{ m}$ (Figure 21.16). A charge $q = 25.0 \mu C$ is applied to each ball. Then the two balls hang at rest, and each supporting rope has an angle of 25.0° with respect to the vertical (Figure 21.16a). What is the mass of each ball?

**SOLUTION**

**THINK**

Each charged ball has three forces acting on it: the force of gravity, the repulsive electrostatic force, and the tension in the supporting rope. Using the
first condition for static equilibrium from Chapter 11, we know that the sum of all the forces on each ball must be zero. We can resolve the components of the three forces and set them equal to zero, allowing us to solve for the mass of the charged balls.

**SKETCH**
A free-body diagram for the left-hand ball is shown in Figure 21.16b.

**RESEARCH**
The condition for static equilibrium says that the sum of the $x$-components of the three forces acting on the ball must equal zero and the sum of $y$-components of these forces must equal zero. The sum of the $x$-components of the forces is

$$T \sin \theta - F_e = 0, \quad (i)$$

where $T$ is the magnitude of the string tension, $\theta$ is the angle of the string relative to the vertical, and $F_e$ is the magnitude of the electrostatic force. The sum of the $y$-components of the forces is

$$T \cos \theta - F_g = 0. \quad (ii)$$

The force of gravity, $F_g$, is just the weight of the charged ball:

$$F_g = mg, \quad (iii)$$

where $m$ is the mass of the charged ball. The electrostatic force the two balls exert on each other is given by

$$F_e = k \frac{q^2}{d^2}, \quad (iv)$$

where $d$ is the distance between the two balls. We can express the distance between the two balls in terms of the length of the string, $\ell$, by looking at Figure 21.16a. We see that

$$\sin \theta = \frac{d}{2\ell}.$$ 

We can then express the electrostatic force in terms of the angle with respect to the vertical, $\theta$, and the length of the string, $\ell$:

$$F_e = k \frac{q^2}{\left(2\ell \sin \theta\right)^2} = \frac{kq^2}{4\ell^2 \sin^2 \theta}. \quad (v)$$

**SIMPLIFY**
We divide equation (i) by equation (ii):

$$\frac{T \sin \theta - F_e}{T \cos \theta} = \frac{F_e}{F_g},$$

eliminating the (unknown) string tension and obtaining

$$\tan \theta = \frac{F_e}{F_g}.$$ 

Substituting from equations (iii) and (v) for the force of gravity and the electrostatic force, we get

$$\tan \theta = \frac{kq^2}{mg} \frac{1}{4\ell^2 \sin^2 \theta} = \frac{kq^2}{4mg \ell^2 \sin^2 \theta}.$$ 

Solving for the mass of the ball, we obtain

$$m = \frac{kq^2}{4g \ell^2 \sin^2 \theta \tan \theta}.$$
**CALCULATE**

Putting in the numerical values gives

\[
m = \frac{8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2 \times 25.0 \mu\text{C}^2}{4 \left(9.81 \text{ m/s}^2 \times 1.50 \text{ m} \times \sin^2 25.0^\circ \tan 25.0^\circ \right)} = 0.764116 \text{ kg}.
\]

**ROUND**

We report our result to three significant figures:

\[m = 0.764 \text{ kg}.
\]

**DOUBLE-CHECK**

To double-check, we make the small-angle approximations that \(\sin \theta \approx \tan \theta \approx \theta \) and \(\cos \theta \approx 1\). The tension in the string then approaches \(mg\), and we can express the \(x\)-components of the forces as

\[T \sin \theta = mg\theta = E = k \frac{q^2}{d^2} = k \frac{q^2}{(2\theta)^2}.
\]

Solving for the mass of the charged ball, we get

\[
m = \frac{kq^2}{4 \epsilon_0^2 \theta^2} = \frac{8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2 \times 25.0 \mu\text{C}^2}{4 \left(9.81 \text{ m/s}^2 \times 1.50 \text{ m} \times (0.436 \text{ rad})^2 \right)} = 0.768 \text{ kg},
\]

which is close to our answer.

---

**Electrostatic Precipitator**

An application of electrostatic charging and electrostatic forces is the cleaning of emissions from coal-fired power plants. A device called an **electrostatic precipitator** (ESP) is used to remove ash and other particulates resulting from the burning of coal to generate electricity. Its operation is illustrated in Figure 21.17.

The ESP consists of wires and plates, with the wires held at a high negative voltage relative to a series of plates held at a positive voltage. (Here the term **voltage** is used colloquially; in Chapter 23, the concept will be defined in terms of electric potential difference.) In Figure 21.17, the exhaust from the coal-burning process enters the ESP from the left. Particulates passing near the wires pick up a negative charge. These particles are then attracted to one of the positive plates and stick there. The gas continues through the ESP, leaving the ash and other particulates behind. The accumulated material is then shaken off the plates to a hamper below. This waste can be used for many purposes, including construction materials and fertilizer. Figure 21.18 shows an example of a coal-fired power plant that incorporates an ESP.

---

**21.2 Self-Test Opportunity**

A positive point charge \(+q\) is placed at point \(P\), to the right of two charges \(q_1\) and \(q_2\), as shown in the figure. The net electrostatic force on the positive charge \(+q\) is found to be zero. Identify each of the following statements as true or false.

- a) Charge \(q_2\) must have the opposite sign from \(q_1\) and be smaller in magnitude.
- b) The magnitude of charge \(q_1\) must be smaller than the magnitude of charge \(q_2\).
- c) Charges \(q_1\) and \(q_2\) must have the same sign.
- d) If \(q_1\) is negative, then \(q_2\) must be positive.
- e) Either \(q_1\) or \(q_2\) must be positive.

---

**21.8 In-Class Exercise**

Consider three charges placed along the \(x\)-axis, as shown in the figure.

\[q_1, q_2, q_3\]

The values of the charges are \(q_1 = -8.10 \mu\text{C}, q_2 = 2.16 \mu\text{C},\) and \(q_3 = 2.16 \mu\text{C}\). The distance between \(q_1\) and \(q_2\) is \(d_1 = 1.71 \text{ m}\). The distance between \(q_1\) and \(q_3\) is \(d_3 = 2.62 \text{ m}\). What is the magnitude of the total electrostatic force exerted on \(q_3\) by \(q_1\) and \(q_2\)?

- a) \(2.77 \cdot 10^{-4} \text{ N}\)
- b) \(2.22 \cdot 10^{-4} \text{ N}\)
- c) \(7.92 \cdot 10^{-4} \text{ N}\)
- d) \(6.71 \cdot 10^{-5} \text{ N}\)
- e) \(1.44 \cdot 10^{-5} \text{ N}\)

---

**FIGURE 21.17** Operation of an electrostatic precipitator used to clean the exhaust gas of a coal-fired power plant. The view is from the top of the device.
Laser Printer

Another example of a device that applies electrostatic forces is the laser printer. The operation of a laser printer is illustrated in Figure 21.19. The paper path follows the blue arrows. Paper is taken from the paper tray or fed manually through the alternate paper feed. The paper passes over a drum where the toner is placed on the surface of the paper and then passes through a fuser that melts the toner and permanently affixes it to the paper.

The drum consists of a metal cylinder coated with a special photosensitive material; originally amorphous selenium was used but has been replaced with an organic material. The photosensitive surface is an insulator that retains charge in the absence of light, but discharges quickly if light is incident on the surface. The drum rotates so that its surface speed is the same as the speed of the moving paper. The basic principle of the operation of the drum is illustrated in Figure 21.20.

The drum is negatively charged with electrons using a wire held at high voltage. Then laser light is directed at the surface of the drum. Wherever the laser light strikes the surface of the drum, the surface at that point is discharged. A laser is used because its beam is narrow and remains focused. A line of the image being printed is written one pixel (picture element or dot) at a time using a laser beam directed by a moving mirror and a lens. A typical laser printer can write 300 pixels per inch, with many printers being able to write 600 or 1200 pixels per inch. The surface of the drum then passes by a roller that picks up toner from the toner cartridge. Toner consists of small, black, insulating particles composed of a plastic-like material. The toner roller is charged to the same negative voltage as the drum. Therefore, wherever the surface of the drum has been discharged, electrostatic forces deposit toner on the surface of the drum. Any portion of the drum surface that has not been exposed to the laser will not pick up toner.

As the drum rotates, it next comes in contact with the paper. The toner is then transferred from the surface of the drum to the paper. Some printers charge the paper positively to help attract the negatively charged toner. As the drum rotates, any remaining toner is scraped off and the surface is neutralized with an erase light or a rotating erase drum in preparation for printing the next image. The paper then continues on to the fuser, which melts the toner, producing a permanent image on the paper. Finally the paper exits the printer.
Coulomb’s Law and Newton’s Law of Gravitation

Coulomb’s Law describing the electrostatic force between two electric charges, \( F_e \), has a similar form to Newton’s Law describing the gravitational force between two masses, \( F_g \):

\[
F_e = k \frac{|q_1 q_2|}{r^2} \quad \text{and} \quad F_g = G \frac{m_1 m_2}{r^2},
\]

where \( m_1 \) and \( m_2 \) are the two masses, \( q_1 \) and \( q_2 \) are the two electric charges, and \( r \) is the distance of separation. Both forces vary with the inverse square of the distance. The electric force can be attractive or repulsive because charges can have positive or negative signs. (See Figure 21.13a and b.) The gravitational force is always attractive because there is only one kind of mass. (For the gravitational force, only the case depicted in Figure 21.13b is possible.) The relative strengths of the forces are given by the proportionality constants \( k \) and \( G \).

**Example 21.4  Forces between Electrons**

Let’s evaluate the relative strengths of the two interactions by calculating the ratio of the electrostatic force and the gravitational force that two electrons exert on each other. This ratio is given by

\[
\frac{F_e}{F_g} = \frac{k q_e^2}{G m_e^2}.
\]

Because the dependence on distance is the same in both forces, there is no dependence on distance in the ratio of the two forces—it cancels out. The mass of an electron is \( m_e = 9.109 \cdot 10^{-31} \) kg, and its charge is \( q_e = -1.602 \cdot 10^{-19} \) C. Using the value of Coulomb’s constant given in equation 21.7, \( k = 8.99 \cdot 10^9 \) N m^2/C^2, and the value of the universal gravitational constant, \( G = 6.67 \cdot 10^{-11} \) N m^2/kg^2, we find numerically

\[
\frac{F_e}{F_g} = \frac{(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2)(1.602 \cdot 10^{-19} \text{ C})^2}{(6.67 \cdot 10^{-11} \text{ N m}^2/\text{kg}^2)(9.109 \cdot 10^{-31} \text{ kg})^2} = 4.2 \cdot 10^{42}.
\]

Therefore, the electrostatic force between electrons is stronger than the gravitational force between them by more than 42 orders of magnitude.

Despite the relative weakness of the gravitational force, on the astronomical scale, gravity is the only force that matters. The reason for this dominance is that all stars, planets, and other objects of astronomical relevance carry no net charge. Therefore, there is no net electrostatic interaction between them, and gravity dominates.

Coulomb’s Law of electrostatics applies to macroscopic systems down to the atom, though subtle effects in atomic and subatomic systems require use of a more sophisticated approach called quantum electrodynamics. Newton’s law of gravitation fails in subatomic systems and also must be modified for astronomical systems, such as the precessional motion of Mercury around the Sun. These fine details of the gravitational interaction are governed by Einstein’s theory of general relativity.

The similarities between the gravitational and electrostatic interactions will be covered further in the next two chapters, which address electric fields and electric potential.

**What We Have Learned**

- There are two kinds of electric charge, positive and negative. Like charges repel, and unlike charges attract.
- The quantum (elementary quantity) of electric charge is \( e = 1.602 \cdot 10^{-19} \) C.
- The electron has charge \( q_e = -e \) and the proton has charge \( q_p = +e \). The neutron has zero charge.
- The net charge of an object is given by \( e \) times the number of protons, \( N_p \), minus \( e \) times the number of electrons, \( N_e \), that make up the object: \( q = e \cdot (N_p - N_e) \).
Problem 21.2  Bead on a Wire

Problem
A bead with charge \( q_1 = +1.28 \ \mu C \) is fixed in place on an insulating wire that makes an angle of \( \theta = 42.3^\circ \) with respect to the horizontal (Figure 21.21a). A second bead with charge \( q_2 = -5.06 \ \mu C \) slides without friction on the wire. At a distance \( d = 0.380 \) m between the beads, the net force on the second bead is zero. What is the mass, \( m_2 \), of the second bead?
SOLUTION

THINK
The force of gravity pulling the bead of mass $m_2$ down the wire is compensated by the attractive electrostatic force between the positive charge on the first bead and the negative charge on the second bead. The second bead can be thought of as sliding on an inclined plane.

SKETCH
Figure 21.21b shows a free-body diagram of the forces acting on the second bead. We have defined a coordinate system in which the positive $x$-direction is down the wire. The force exerted on $m_2$ by the wire can be omitted because this force has only a $y$-component, and we can solve the problem by analyzing just the $x$-components of the forces.

RESEARCH
The attractive electrostatic force between the two beads balances the component of the force of gravity that acts on the second bead down the wire. The electrostatic force acts in the negative $x$-direction and its magnitude is given by

$$F_e = k \frac{|q_1 q_2|}{d^2}. \quad (i)$$

The $x$-component of the force of gravity acting on the second bead corresponds to the component of the weight of the second bead that is parallel to the wire. Figure 21.21b indicates that the component of the weight of the second bead down the wire is given by

$$F_g = m_2 g \sin \theta. \quad (ii)$$

SIMPLIFY
For equilibrium, the electrostatic force and the gravitational force are equal: $F_e = F_g$. Substituting the expressions for these forces from equations (i) and (ii) yields

$$k \frac{|q_1 q_2|}{d^2} = m_2 g \sin \theta.$$

Solving this equation for the mass of the second bead gives us

$$m_2 = \frac{k |q_1 q_2|}{d^2 g \sin \theta}.$$

CALCULATE
We put in the numerical values and get

$$m_2 = \frac{k |q_1 q_2|}{d^2 g \sin \theta} = \frac{\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right) \left(1.28 \mu\text{C}\right) \left(5.06 \mu\text{C}\right)}{\left(0.380 \text{ m}\right)^2 \left(9.81 \text{ m/s}^2\right) \left(\sin 42.3^\circ\right)} = 0.0610746 \text{ kg}.$$

ROUND
We report our result to three significant figures:

$$m_2 = 0.0611 \text{ kg} = 61.1 \text{ g}.$$

DOUBLE-CHECK
To double-check, let’s calculate the mass of the second bead assuming that the wire is vertical, that is, $\theta = 90^\circ$. We can then set the weight of the second bead equal to the electrostatic force between the two beads:

$$k \frac{|q_1 q_2|}{d^2} = m_2 g.$$
Solving for the mass of the second bead, we obtain
\[ m_2 = \frac{kq_1q_2}{d^2g} = \left( \frac{8.99 \times 10^9 \text{ N m}^2/\text{C}^2}{(0.380 \text{ m})^2} \right) \left( 1.28 \mu\text{C} \right) \left( 5.06 \mu\text{C} \right) = 0.0411 \text{ kg}. \]

As the angle of the wire relative to the horizontal decreases, the calculated mass of the second bead will increase. Our result of 0.0611 kg is somewhat higher than the mass that can be supported with a vertical wire, so it seems reasonable.

**Solved Problem 21.3** Four Charged Objects

Consider four charges placed at the corners of a square with side length 1.25 m, as shown in Figure 21.22a.

**Problem**

What are the magnitude and direction of the electrostatic force on \( q_4 \) resulting from the other three charges?

**Solution**

**Think**

The electrostatic force on \( q_4 \) is the vector sum of the forces resulting from its interactions with the other three charges. Thus, it is important to avoid simply adding the individual force magnitudes algebraically. Instead we need to determine the individual force components in each spatial direction and add those to find the components of the net force vector. Then we need to calculate the length of this net force vector.

**Sketch**

Figure 21.22b shows the four charges in an \( xy \)-coordinate system with its origin at the location of \( q_2 \).

**Research**

The net force on \( q_4 \) is the vector sum of the forces \( \vec{F}_{1\rightarrow 4}, \vec{F}_{2\rightarrow 4}, \) and \( \vec{F}_{3\rightarrow 4} \). The \( x \)-component of the summed forces is

\[ F_x = k \frac{q_1q_4}{d^2} + k \frac{q_2q_4}{(\sqrt{2}d)^2} \cos 45^\circ = \frac{kq_4}{d^2} \left( q_1 + \frac{q_2}{2} \cos 45^\circ \right), \] (i)

where \( d \) is the length of a side of the square and, as Figure 21.22b indicates, the \( x \)-component of \( \vec{F}_{3\rightarrow 4} \) is zero. The \( y \)-component of the summed forces is

\[ F_y = k \frac{q_1q_4}{\sqrt{2}d^2} \sin 45^\circ - k \frac{q_3q_4}{\sqrt{2}d^2} \sin 45^\circ = \frac{kq_4}{d^2} \left( \frac{q_1}{2} \sin 45^\circ + q_3 \right). \] (ii)
where, as Figure 21.22b indicates, the y-component of $\vec{F}_{1\rightarrow 4}$ is zero.

The magnitude of the net force is given by

$$F = \sqrt{F_x^2 + F_y^2},$$

and the angle of the net force is given by

$$\tan \theta = \frac{F_y}{F_x}.$$

**Simplify**

We substitute the expressions for $F_x$ and $F_y$ from equations (i) and (ii) into equation (iii):

$$F = \sqrt{\frac{k q_1}{d^2} \left( q_1 + \frac{q_2}{2} \cos 45^{\circ} \right)^2 + \frac{k q_2}{d^2} \left( \frac{q_2}{2} \sin 45^{\circ} + q_3 \right)^2}.$$

We can rewrite this as

$$F = \frac{k q_1}{d^2} \sqrt{\left( q_1 + \frac{q_2}{2} \cos 45^{\circ} \right)^2 + \left( \frac{q_2}{2} \sin 45^{\circ} + q_3 \right)^2}.$$

For the angle of the force, we get

$$\theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{\frac{q_2}{2} \sin 45^{\circ} + q_3}{\frac{q_2}{2} \cos 45^{\circ} + q_3} \right) = \tan^{-1} \left( \frac{\frac{q_2}{2} \sin 45^{\circ} + q_3}{\frac{q_2}{2} \cos 45^{\circ} + q_3} \right) = \tan^{-1} \left( \frac{\frac{q_2}{2} \sin 45^{\circ} + q_3}{\frac{q_2}{2} \cos 45^{\circ} + q_3} \right).$$

**Calculate**

Putting in the numerical values, we get

$$\frac{q_2}{2} \sin 45^{\circ} = \frac{q_2}{2} \cos 45^{\circ} = \frac{2.50 \mu C}{2 \sqrt{2}} = 0.883883 \mu C.$$

The magnitude of the force is then

$$F = \left( 8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2 \right) \left( 4.50 \mu C \right) \left( 0.883883 \mu C \left( 1.50 \mu C + 0.883883 \mu C \right)^2 + \left( 0.883883 \mu C - 3.50 \mu C \right)^2 \right)$$

$$= 0.0916379 \text{ N}.$$

For the direction of the force, we obtain

$$\theta = \tan^{-1} \left( \frac{\frac{q_2}{2} \sin 45^{\circ} + q_3}{\frac{q_2}{2} \cos 45^{\circ} + q_3} \right) = \tan^{-1} \left( \frac{0.883883 \mu C - 3.50 \mu C}{1.50 \mu C + 0.883883 \mu C} \right) = -47.6593^{\circ}.$$

**Round**

We report our results to three significant figures:

$$F = 0.0916 \text{ N}$$

and

$$\theta = -47.7^{\circ}.$$
DOUBLE-CHECK

To double-check our result, we calculate the magnitude of the three forces acting on \( q_4 \). For \( F_{1-4} \), we get

\[
F_{1-4} = k \frac{q_4 q_1}{r_{14}^2} = \frac{8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \left(1.50 \mu\text{C}\right) \left(4.50 \mu\text{C}\right)}{\left(1.25 \text{ m}\right)^2} = 0.0388 \text{ N}.
\]

For \( F_{2-4} \), we get

\[
F_{2-4} = k \frac{q_4 q_2}{r_{24}^2} = \frac{8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \left(2.50 \mu\text{C}\right) \left(4.50 \mu\text{C}\right)}{\sqrt{2} \left(1.25 \text{ m}\right)^2} = 0.0324 \text{ N}.
\]

For \( F_{3-4} \), we get

\[
F_{3-4} = k \frac{q_4 q_3}{r_{34}^2} = \frac{8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \left(3.50 \mu\text{C}\right) \left(4.50 \mu\text{C}\right)}{\left(1.25 \text{ m}\right)^2} = 0.0906 \text{ N}.
\]

All three of the magnitudes of the individual forces are of the same order as our result for the net force. This gives us confidence that our answer is not off by a large factor.

The direction we obtained also seems reasonable, because it orients the resulting force downward and to the right, as could be expected from looking at Figure 21.22b.

MULTIPLE-CHOICE QUESTIONS

21.1 When a metal plate is given a positive charge, which of the following is taking place?

a) Protons (positive charges) are transferred to the plate from another object.

b) Electrons (negative charges) are transferred from the plate to another object.

c) Electrons (negative charges) are transferred from the plate to another object, and protons (positive charges) are also transferred to the plate from another object.

d) It depends on whether the object conveying the charge is a conductor or an insulator.

21.2 The force between a charge of 25 \( \mu\text{C} \) and a charge of \(-10 \mu\text{C}\) is 8.0 N. What is the separation between the two charges?

a) 0.28 m  

b) 0.53 m  

d) 0.15 m

21.3 A charge \( q_1 \) is positioned on the x-axis at \( x = a \). Where should a charge \( q_2 = -4q_1 \) be placed to produce a net electrostatic force of zero on a third charge, \( q_3 = Q_1 \), located at the origin?

a) at the origin  

b) at \( x = 2a \)  

c) at \( x = -2a \)  

d) at \( x = -a \)

21.4 Which one of these systems has the most negative charge?

a) 2 electrons  

d) \( N \) electrons and \( 1 \) electron

b) 3 electrons and 1 proton  

\( N - 3 \) protons

c) 5 electrons and 5 protons  

e) 1 electron

21.5 Two point charges are fixed on the x-axis: \( q_1 = 6.0 \mu\text{C} \) is located at the origin, \( O \), with \( x_1 = 0.0 \text{ cm} \), and \( q_2 = -3.0 \mu\text{C} \) is located at point \( A \), with \( x_2 = 8.0 \text{ cm} \). Where should a third charge, \( q_3 \), be placed on the x-axis so that the total electrostatic force acting on it is zero?

a) 19 cm  

c) \( 0.0 \text{ cm} \)  

e) \(-19 \text{ cm}\)

b) 27 cm  

d) \( 8.0 \text{ cm} \)

21.6 Which of the following situations produces the largest net force on the charge \( Q \)?

a) Charge \( Q = 1 \text{ C} \) is 1 m from a charge of \(-2 \text{ C}\).

b) Charge \( Q = 1 \text{ C} \) is 0.5 m from a charge of \(-1 \text{ C}\).

c) Charge \( Q = 1 \text{ C} \) is halfway between a charge of \(-1 \text{ C} \) and a charge of 1 C that are 2 m apart.

d) Charge \( Q = 1 \text{ C} \) is halfway between two charges of \(-2 \text{ C} \) that are 2 m apart.

e) Charge \( Q = 1 \text{ C} \) is a distance of 2 m from a charge of \(-4 \text{ C} \).

21.7 Two protons placed near one another with no other objects close by would

a) accelerate away from each other.

b) remain motionless.

c) accelerate toward each other.

d) be pulled together at constant speed.

e) move away from each other at constant speed.
21.8 Two lightweight metal spheres are suspended near each other from insulating threads. One sphere has a net charge; the other sphere has no net charge. The spheres will:

a) attract each other.
b) exert no net electrostatic force on each other.
c) repel each other.
d) do any of these things depending on the sign of the charge on the one sphere.

21.9 A metal plate is connected by a conductor to a ground through a switch. The switch is initially closed. A charge +Q is brought close to the plate without touching it, and then the switch is opened. After the switch is opened, the charge +Q is removed. What is the charge on the plate then?

21.10 You bring a negatively charged rubber rod close to a grounded conductor without touching it. Then you disconnect the ground. What is the sign of the charge on the conductor after you remove the charged rod?

a) negative  

b) positive  

c) no charge  

d) cannot be determined from the given information

21.11 If two charged particles (the charge on each is Q) are separated by a distance d, there is a force F between them. What is the force if the magnitude of each charge is doubled and the distance between them changes to 2d?

21.12 Suppose the Sun and the Earth were each given an equal amount of charge of the same sign, just sufficient to cancel their gravitational attraction. How many times the charge on an electron would that charge be? Is this number a large fraction of the number of charges of either sign in the Earth?

21.13 It is apparent that the electrostatic force is extremely strong, compared to gravity. In fact, the electrostatic force is the basic force governing phenomena in daily life—the tension in a string, the normal forces between surfaces, friction, chemical reactions, etc.—except weight. Why then did it take so long for scientists to understand this force? Newton came up with his gravitational law long before electricity was even crudely understood.

21.14 Occasionally, people who gain static charge by shuffling their feet on the carpet will have their hair stand on end. Why does this happen?

21.15 Two positive charges, each equal to Q, are placed a distance 2d apart. A third charge, –0.2Q, is placed exactly halfway between the two positive charges and is displaced a distance x ≪ d perpendicular to the line connecting the positive charges. What is the force on this charge? For x ≪ d, how can you approximate the motion of the negative charge?

21.16 Why does a garment taken out of a clothes dryer sometimes cling to your body when you wear it?

21.17 Two charged spheres are initially a distance d apart. The magnitude of the force on each sphere is F. They are moved closer to each other such that the magnitude of the force on each of them is 9F. By what factor has the difference between the two spheres changed?

21.18 How is it possible for one electrically neutral atom to exert an electrostatic force on another electrically neutral atom?

21.19 The scientists who first contributed to the understanding of the electrostatic force in the 18th century were well aware of Newton’s law of gravitation. How could they deduce that the force they were studying was not a variant or some manifestation of the gravitational force?

21.20 Two charged particles move solely under the influence of the electrostatic forces between them. What shapes can their trajectories have?

21.21 Rubbing a balloon causes it to become negatively charged. The balloon then tends to cling to the wall of a room. For this to happen, must the wall be positively charged?

21.22 Two electric charges are placed on a line, as shown in the figure. Is it possible to place a charged particle (that is free to move) anywhere on the line between the two charges and have it not move?

21.23 Two electric charges are placed on a line as shown in the figure. Where on the line can a third charge be placed so that the force on that charge is zero? Does the sign or the magnitude of the third charge make any difference to the answer?

21.24 When a positively charged rod is brought close to a neutral conductor without touching it, will the rod experience an attractive force, a repulsive force, or no force at all? Explain.

21.25 When you exit a car and the humidity is low, you often experience a shock from static electricity created by sliding across the seat. How can you discharge yourself without experiencing a painful shock? Why is it dangerous to get back into your car while fueling your car?
PROBLEMS

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Section 21.2

21.26 How many electrons are required to yield a total charge of 1 C?

21.27 The faraday is a unit of charge frequently encountered in electrochemical applications and named for the British physicist and chemist Michael Faraday. It consists of 1 mole of elementary charges. Calculate the number of coulombs in 1 faraday.

21.28 Another unit of charge is the electrostatic unit (esu). It is defined as follows: Two point charges, each of 1 esu and separated by 1 cm, exert a force of exactly 1 dyne on each other: 1 dyne = 1 g cm/s² = 1 · 10⁻⁵ N.

a) Determine the relationship between the esu and the coulomb.
b) Determine the relationship between the esu and the elementary charge.

21.29 A current of 5 mA is enough to make your muscles twitch. Calculate how many electrons flow through your skin if you are exposed to such a current for 10 s.

• 21.30 How many electrons does 1.00 kg of water contain?

• 21.31 The Earth is constantly being bombarded by cosmic rays, which consist mostly of protons. These protons are incident on the Earth’s atmosphere from all directions at a rate of 1245.0 protons per square meter per second. Assuming that the depth of Earth’s atmosphere is 120 km, what is the total charge incident on the atmosphere in 5 min? Assume that the radius of the surface of the Earth is 6378 km.

• 21.32 Performing an experiment similar to Millikan’s oil drop experiment, a student measures these charge magnitudes:

3.26 · 10⁻¹⁹ C  5.09 · 10⁻¹⁹ C  1.53 · 10⁻¹⁹ C
6.39 · 10⁻¹⁹ C  4.66 · 10⁻¹⁹ C

Find the charge on the electron using these measurements.

Section 21.3

• 21.33 A silicon sample is doped with phosphorus at 1 part in 10⁵. Phosphorus acts as an electron donor, providing one free electron per atom. The density of silicon is 2.33 g/cm³, and its atomic mass is 28.09 g/mol.

a) Calculate the number of free (conduction) electrons per unit volume of the doped silicon.
b) Compare the result from part (a) with the number of conduction electrons per unit volume of copper wire (assume each copper atom produces one free (conduction) electron). The density of copper is 8.96 g/cm³, and its atomic mass is 63.54 g/mol.

Section 21.5

21.34 Two charged spheres are 8 cm apart. They are moved closer to each other enough that the force on each of them increases four times. How far apart are they now?

21.35 Two identically charged particles separated by a distance of 1.0 m repel each other with a force of 1 N. What is the magnitude of the charges?

21.36 How far must two electrons be placed on the Earth’s surface for there to be an electrostatic force between them equal to the weight of one of the electrons?

21.37 In solid sodium chloride (table salt), chloride ions have one more electron than they have protons, and sodium ions have one more proton than they have electrons. These ions are separated by about 0.28 nm. Calculate the electrostatic force between a sodium ion and a chloride ion.

21.38 In gaseous sodium chloride, chloride ions have one more electron than they have protons, and sodium ions have one more proton than they have electrons. These ions are separated by about 0.24 nm. Suppose a free electron is located 0.48 nm above the midpoint of the sodium chloride molecule. What are the magnitude and the direction of the electrostatic force the molecule exerts on it?

21.39 Calculate the magnitude of the electrostatic force the two up quarks inside a proton exert on each other if they are separated by a distance of 0.9 fm.

21.40 A –4.0-μC charge lies 20.0 cm to the right of a 2.0-μC charge on the x-axis. What is the force on the 2.0-μC charge?

• 21.41 Two initially uncharged identical metal spheres, 1 and 2, are connected by an insulating spring (unstretched length L₀ = 1.00 m, spring constant k = 25.0 N/m), as shown in the figure. Charges +q and –q are then placed on the spheres, and the spring contracts to length L = 0.635 m. Recall that the force exerted by a spring is Fₛ = kΔx, where Δx is the change in the spring’s length from its equilibrium length. Determine the charge q. If the spring is coated with metal to make it conducting, what is the new length of the spring?

21.42 A point charge +3q is located at the origin, and a point charge –q is located on the x-axis at D = 0.500 m. At what location on the x-axis will a third charge, q₀, experience no net force from the other two charges?

• 21.43 Identical point charges Q are placed at each of the four corners of a rectangle measuring 2.0 m by 3.0 m. If Q = 32 μC, what is the magnitude of the electrostatic force on any one of the charges?
•21.44 Charge \( q_1 = 1.4 \cdot 10^{-8} \text{ C} \) is placed at the origin. Charges \( q_2 = -1.8 \cdot 10^{-8} \text{ C} \) and \( q_3 = 2.1 \cdot 10^{-8} \text{ C} \) are placed at points (0.18 m, 0 m) and (0 m, 0.24 m), respectively, as shown in the figure. Determine the net electrostatic force (magnitude and direction) on charge \( q_2 \).

\[ \begin{array}{c}
\text{Charge } q_1 = 1.4 \cdot 10^{-8} \text{ C} \\
\text{Charge } q_2 = -1.8 \cdot 10^{-8} \text{ C} \\
\text{Charge } q_3 = 2.1 \cdot 10^{-8} \text{ C}
\end{array} \]

•21.45 A positive charge \( Q \) is on the \( y \)-axis at a distance \( a \) from the origin, and another positive charge \( q \) is on the \( x \)-axis at a distance \( b \) from the origin.

a) For what value(s) of \( b \) is the \( x \)-component of the force on \( q \) a maximum?

b) For what value(s) of \( b \) is the \( x \)-component of the force on \( q \) a minimum?

•21.46 Find the magnitude and direction of the electrostatic force acting on the electron in the figure.

•21.47 In a region of two-dimensional space, there are three fixed charges: +1 mC at (0,0), –2 mC at (17 mm, -5 mm), and +3 mC at (-2 mm, 11 mm). What is the net force on the –2-mC charge?

•21.48 Two cylindrical glass beads each of mass \( m = 10 \text{ mg} \) are set on their flat ends on a horizontal insulating surface separated by a distance \( d = 2 \text{ cm} \). The coefficient of static friction between the beads and the surface is \( \mu_s = 0.2 \). The beads are then given identical charges (magnitude and sign). What is the minimum charge needed to start the beads moving?

•21.49 A small ball with a mass of 30 g and a charge of \(-0.2 \mu\text{C}\) is suspended from the ceiling by a string. The ball hangs at a distance of 5.0 cm above an insulating floor. If a second small ball with a mass of 50 g and a charge of 0.4 \( \mu \text{C} \) is rolled directly beneath the first ball, will the second ball leave the floor? What is the tension in the string when the second ball is directly beneath the first ball?

•21.50 A +3-mC charge and a –4-mC charge are fixed in position and separated by 5 m.

a) Where could a +7-mC charge be placed so that the net force on it is zero?

b) Where could a –7-mC charge be placed so that the net force on it is zero?

•21.51 Four point charges, \( q \), are fixed to the four corners of a square that is 10.0 microns on a side. An electron is suspended above a point at which its weight is balanced by the electrostatic force due to the four electrons, at a distance of 15 nm above the center of the square. What is the magnitude of the fixed charges? Express both in coulombs and as a multiple of the electron’s charge.

•21.52 The figure shows a uniformly charged thin rod of length \( L \) that has total charge \( Q \). Find an expression for the magnitude of the electrostatic force acting on an electron positioned on the axis of the rod at a distance \( d \) from the midpoint of the rod.

\[ \begin{array}{c}
\text{Uniformly charged rod of length } L \\
\text{Charge } Q \text{ on the rod}
\end{array} \]

•21.53 A negative charge, \( -q \), is fixed at the coordinate (0,0). It is exerting an attractive force on a positive charge, +\( q \), that is initially at coordinate \( (x,0) \). As a result, the positive charge accelerates toward the negative charge. Use the binomial expansion \((1+x)^n \approx 1+nx\), for \( x \ll 1 \), to show that when the positive charge moves a distance \( \delta \ll x \) closer to the negative charge, the force that the negative charge exerts on it increases by \( \Delta F = 2kq^2/\delta^2 \).

•21.54 Two equal magnitude negative charges \((–q \text{ and } –q)\) are fixed at coordinates \((-\delta,0)\) and \((0,\delta)\). A positive charge of the same magnitude, \( q \), and with mass \( m \) is placed at coordinate \((0,0)\), midway between the two negative charges. If the positive charge is moved a distance \( \delta \ll d \) in the positive \( y \)-direction and then released, the resulting motion will be that of a harmonic oscillator—the positive charge will oscillate between coordinates \((0,\delta)\) and \((0,-\delta)\). Find the net force acting on the positive charge when it moves to \((0,\delta)\) and use the binomial expansion \((1+x)^n \approx 1+nx\), for \( x \ll 1 \), to find an expression for the frequency of the resulting oscillation. (Hint: Keep only terms that are linear in \( \delta \).)

Section 21.6

21.55 Suppose the Earth and the Moon carried positive charges of equal magnitude. How large would the charge need to be to produce an electrostatic repulsion equal to 1% of the gravitational attraction between the two bodies?

21.56 The similarity of form of Newton’s law of gravitation and Coulomb’s Law caused some to speculate that the force of gravity is related to the electrostatic force. Suppose that gravitation is entirely electrical in nature—that an excess charge \( Q \) on the Earth and an equal and opposite excess charge \(-Q\) on the Moon are responsible for the gravitational force that causes the observed orbital motion of the Moon about the Earth. What is the required size of \( Q \) to reproduce the observed magnitude of the gravitational force?

21.57 In the Bohr model of the hydrogen atom, the electron moves around the one-proton nucleus on circular orbits of well-determined radii, given by \( r_n = n^2 a_0 \) where \( n = 1, 2, 3, \ldots \) is an integer that defines the orbit and \( a_0 = 5.29 \cdot 10^{-11} \text{m} \) is the radius of the first (minimum) orbit, called the Bohr radius. Calculate the force of electrostatic interaction between the electron and the proton in the hydrogen atom for the first orbit.
four orbits. Compare the strength of this interaction to the
gravitational interaction between the proton and the electron.

**21.58** Some of the earliest atomic models held that the orbital
velocity of an electron in an atom could be correlated with the
radius of the atom. If the radius of the hydrogen atom is $10^{-10}$ m and
the electrostatic force is responsible for the circular motion of the electron, what is the electron’s orbital velocity? What is
the kinetic energy of this orbital electron?

**21.59** For the atom described in Problem 21.58, what is the
ratio of the gravitational force between electron and proton
to the electrostatic force? How does this ratio change if the
radius of the atom is doubled?

**21.60** In general, astronomical objects are not exactly elec-
trically neutral. Suppose the Earth and the Moon each carry
a charge of $–1.00 \cdot 10^6$ C (this is approximately correct; a more
precise value is identified in Chapter 22).

a) Compare the resulting electrostatic repulsion with the
gravitational attraction between the Moon and the Earth.
Look up any necessary data.
b) What effects does this electrostatic force have on the size,
shape, and stability of the Moon’s orbit around the Earth?

**Additional Problems**

**21.61** Eight 1-μC charges are arrayed along the y-axis located
every 2 cm starting at $y = 0$ and extending to $y = 14$ cm. Find
the force on the charge at $y = 4$ cm.

**21.62** In a simplified Bohr model of the hydrogen atom,
an electron is assumed to be traveling in a circular orbit of
radius of about $5.2 \cdot 10^{-11}$ m around a proton. Calculate the
speed of the electron in that orbit.

**21.63** The nucleus of a carbon-14 atom (mass = 14 amu) has
diameter of 3 fm. It has 6 protons and a charge of $+6e$.

a) What is the force on a proton located at 3 fm from the surface
of this nucleus? Assume that the nucleus is a point charge.
b) What is the proton’s acceleration?

**21.64** Two charged objects experience a mutual repulsive
force of 0.10 N. If the charge of one of the objects is reduced
by half and the distance separating the objects is doubled, what is the new force?

**21.65** A particle (charge = $+19.0 \mu C$) is located on the x-axis
at $x = –10.0$ cm, and a second particle (charge = $–57.0 \mu C$) is
placed on the x-axis at $x = +20.0$ cm. What is the magnitude
of the total electrostatic force on a third particle (charge = $–3.80 \mu C$) placed at the origin ($x = 0$)?

**21.66** Three point charges are positioned on the x-axis:
$+64.0 \mu C$ at $x = 0.00$ cm, $+80.0 \mu C$ at $x = 25.0$ cm, and
$–160.0 \mu C$ at $x = 50.0$ cm. What is the magnitude of the
electrostatic force acting on the $+64.0-\mu C$ charge?

**21.67** From collisions with cosmic rays and from the solar
wind, the Earth has a net electric charge of approximately
$–6.8 \cdot 10^9$ C. Find the charge that must be given to a 1.0-g
object for it to be electrostatically levitated close to the Earth’s
surface.

**21.68** Your sister wants to participate in the yearly science
fair at her high school and asks you to suggest some exciting
project. You suggest that she experiment with your recently
created electron extractor to suspend her cat in the air. You
tell her to buy a copper plate and bolt it to the ceiling in her
room and then use your electron extractor to transfer elec-
trons from the plate to the cat. If the cat weighs 7 kg and is
suspended 2 m below the ceiling, how many electrons have
to be extracted from the cat? Assume that the cat and the
metal plate are point charges.

**21.69** A 10-g mass is suspended 5 cm above a noncon-
ducting flat plate, directly above an embedded charge of
$q$ (in coulombs). If the mass has the same charge, $q$, how
much must $q$ be so that the mass levitates (just floats,
neither rising nor falling)? If the charge $q$ is produced by
adding electrons to the mass, by how much will the mass be
changed?

**21.70** Four point charges are placed at the following xy-
coordinates:

- $Q_1 = +1 \text{ mC}$, at $(–3 \text{ cm}, 0 \text{ cm})$
- $Q_2 = +1 \text{ mC}$, at $(+3 \text{ cm}, 0 \text{ cm})$
- $Q_3 = +1.024 \text{ mC}$, at $(0 \text{ cm}, 0 \text{ cm})$
- $Q_4 = +2 \text{ mC}$, at $(0 \text{ cm}, –4 \text{ cm})$

Calculate the net force on charge $Q_4$ due to charges $Q_1$, $Q_2$, and $Q_3$.

**21.71** Three 5-g Styrofoam balls of radius 2 cm are coated
with carbon black to make them conducting and then are
tied to 1-m-long threads and suspended freely from a
common point. Each ball is given the same charge, $q$. At
equilibrium, the balls form an equilateral triangle with sides
of length 25 cm in the horizontal plane. Determine $q$.

**21.72** Two point charges lie on the x-axis. If one point charge
is $6.0 \mu C$ and lies at the origin and the other is $–2.0 \mu C$
and lies at 20.0 cm, at what position must a third charge be
placed to be in equilibrium?

**21.73** Two beads with charges $q_1 = q_2 = +2.67 \mu C$ are on
an insulating string that hangs straight down from the
ceiling as shown in the figure. The lower bead is fixed in place
on the end of the string and has a mass $m_1 = 0.280$ kg. The
second bead slides without friction on the
string. At a distance $d = 0.360$ m between
the centers of the beads, the force of the
Earth’s gravity on $m_1$ is balanced by the
electrostatic force between the two beads.
What is the mass, $m_2$, of the second bead?

**21.74** Find the net force on a 2.0-C charge at the origin of
an xy-coordinate system if there is a $+5.0-C$ charge at $(3 \text{ m}, 0)$
and a $–3.0-C$ charge at $(0, 4 \text{ m})$.

**21.75** Two spheres, each of mass $M = 2.33$ g, are attached
by pieces of string of length $L = 45$ cm to a common point.
The strings initially hang straight down, with the spheres just touching one another. An equal amount of charge, \( q \), is placed on each sphere. The resulting forces on the spheres cause each string to hang at an angle of \( \theta = 10.0^\circ \) from the vertical. Determine \( q \), the amount of charge on each sphere.

\[ \bullet \text{21.76} \] A point charge \( q_1 = 100 \text{ nC} \) is at the origin of an \( xy \)-coordinate system, a point charge \( q_2 = -80 \text{ nC} \) is on the \( x \)-axis at \( x = 2.0 \text{ m} \), and a point charge \( q_3 = -60 \text{ nC} \) is on the \( y \)-axis at \( y = -2.0 \text{ m} \). Determine the net force (magnitude and direction) on \( q_1 \).

\[ \bullet \text{21.77} \] A positive charge \( q_1 = 1 \mu \text{C} \) is fixed at the origin, and a second charge \( q_2 = -2 \mu \text{C} \) is fixed at \( x = 10 \text{ cm} \). Where along the \( x \)-axis should a third charge be positioned so that it experiences no force?

\[ \bullet \text{21.78} \] A bead with charge \( q_1 = 1.27 \mu \text{C} \) is fixed in place at the end of a wire that makes an angle of \( \theta = 51.3^\circ \) with the horizontal. A second bead with mass \( m_2 = 3.77 \text{ g} \) and a charge of \( 6.79 \mu \text{C} \) slides without friction on the wire. What is the distance \( d \) at which the force of the Earth’s gravity on \( m_2 \) is balanced by the electrostatic force between the two beads? Neglect the gravitational interaction between the two beads.

\[ \bullet \text{21.79} \] In the figure, the net electrostatic force on charge \( Q_A \) is zero. If \( Q_A = +1 \text{ nC} \), determine the magnitude of \( Q_0 \).

\[ +Q_0 \quad a \quad -1 \text{nC} \]

\[ \bullet \text{21.80} \] Two balls have the same mass of \( 0.681 \text{ kg} \) and identical charges of \( 18.0 \mu \text{C} \). They hang from the ceiling on strings of identical length as shown in the figure. If the angle with respect to the vertical of the strings is \( 20.0^\circ \), what is the length of the strings?

\[ \bullet \text{21.81} \] As shown in the figure, charge 1 is \( 3.94 \mu \text{C} \) and is located at \( x_1 = -4.7 \text{ m} \), and charge 2 is \( 6.14 \mu \text{C} \) and is at \( x_2 = 12.2 \text{ m} \). What is the \( x \)-coordinate of the point at which the net force on a point charge of \( 0.300 \mu \text{C} \) is zero?